Final Paper

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Paper:
Discrete Structure

Summer-20 Final Term Assignment

Subject: Discrete Structure

Note: Attempt all Questions. All questions carry equal marks. **(50)**

Question No. 1: **(10)**

a) Find the 36th term of the arithmetic sequence whose 3rd term is 7 and 8^{th} term is 17.

Question No. 2: **(10)**

Find ***fog*(x)** and ***gof(x)*** of the functions f(x) = 2*x* + 3 and *g*(*x*) = $-x^2 + 5$

Question No. 3: **(10)**

Prove by mathematical induction that the statement is true for all integers n ≥ 1 **(10)**

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Question No. 4: **(10)**

Discuss different types of relations with example in detail.

Question No. 5 **(10)**

Suppose that an automobile license plate has three letters followed by three digits.

a. How many different license plates are possible?

b. How many license plates could begin with A and end on 0.

c. How many license plates begin with PQR

Question 1:

Question #01:-

Answer:-

→ Let a be the first term and d be the common difference of the arithematic sequence. Then

$\therefore \quad a_n = a + (n-1)d \quad , n \geq 1$

$\Rightarrow a_3 = a + (3-1)d$

$a_8 = a + (8-1)d$

→ Given that $a_3 = 7$ and $a_8 = 17$

So we get

$7 = a + 2d$ —— ①

$17 = a + 7d$ —— ②

Subtracting ① from ②

$$17 = \cancel{a} + 7d$$
$$\underline{-7 = -\cancel{a} - 2d}$$
$$10 = 5d$$

$\Rightarrow \boxed{d = 2}$

Now putting $d = 2$ in eq ①

we get

$$7 = a + 2(2)$$

$$7 = a + 4$$

$$\boxed{a = 3}$$

Since, $a_n = a + (n-1)d$

putting the values of a & d

$$a_n = 3 + (n-1)2$$

→ we have to find 36^{th} term

so, putting $n = 36$

$$a_{36} = 3 + (36-1)2$$

$$= 3 + 70$$

$$\boxed{a_{36} = 73}$$

Question 2:

Question # 02:-

Answer:-

$\longrightarrow \quad fog(x) = f(g(x))$

$= f(-x^2+5)$

$= 2(\ )+3 \longrightarrow$ setting up to insert output

$= 2(-x^2+5)+3$

$= -2x^2+10+3$

$\boxed{= -2x^2+13}$

$\longrightarrow \quad gof(x) = g(f(x))$

$= g(2x+3)$

$= -(2x+3)^2+5$

$= -(4x^2+12x+9)+5$

$= -4x^2-12x-9+5$

$\boxed{= -4x^2-12x-4}$

Question 3:

Question #03:-

Answer:-

Let the given statement be $P(n)$

$$\Rightarrow P(n): 1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

For $n=1$,

$$P(1): 1 = \frac{1(1+1)(2(1)+1)}{6}$$

$$1 = \frac{1(2)(3)}{6}$$

$$1 = \frac{6}{6} \Rightarrow 6/6 = 1$$

$\therefore$ Which is true.

Lets assume that $P(k)$ is true for positive integers k. Which is

$$1^2+2^2+3^2\cdots+k^2 = \frac{k(k+1)(2k+1)}{6} \quad \text{— } 1$$

Now we have to prove that $P(k+1)$ is also true.

We have,

$$(1^2+2^2+3^2+\cdots+k^2)+(k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6}+(k+1)^2 \quad \left[\begin{matrix}\text{using} \\ \text{eq } 1\end{matrix}\right]$$

$$= \frac{k(k+1)(2k+1)+6(k+1)^2}{6}$$

$$= \frac{(k+1)(2k^2+7k+6)}{6}$$

$$= \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

Thus $P(k+1)$ is true, whenever $P(k)$ is true.

Hence, from the principle of mathematical induction, the statement $P(n)$ is true for all natural numbers n.

Question 4:

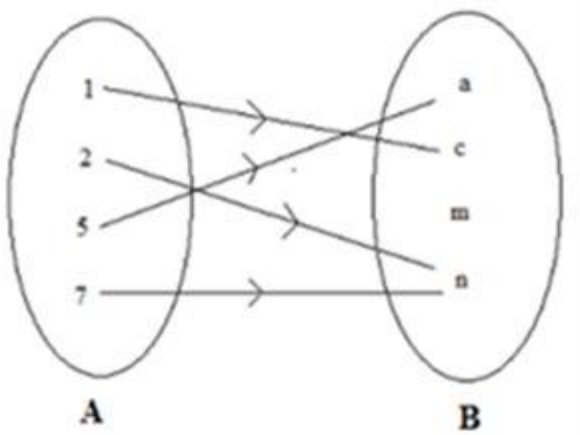
Relations Definition

A relation in arithmetic defines the link between 2 completely different sets of knowledge. If 2 sets ar thought-about, the relation between them are going to be established if there's a association between the weather of 2 or additional non-empty sets.

In the morning assembly at colleges, students ar imagined to interchange a queue in ascending order of the heights of all the scholars. This defines an ordered relation between the scholars and their heights.

Therefore, we can say,

'A set of ordered pairs is outlined as a relation.'



This mapping depicts a relation from set A into set B. A relation from A to B could be a set of A x B. The ordered pairs ar (1, c), (2, n), (5, a), (7, n). For outlining a relation, we have a tendency to use the notation wherever,

Set represents the domain.

Set represents the vary.

Sets and Relations

Sets and relation are interconnected with one another. The relation defines the relation between 2 given sets.

If there are 2 sets out there, then to ascertain if there's any association between the 2 sets, we have a tendency to use relations.

For example, an empty relation denotes none of the weather within the 2 sets is same.

Let us discuss the opposite varieties of relations here.

Relations in arithmetic

In Maths, the relation is that the relationship between 2 or additional set of values.

Suppose, x and y ar 2 sets of ordered pairs. And set x has relation with set y, then the values of set x ar known as domain whereas the values of set y ar known as vary.

Types of Relations

There are eight main varieties of relations that include:

- Empty Relation
- Universal Relation
- Identity Relation
- Inverse Relation
- Reflexive Relation
- Symmetric Relation
- Transitive Relation
- Equivalence Relation

Empty Relation

An empty relation (or void relation) is one during which there's no relation between any parts of a collection. as an example, if set A = then, one amongst the void relations will be R = where, |x – y| = eight. For empty relation,

$R = \phi \subset A \times A$

Universal Relation

A universal (or full relation) could be a form of relation during which each component of a collection is expounded to every different. take into account set A = . currently one amongst theuniversal relations are going to be R = where, |x – y| ≥ zero. For universal relation,

$R = A \times A$

Identity Relation

In an identity relation, each component of a collection is expounded to itself solely. as an example, in a very set A = , the identity relation are going to be I = , , . For identity relation,

Inverse Relation

Inverse relation is seen once a collection has parts that ar inverse pairs of another set. as an example if set A = , then inverse relation are going to be R-1 = . So, for an inverse relation,

Reflexive Relation

In a reflexive relation, each component maps to itself. as an example, take into account a collection A = . Currently an example of reflexive relation are going to be R = . The reflexive relation is given by-

$(a, a) \in R$

Symmetric Relation

In a symmetrical relation, if a=b is true then b=a is additionally true. In different words, a relation R is symmetrical on condition that (b, a) ∈ R is true once (a,b) ∈ R. associate degree example of symmetrical relation are going to be R = for a collection A = . So, for a symmetrical relation,

$aRb \Rightarrow bRa, \forall a, b \in A$

Transitive Relation

For transitive relation, if (x, y) ∈ R, (y, z) ∈ R, then (x, z) ∈ R. For a transitive relation,

aRb and $bRc \Rightarrow aRc \; \forall a, b, c \in A$

Equivalence Relation

If a relation is reflexive, symmetrical and transitive at an equivalent time it's referred to as an equivalence relation.

Question 5: (a,b,c)

Question#05:- (a)

answer:-

→ There are 26 letters in alphabets

→ And there are 10 digits 0 – 9

Each letter has 26 posibilitées and each digit has 10 posibilities

Thus calculating possibilitées as 3 lett are followed by 3 digits

$$\underbrace{26 \times 26 \times 26}_{\text{alphabets}} \times \underbrace{10 \times 10 \times 10}_{\text{digits}}$$

→ By calculating we get,

$\boxed{17,576000 \text{ Possibilities}}$

Question #05 :- (b)

Answer :-

→ As the sequence starts with "A" and ends with "O"

→ So we have to get the possibilities for 2^{nd}, 3^{rd}, 4^{th} and 5^{th} place.

The possibilities are

$$\underbrace{1}_{\text{A}} \times \underbrace{26 \times 26}_{\text{alphabets}} \times \underbrace{10 \times 10}_{\text{digits}} \times \underbrace{1}_{\text{O}}$$

⇒ By calculating we get,

$\boxed{67600 \text{ Possibilities}}$

Question #05:- (c)

Answer:-

→ Plates begin with PQR

so we have

$$\underbrace{1 \times 1 \times 1}_{\substack{\text{alphabets} \\ \text{(PQR)}}} \times \underbrace{10 \times 10 \times 10}_{\text{digits}}$$

By calculating this we get,

$\boxed{1000 \text{ Possibilities}}$