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SECTION "A"

SUBJECT DIFFENTIAL EQUITION

SEMESTER 4th

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Question 01

Solve the following objective type questions.

01 The order of a matrix A is $m \times p$ and the order of is $p \times n$. Then the order of matrix AB is ?

→ $\boxed{AB = m \times n}$

02 The numbers of non-zero rows in an Echelon form ?

→ $\boxed{\text{One}}$

03 If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is Singular matrix then $a = ?$

→ $\boxed{8}$

04 If $A = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$ Then $|A| = ?$

→ $\boxed{3}$

05 The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

→ $\boxed{\text{Scalar matrix}}$

06 Solution of $\frac{dy}{dx} + 2xy = y$?

→ $\boxed{\ln y = x - x^2 + c}$

07 The Order and degree of differential equtiom $\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is ?

→ $\boxed{\text{order} = 1 \quad \text{Degree} = 3}$

08 The order and degree of differential equation

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right) \text{ is ?}$$

→ $\boxed{\text{Order} = 2 \quad \text{Degree} = 1}$

09 The Differential equation $2\frac{dy}{dx} + x^2y = 2x + 3, y(0) = 5 = ?$

→ $\boxed{5 = C}$

Then $y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3 y}{6} + 5$

10 $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

→ $|A| = bc^2 - b^2c - ac^2 + ab^2 + a^2c - a^2b$

①

Question 2

(i)

Express the determinant.

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product of factors which are linear in a, b, c..

Solution:-

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expend by row 1 "R1"

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a.(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2cb^3 - a^3b^2c$$

NOW!

we take abc Common

$$= abc(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$$

$$\boxed{= abc[bc(c-b) - ac(c-a) + ab\,\cancel{b}\,(b-a)]}$$

(Question 2nd)

(ii)

Find the Eigen value

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Solution:- $\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$

Charectarislic equation $|A-\lambda I| = 0 \rightarrow (A)$

$$= \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

NOW!

We take detesminant

$|A-\lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expend by R_1

$$2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \rightarrow (B)$$

③

④

AGAIN:

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ Expend by R_1

$$= 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\Big[\big((3-\lambda)(2-\lambda) - (-1)(-1)\big) + 1\big((-1)(2-\lambda) - (-1)(-1)\big) - 1\big((-1)(-1) - (-1)(3-\lambda)\big)\Big]$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (-2+\lambda-1) - (+1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5) + (-3+\lambda) - (4-\lambda)$$

$$= 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda$$

$$= \boxed{-\lambda^3 + 8\lambda^2 - 18\lambda + 8} \rightarrow (a)$$

$$\Rightarrow +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expend by C_1

$$-1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$= -1\,(6-3\lambda-2\lambda+\lambda^2-1) + 1\,(-2+\lambda-1)$$

$$= -\lambda^2 + 5\lambda - 5 - 3 + \lambda$$

$$= \boxed{-\lambda^2 + 6\lambda - 8} \rightarrow (b)$$

$$\Rightarrow -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expend by C_1

5

$$\Rightarrow -\left[-1\begin{vmatrix}-1 & -1\\ -1 & 2-\lambda\end{vmatrix} - (-1)\begin{vmatrix}3-\lambda & -1\\ -1 & 2-\lambda\end{vmatrix} + 0\right]$$

$$= -\left[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right.$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$\boxed{= -\lambda^2+6\lambda-8} \rightarrow c$$

NOW! put a b and c in B

$$(2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right] -\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$$

$$= -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2$$
$$+6\lambda-8-\lambda^2+16\lambda-8$$

$$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2-36\lambda$$
$$-8\lambda+6\lambda+6\lambda+16-16$$

$$= \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$$

By Synthatic division

We get :-

$$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$$

$$(\lambda=0)$$

$$\lambda-2=0 \Rightarrow \lambda=2$$

$$\lambda^2-8\lambda+16=0$$

By Factolization method.

$$\lambda^2-4\lambda-4\lambda+16=0$$

$$\lambda(\lambda-4)-4(\lambda-4)=0$$

$$(\lambda-4)(\lambda-4)$$

(5)

6

$\Rightarrow (\lambda-4)(\lambda-4)$

$\lambda = 4 \quad , \lambda = 4$

$\boxed{\lambda_1 = 0 , \lambda_2 = 2, \lambda_3 = 4 , \lambda_4 = 4}$

7

(Question 03)

The rate of change in the form of differential equation is given by $(x^2+3y^2)dx - 2xy\,dy = 0$ Find the general Solution at $x = 2$ and $y = 6$.

$x = 2$ and $y = 6$

Solution:

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$x = 2$, $y = 6$

$$\Rightarrow (x^2+3y^2)\,dx = 2xy\,dy$$

Divide both side by $2xy\,dx$

We get :-

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^{\cancel{2}}}{2\cancel{x}y} + \frac{3y^{\cancel{2}}}{2x\cancel{y}}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right]} \text{ — (i)}$$

Let $y = vx$

Diff:

$$dy = v\,dx + x\,dv$$

Dividing by dx

$$\boxed{\frac{dy}{dx} = v + x + x\frac{dv}{dx}} \text{ — (a)}$$

NOW! Putt ⓐ in ⓘ

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{\cancel{x}v} + 3\frac{v\cancel{x}}{\cancel{x}}\right]$$

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiplying both sides by "2"

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

Multiplying both sides by $\frac{dx}{dv}$ we get:-

$$2x\,dv = \frac{1+v^2}{v}dx$$

Multiplying both sides by $\frac{v}{x(1+v^2)}$ we get:-

$$\frac{v}{1+v^2}dv = \frac{1}{x}dx$$

Take integration ($\int$) on both sides

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

Take "e" on Both Sides

$$e^{\ln|1+v^2|} = e^{\ln|xc|}$$

$$1+v^2 = xc$$

Putt $v = y/x$

$$1+(y/x)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$\boxed{x^2+y^2 = x^3c} \text{ —— } (\star)$$

Putt $x=2$, $y=6$ in eq $(\star)$

$$(4)+(36) = 8c$$

$$c = = 40/8$$

$c = 5 \rightarrow$ Putt in eq $(\star)$

So!

$$x^2+y^2 = 5x^3$$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x-1)$$

Taking Squre root "$\sqrt{\ }$" on both sides.

$y = +x\sqrt{5x-1}$, $y = -x\sqrt{5x-1}$

(OR)

$\boxed{y = \pm x\sqrt{5x-1}}$ Answer.

Rough_work

① $m \times n$

② one

③ $\begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix} = a \times 1 - 2 \times 4$

$a - 8 = 0$

$a = 8$

④

$|A| = 2i(-i) - i(i)$

$= -2i^2 - i^2$

$= -2(-1) - (-1)$

$= 2 + 1$

$= 3$

⑥ $\frac{dy}{dx} + 2xy = y$

$\frac{dy}{dx} = y - 2xy$

$\frac{dy}{dx} = y(1-2x)$

$\frac{dy}{y} = (1-2x)dx$

Take $\int$

$\ln y = x - \cancel{2}\frac{x^2}{\cancel{2}} + c$

$\ln y = x - x^2 + c$

Order 1 Degree 3

Order 2 Degree = 1

$b^2 c - \cancel{ab^2} - a^2(c^2 - b^2) + c$

$b^2 c(c-b) - a(c-b) + a^2(c-b)$

$(c-b)(bc - a + a^2)$

$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

Exp by R.

$|A| = +1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - a\begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} + a^2\begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$

$|A| = bc^2 - b^2c - ac^2 + ab^2 + a^2c - a^2b$