Subject: Date:

Name: Hadeed ul Haq
ID: 6575
Subject: Linear Algebra

My ID: 6575

Q:1 Determine if the following system is consistent or not:

$$x_1 - 7x_2 + x_3 = 0$$
$$2x_2 - 8x_3 = 8$$
$$5x_1 - 5x_3 = 10$$

$$A = \begin{bmatrix} 1 & -7 & 1 \\ 0 & 2 & -8 \\ 5 & 0 & -5 \end{bmatrix}, \quad X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 8 \\ 10 \end{bmatrix}$$

Now $[A:B] = \left[\begin{array}{ccc|c} 1 & -7 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right]$ $R_3 - 5R_1$

$$= \left[\begin{array}{ccc|c} 1 & -7 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 0 & 35 & -10 & 10 \end{array}\right] \quad \tfrac{1}{2} R_2$$

Subject: Date:

$$= \begin{bmatrix} 1 & -7 & 1 & : & 0 \\ 0 & 1 & -4 & : & 4 \\ 0 & 35 & -10 & : & 10 \end{bmatrix} \quad R_3 - 35R_2$$

$$= \begin{bmatrix} 1 & -7 & 1 & : & 0 \\ 0 & 1 & -4 & : & 4 \\ 0 & 0 & 130 & : & -130 \end{bmatrix}$$

Now;

Rank of $[A:B]$ = Rank of $[A] = 3$

No of Variables $= 3$

Solution:

The given matrix is consistent and has unique solutions

$x_1 - 7x_2 + x_3 = 0$ — ①

$x_2 - 4x_3 = 4$ — ②

$130x_3 = -130$ — ③

So;

$$x_3 = \frac{-130}{130}$$

$$\boxed{x_3 = -1}$$

From eq (2)

$x_2 - 4(-1) = 4$

$x_2 + 4 = 4$

$x_2 = 4 - 4$

$$\boxed{x_2 = 0}$$

Now eq 1

$x_1 - 7(0) + (-1) = 0$

$x_1 - 0 - 1 = 0$

Subject: Date:

$\boxed{x_1 = 1}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Q2 Find the inverse of $A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 5 \\ 5 & -2 & 7 \end{bmatrix}$ by adjoint method:

$$|A| = \begin{vmatrix} 3 & 4 & 5 \\ 2 & -1 & 5 \\ 5 & -2 & 7 \end{vmatrix}$$

$$= 3\begin{vmatrix} -1 & 5 \\ -2 & 7 \end{vmatrix} - 4\begin{vmatrix} 2 & 5 \\ 5 & 7 \end{vmatrix} + 5\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}$$

$$= 3[-7+10] - 4[14-25] + 5[-4+5]$$

$$= 3(3) - 4(-11) + 5(1)$$

$$= 9 + 44 + 5$$

$$|A| = 58$$

Cofactor of $A_{11} = (-1)^{1+1} M_{11}$

$$= (-1)^2 \begin{vmatrix} -1 & 5 \\ -2 & 7 \end{vmatrix}$$

$$= 1[-7+10]$$

$$= 1(3)$$

$$= 3$$

Cofactor of $A_{12} = (-1)^{1+2} M_{12}$

$$= (-1)^3 \begin{vmatrix} 2 & 5 \\ 5 & 7 \end{vmatrix}$$

$$= -1[14-25]$$

$$= -1(-11)$$

Subject: Date:

$= 11$

Cofactor of $A_{13} = (-1)^{1+3} M_{13}$

$= (-1)^4 \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}$

$= 1 [-4+5]$

$= 1(1)$

$= 1$

Cofactor of $A_{21} = (-1)^{2+1} M_{21}$

$= (-1)^3 \begin{vmatrix} 4 & 5 \\ -2 & 7 \end{vmatrix}$

$= -1[28+10]$

$= -1(38)$

$= -38$

Co factor of $A_{22} = (-1)^{2+2} M_{22}$

$= (-1)^4 \begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix}$

$= 1[21-25]$

$= 1(-4)$

$= -4$

Co factor of $A_{23} = (-1)^{2+3} M_{23}$

$= (-1)^5 \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix}$

$= -1[-6-20]$

$= -1(-26)$

$= 26$

Co factor of $A_{31} = (-1)^{3+1} M_{31}$

$= (-1)^4 \begin{vmatrix} 4 & 5 \\ -1 & 5 \end{vmatrix}$

Subject: Date:

$= 1[20+5]$
$= 1(25)$
$= 25$

Cofactor of $A_{32} = (-1)^{3+2} M_{32}$

$= (-1)^5 \begin{vmatrix} 3 & 5 \\ 2 & 5 \end{vmatrix}$

$= -1[15-10]$
$= -1(5)$
$= -5$

Cofactor of $A_{33} = (-1)^{3+3} M_{33}$

$= (-1)^6 \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix}$

$= 1[-3 \ -8]$
$= 1[-11]$
$= -11$

$$\text{Adj } A = \begin{bmatrix} 3 & 11 & 1 \\ -38 & -4 & 26 \\ 25 & -5 & -11 \end{bmatrix}^t$$

$$= \begin{bmatrix} 3 & -38 & 1 \\ 11 & -4 & -5 \\ 1 & 5 & -11 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj}(A)}{|A|}$$

$$= \frac{\begin{bmatrix} 3 & -38 & 25 \\ 11 & -4 & -5 \\ 1 & 26 & -11 \end{bmatrix}}{58}$$

Subject: Date:

$$= \begin{bmatrix} 3/58 & -38/58 & 25/58 \\ 11/58 & -4/58 & -5/58 \\ 1/58 & 26/58 & -11/58 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} 3/58 & 19/29 & 25/58 \\ 11/58 & -2/29 & -5/58 \\ 1/58 & 13/29 & -11/58 \end{bmatrix}}$$

Q:3 Solve the following systems of linear equations by Guass-Jordan method.

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + 2y - 3z = 14$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right] \quad \frac{1}{2} R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{array}\right] \quad \begin{matrix} R_2 - R_1 \\ R_3 - 3R_1 \end{matrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -1 & -9 & 13 \end{array}\right] \quad 2R_3 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 18 & -22 \end{array}\right] \rightarrow \begin{cases} 1/2\, R_2 \\ 1/18\, R_3 \end{cases}$$

Subject: Date:

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11/9 \end{array}\right] \quad R_1 - R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11/9 \end{array}\right] \quad R_1 - 2R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 59/9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -11/9 \end{array}\right]$$

Now;

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 59/9 \\ 2 \\ -11/9 \end{bmatrix}$$

Q: 4 Show that this matrix is diagonisable

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

The characteristics roots are:

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 4-\lambda & 2 & -2 \\ -5 & 3-\lambda & 2 \\ -2 & 4 & 1-\lambda \end{vmatrix} = 0$$

Now;

Subject: Date:

Solve for R_1 =

$$4-\lambda \begin{vmatrix} 3-\lambda & 2 \\ 4 & 1-\lambda \end{vmatrix} - 2 \begin{vmatrix} -5 & 2 \\ -2 & 1-\lambda \end{vmatrix}$$

$$-2 \begin{vmatrix} -5 & 3-\lambda \\ -2 & 4 \end{vmatrix} = 0$$

$4-\lambda\,[(3-3\lambda-\lambda+\lambda^2)-8]-2[-5+5\lambda+4]$

$-2[-20+6-2\lambda]=0$

$4-\lambda\,[3-4\lambda+\lambda^2-8]-2[5\lambda-1]-2[-14-2\lambda]=0$

$4-\lambda\,[-5-4\lambda+\lambda^2]-2[5\lambda-1]-2[-14-2\lambda]=0$

$-20+5\lambda+16\lambda+4\lambda^2+4\lambda^2-\lambda^3-10\lambda+2$

$+28+4\lambda=0$

$10-17\lambda+8\lambda^2-\lambda^3=0$

$\lambda^3-8\lambda^2+17\lambda-10=0$

Solving we get

$\boxed{\lambda = 1, 2, 5}$ eiger values

At $\lambda = 1$

$[A-1\lambda]\,X_1=0$

$$\begin{bmatrix} 4-1 & 2 & -2 \\ -5 & 3-1 & 2 \\ -2 & 4 & 1-1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & -2 \\ -5 & 2 & 2 \\ -2 & 4 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Subject: Date:

$$3x_1 + 2x_2 - 2x_3 = 0 \quad - 1$$
$$-5x_1 + 2x_2 + 2x_3 = 0 - 2$$
$$-2x_1 + 4x_2 = 0$$

$$4x_2 = 2x_1$$
$$\boxed{x_1 = 2x_2}$$

In eq (i)

$$3(2x_2) + 2x_2 - 2x_3 = 0$$
$$6x_2 + 2x_2 - 2x_3 = 0$$
$$8x_2 - 2x_3 = 0$$

$$8x_2 = 2x_3$$
$$4x_2 = x_3$$

In eq (ii)

$$-5(2x_2) + 2x_2 + 2x_3 = 0$$
$$-10x_2 + 2x_2 + 2x_3 = 0$$
$$8x_2 + 2x_3 = 0$$

$$\text{So } X_1 = \begin{bmatrix} 2x_2 \\ x_2 \\ 4x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

A $\lambda = 2$

$$\begin{bmatrix} 4-2 & 2 & -2 \\ -5 & 3-2 & 2 \\ -2 & 4 & 1-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & -2 \\ -5 & 1 & 2 \\ -2 & 4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x_1 + 2x_2 - 2x_3 = 0 - 1$$
$$-5x_1 + x_2 + 2x_3 = 0 - 2$$
$$-2x_1 + 4x_2 - x_3 = 0 - 3$$

Subject: Date:

From eq (i)

$$X_1 + X_2 - X_3 = 0$$

In eq(ii);

$$-5x_1 + x_2 + 2(x_1 + x_2) = 0$$

$$-5x_1 + x_2 + 2x_1 + 2x_2 = 0$$

$$-3x_1 + 3x_2 = 0$$

$$3x_1 = 3x_2$$

$$\boxed{x_1 = x_2}$$

$$x_3 = x_2 + x_2$$

$$\boxed{X_3 = 2x_2}$$

Now

$$X_2 = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} X_2 \\ X_2 \\ 2X_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

At $\lambda = 5$

$$\begin{bmatrix} 4-5 & 2 & -2 \\ -5 & 3-5 & 2 \\ -2 & 4 & +1-5 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 & 2 \\ -5 & -2 & 2 \\ -2 & 4 & -4 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-X_1 + 2X_2 + 2x_3 = 0 \quad -1$$

$$-5x_1 - 2x_2 + 2x_3 = 0 \quad - 2$$

$$-2x_1 + 4x_2 - 4x_3 = 0 \quad - 3$$

In eq 3;

$$-x_1 - 2x_2 - x_3 = 0 \quad - 4$$

$$-x_1 = +x_3 - 2x_2$$

Subject: Date:

In eq 2

$$-5(-x_3+2x_2)-2x_2+2x_3=0$$
$$+5x_3-10x_2-2x_2+2x_3=0$$
$$7x_3-12x_2=0$$
$$7x_3=12x_2$$
$$\boxed{x_3=\frac{12}{7}x_2}$$

In eq 4.

$$-x_1+2x_2-\left(\frac{12}{7}x_2\right)=0$$
$$-7x_1+14x_2-12x_2=0$$
$$-7x_1+2x_2=0$$
$$7x_1=2x_2$$
$$\boxed{x_1=\frac{2}{7}x_2}$$

So; $X_3=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}2/7\,x_2\\x_2\\12/7\,x_2\end{bmatrix}=\begin{bmatrix}2/7\\1\\12/7\end{bmatrix}$

So $P=[X_1, X_2\ \ X_3]$

$$\begin{bmatrix}2 & 1 & 2/7\\1 & 1 & 1\\4 & 2 & 12/7\end{bmatrix}$$

Now

diagonal matrix of

$D= P^{-1}AP = D$

Subject: Date:

$$= \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

Q:5 Determine if the following homogeneous system has a non-trivial solution then describe solution set

$$3x_1 + 5x_2 - 4x_3 = 0$$
$$-3x_1 - 25x_2 + 4x_3 = 0$$
$$6x_1 + x_2 - 8x_3 = 0$$

Non trivial solution

$$\begin{bmatrix} 3 & 5 & -4 \\ -3 & -25 & +4 \\ 6 & 1 & -8 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Reduce into echolon form

$$\begin{bmatrix} 1 & 5/3 & -4/3 \\ -3 & 25 & 4 \\ 6 & 1 & -8 \end{bmatrix} \quad \begin{matrix} 1/3\, R_1 \\ R_2 + 3R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 5/3 & -4/3 \\ 0 & -20 & 0 \\ 6 & 1 & -8 \end{bmatrix} \quad R_3 - 6R_1$$

Subject: Date:

$$\begin{bmatrix} 1 & 5/3 & -4/3 \\ 0 & -20 & 0 \\ 0 & -9 & 0 \end{bmatrix} \quad -1/20\, R_2$$

$$\begin{bmatrix} 1 & 5/3 & -4/3 \\ 0 & 1 & 0 \\ 0 & -9 & 0 \end{bmatrix} \quad R_3 + 9R_2$$

$$\begin{bmatrix} 1 & 5/3 & -4/3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

In this question

$n = 3, \; r = 2$

$n > r$

So, it is homogenous eq of non trivial equation

$x + \frac{5}{3} y - \frac{4}{3} z = 0$

$\boxed{y = 0}$

So let $z = t$

$x + \frac{5}{3}(0) - \frac{4}{3}(t) = 0$

$$\left.\begin{aligned} x &= \frac{4}{3} t \\ y &= 0 \\ z &= t \end{aligned}\right\} \text{non trivial solution}$$

Subject: Date:

Q# 6

Reduce the matrix into normal form and find its rank

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

To find rank

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix} \quad \begin{matrix} R_2 - 3R_1 \\ R_3 - R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -3 \end{bmatrix} \quad -2R_3 + R_2$$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

So; there is one zero row at the bottom and two rows are now zero

Then Rank of matrix is 2