

## Question No 1

Find the root of the equation

$$x^3 + 3.993 \times 10^{-4} = 0.165x^2$$

Use Newton Raphson method with  $x_0 = 0.02$

Solution :-

Rearranging the equation.

$$x^3 - 0.165x^2 + 0.00039 = 0$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = 3x^2 - 0.33x = 0$$

Find out  $x_1 = ?$

Data :-

$$F(x_0) = 0.00033$$

$$F'(x_0) = -0.0054$$

Sol :-

$$x_1 = x_0 - \frac{F(x_0)}{F'(x_0)}$$

$$x_1 = 0.02 - \frac{0.00033}{-0.0054}$$

$$x_1 = 0.081$$

find out  $x_2 = ?$

**Data:**

$$f(x_1) = -0.00016$$

$$f'(x_1) = -0.0070$$

Sol:-

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 0.081 - \frac{(-0.00016)}{-0.0070}$$

$$x_2 = 0.058$$

find out  $x_3 = ?$

**Data:**

$$f(x_2) = \frac{0.0000030}{-0.0090}$$

$$f'(x_2) = 0.0090$$

Sol:-

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 0.058 - \frac{0.0000030}{(-0.0090)}$$

$$x_3 = 0.061$$

## Question No 2.

Use the number  $x_0 = 2, x_1 = 2.75, x_2 = 4$  to find the Lagrange interpolation polynomial for  $f(x) = \frac{1}{x}$  at  $x = 3$ .

Answer:-

$$\begin{array}{ll} x_0 = 2, & y_0 = 0.5 \\ x_1 = 2.75 & y_1 = 0.36 \\ x_2 = 4 & y_2 = 0.25 \end{array}$$

As we know that

Lagrange Interpolation formula

$$y = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} y_0$$

$$\begin{array}{l|l} x_0 = 2 & y_0 = 0.5 \\ x_1 = 2.75 & y_1 = 0.36 \\ x_2 = 4 & y_2 = 0.25 \end{array}$$

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

Putting the values in the above formula.

$$\downarrow = \frac{(3 - 2.75)(3 - 4)}{(2 - 2.75)(2 - 4)} \quad (0.5)$$

$$+ \frac{(3 - 2)(3 - 4)}{(2.75 - 2)(2.75 - 4)} \quad (0.36)$$

$$+ \frac{(3 - 2)(3 - 2.75)}{(4 - 2)(4 - 2.75)} \quad (0.25)$$

$$\downarrow = (-0.833) + 0.384 + 0.025$$

$$\downarrow = -0.424$$

### Question No 3.

Complete the divided difference table for the given data and construct the interpolating Polynomial that uses all this data.

$x =$	1.0	1.3	1.6
$y =$	0.7651977	0.6200860	0.2818186

$x =$	2.2
$y =$	0.1103623

$x_i$	$F(x_i)$	$F(x_{i-1}, x_i)$	$F(x_{i-2}, x_{i-1}, x_i)$	$F(x_{i-3}, \dots, x_i)$	$F(x_{i-4}, \dots, x_i)$
1	0.7651977				
2.3	0.6200860	-0.4837056			
1.6	0.4554022	-0.548946	-0.108734		
1.9	0.2818186	-0.578612	-0.0494433	0.065785	
2.2	0.1103623	-0.571521	0.006818	0.06251255	-0.0028049

$$1. F(x_0, x_1) = \frac{F(x_1) - F(x_0)}{x_1 - x_0}$$

$$= \frac{0.6200860 - 0.7651977}{1.3 - 1}$$

$$F(x_0, x_1) = -0.4837056$$

$$2. F(x_1, x_2) = \frac{F(x_2) - F(x_1)}{x_2 - x_1}$$

$$= \frac{0.4554022 - 0.6200860}{1.6 - 1.3}$$

$$F(x_1, x_2) = -0.548946$$

$$3. F(x_2, x_3) = \frac{F(x_3) - F(x_2)}{x_3 - x_2}$$

$$= \frac{0.2818186 - 0.4554022}{1.9 - 1.6}$$

$$F(x_2, x_3) = 0.578612$$

$$4. F(x_3, x_4) = \frac{F(x_4) - F(x_3)}{x_4 - x_3}$$

$$= \frac{0.1103623 - 0.2818186}{2.2 - 1.9}$$

$$F(x_3, x_1) = -0.571521$$

(3)

Second divided difference.

$$F(x_0, x_1, x_2) = \frac{F(x_1, x_2) - F(x_0, x_1)}{x_2 - x_0}$$

$$= \frac{-0.548946 - (-0.4837056)}{1.6 - 1}$$

$$F(x_0, x_1, x_2) = -0.108734$$

$$F(x_1, x_2, x_3) = \frac{F(x_2, x_3) - F(x_1, x_2)}{x_3 - x_1}$$

$$= \frac{-0.578612 - (-0.548946)}{1.9 - 1.3}$$

$$F(x_1, x_2, x_3) = -0.0494433$$

$$F(x_2, x_3, x_4) = \frac{F(x_3, x_4) - F(x_2, x_3)}{x_4 - x_2}$$

$$= \frac{0.571521 - (-0.578612)}{2.2 - 1.6}$$

$$F(x_2, x_3, x_4) = 0.006818$$

Third divided difference. (9)

$$F(x_0, x_1, x_2, x_3) = \frac{F(x_2, x_2, x_3) - F(x_0, x_1, x_2)}{x_3 - x_0}$$

$$= \frac{0.0494433 - (-0.108734)}{1.9 - 1}$$

$$F(x_0, x_1, x_2, x_3) = 0.0658785$$

$$F(x_1, x_2, x_3, x_4) = \frac{F(x_2, x_3, x_4) - F(x_1, x_2, x_3)}{x_4 - x_1}$$

$$= \frac{0.006818 - (-0.049443)}{2.2 - 1.3}$$

$$F(x_1, x_2, x_3, x_4) = 0.06251255$$

4th divided difference.

$$F(x_0, x_1, x_2, x_3, x_4) = \frac{0.06251255 - 0.0658785}{2.2 - 1}$$

$$F(x_0, x_1, x_2, x_3, x_4) = 0.0028049$$



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$$\begin{aligned} F(x) &= F(x_0) + (x - x_0)F(x_0, x_1) + (x - x_0)(x - x_1)F(x_0, x_1, x_2) \\ &+ (x - x_0)(x - x_1)(x - x_2)F(x_0, x_1, x_2, x_3) + (x - x_0) \\ &(x - x_1)(x - x_2)(x - x_3)F(x_0, x_1, x_2, x_3, x_4) \\ &= 0.7651977 + (x - 1) - 0.4837056 + (x - 1) \\ &(x - 1.3) (-0.108734) + (x - 1)(x - 1.3) \\ &(x - 1.6) (0.0658785) + (x - 1)(x - 1.3) \\ &(x - 1.6) (x - 1.9) (-0.0028049) . \end{aligned}$$