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Final Exam

Q 1 Part (a)

Differentiate $3x^4 - 2x^3 + 5$ with respect to x

sol

$$y = \frac{3x^4 - 2x^3 + 5}{x^3 + 1}$$

Differentiate w.r.t x

$$\frac{dy}{dx} = \frac{(x^3 + 1) \frac{d}{dx} (3x^4 - 2x^3 + 5) - (3x^4 - 2x^3 + 5) \frac{d}{dx} (x^3 + 1)}{(x^3 + 1)^2}$$

$$\frac{dy}{dx} = \frac{(x^3 + 1)(12x^3 - 6x^2) - (3x^4 - 2x^3 + 5)(3x^2)}{(x^3 + 1)^2}$$

$$\frac{dy}{dx} = \frac{12x^6 - 6x^5 + 12x^3 - 6x^2 - 9x^6 + 6x^5 - 15x^2}{(x^3 + 1)^2}$$

$$\frac{dy}{dx} = \frac{3x^6 + 12x^3 - 21x^2 - 3x^2(x^4 + 4x - 7)}{(x^3 + 1)^2}$$

Q1 (Part b)

$$y = \frac{(x^3 + 1)^2}{x^3 - 1}$$

sol

Differentiate. x

$$\frac{dy}{dx} = \frac{(x^3 - 1) \frac{d}{dx} (x^3 + 1)^2 - (x^3 + 1)^2 \frac{d}{dx} (x^3 - 1)}{(x^3 - 1)^2}$$

$$\frac{dy}{dx} = \frac{(x^3 - 1) \cdot 2(x^3 + 1)(3x^2) - (x^3 + 1)^2 (3x^2)}{(x^3 - 1)^2}$$

$$\frac{dy}{dx} = \frac{3x^2(x^3 + 1)(2x^3 - 2 - x^3 - 1)}{(x^3 - 1)^2}$$

$$\frac{dy}{dx} = \frac{3x(x^3 + 1)(x^3 - 3)}{(x^3 - 1)^2} \quad (\text{Answer})$$

Question 2 (Part a)

Find the integration of $\frac{1}{\sqrt{x^5}} dx$

sol

$$= \int \frac{1}{\sqrt{x^5}} dx$$

$$= \int \frac{1}{(x^5)^{1/2}} dx$$

$$= \int \frac{1}{x^{5/2}} dx$$

$$= \int x^{-5/2} dx$$

$$= \frac{x^{-5/2+1}}{-5/2+1} + C$$

$$= \frac{x^{-5/2}}{-5/2} + C$$

$$= \frac{x^{-3/2}}{-3/2} + C$$

$$= -\frac{2}{3} x^{-3/2} + C$$

Answer

$$\boxed{-\frac{2}{3} \frac{1}{x\sqrt{x}} + C}$$

Question 2 (Part b)

Find the integration of $\frac{1}{(8x+7)^8} dx$

sol

$$= \int \frac{1}{(8x+7)^8} dx$$

$$= \int (8x+7)^{-8} dx$$

Multiplying and divided 8

$$= \frac{1}{8} \int (8x+7)^{-8} dx$$

$$= \frac{1}{8} \int \frac{(8x+7)^{-8+1}}{-8+1} + C$$

$$= \frac{1}{8} \int \frac{(8x+7)^{-7}}{-7} + C$$

$$= -\frac{1}{56} (8x+7)^{-7} + C$$

$$= -\frac{1}{56} \cdot \frac{1}{(8x+7)^7} + C$$

Answer.

Question 3 (Part a)

Find the integration of $\int \frac{-x+9}{2x^2-8x+6}$ by partial fractions.

Sol

$$= \int \frac{-x+9}{2x^2-8x+6} dx$$

$$= \frac{-x+9}{2x^2-2x-6x+6} = \frac{-x+9}{2x(x-1)-6(x-1)}$$

$$= \frac{-x+9}{(x-1)(2x-6)} = \frac{A}{(x-1)} + \frac{B}{2x-6}$$

Multiplying both sides ^{by} $(x-1)(2x-6)$

$$x+9 = A(2x-6) + B(x-1) - C(x-1) = 0$$

$$-(-1)+9 = A(2(1)-6) + B(1-1)$$

$$1+9 = (2-6) + 0$$

$$10 = A(-4)$$

$$10 = -4A$$

$$-\frac{10}{4} = A$$

$$= -\frac{5}{2} = A$$

$$2x-6 = 0$$

$$2x = 6$$

$$x = \frac{6}{2}$$

$$x = 3$$

Palaced in (x)

$$-x+9 = A(2x-6) + B(x-1)$$

$$-(3)+9 = A(2(3)-6) + B(3-1)$$

$$6 = 0 + 2B$$

$$6 = 2B$$

$$B = 3$$

$$\frac{-x+9}{2x^2-8x+6} = \frac{-5/2}{(x-1)} + \frac{3}{2x-6}$$

$$\int \frac{-x+9}{2x^2-8x+6} dx = \int \frac{-5/2}{2(x-1)} + \frac{3}{2x-6} dx$$

$$= \frac{-5}{2} \int \frac{1}{x-1} dx + 3 \int \frac{1}{2(x-3)} dx$$

$$\int \frac{-x+9}{2x^2-8x+6} dx = \frac{-5}{2} \int \frac{1}{x-1} dx - \frac{3}{2} \int \frac{1}{x-3} dx$$

$$\int \frac{-x+9}{2x^2-8x+6} dx = \frac{-5}{2} \ln(x-1) - \frac{3}{2} \ln(x-3)$$

(Answer)

Question 3 Part (B)

Find the integration of $\int \frac{4x^2 + 8x}{(x^2+1)(x^2+2x+3)}$ by partial fraction.

sol

$$= \int \frac{4x^2 + 8x}{(x^2+1)(x^2+2x+3)} dx$$

$$= \frac{4x^2 + 8x}{(x^2+1)(x^2+2x+3)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+2x+3}$$

$$4x^2 + 8x = Ax + B(x^2 + 2x + 3) + (Cx + D)(x^2 + 1)$$

$$= 4x^2 + 8x = Ax^3 + 2Ax^2 + 3Ax + Bx^2 + 2Bx + 3B + Cx^3 + Cx + Dx^2 + D$$

Coefficient comparison

$$D = A + C$$

$$-C = A$$

$$-C = 0$$

$$C = 0$$

$$4 = 2A + B$$

$$8 = 4A + 2B$$

$$8 = 2A - 2B$$

$$0 = 2A$$

$$0 = A$$

$$0 = C + D + 3B$$

$$0 = 0 + D + 3(4)$$

$$-12 = D$$

$$8 = 3A = 2B$$

$$8 = 3(0) - 2B$$

$$8 = 2B$$

$$4 = B$$

$$\int \frac{4x^2 + 8x}{(x^2+1)(x^2+2x+3)} dx = \int \frac{4}{x^2+1} dx + \int \frac{-12}{x^2+1} dx$$

Answer

Question = 4

Solve each of the following

Matrix equation
(Part i)

$$x + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$$

Sol

$$x + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$$

$$x = \begin{bmatrix} 5-3 & 1-(-1) \\ -3-2 & 1-2 \end{bmatrix}$$

$$x = \begin{bmatrix} 2 & 2 \\ -5 & -1 \end{bmatrix}$$

(Answer)

Question (4) Part (ii)

$$x = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$$

solⁿ

$$x = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2-4 & 6-8 \\ 1-2 & 5-0 \end{bmatrix}$$

$$x = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$x = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$x = \begin{bmatrix} -2+1 & -2-0 \\ -1-0 & 5-2 \end{bmatrix}$$

$$x = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix} \text{ (Answer)}$$

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Question = 4 (iii)

$$* x + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

sol

$$x + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$x = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2I$$

$$x = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$x = \begin{bmatrix} 3-2 & -1-0 \\ 1-0 & 2-2 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \text{ (Answer)}$$

Question (5)

$$\text{If } A = \begin{pmatrix} 1 & 4 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} -3 & 2 \\ 4 & 0 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

Find $A^2 + Bc$

So,

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 1 \end{pmatrix} \quad B = \begin{pmatrix} -3 & 2 \\ 4 & 0 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

$$A^2 = AA = \begin{pmatrix} 1 & 4 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 2 & 1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 1+8 & 4+4 \\ 2+2 & 1+1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 9 & 8 \\ 4 & 2 \end{pmatrix}$$

 $A^2 + Bc$

$$\begin{pmatrix} 9 & 8 \\ 4 & 2 \end{pmatrix} + \begin{pmatrix} -3 & 2 \\ 4 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 9 & 8 \\ 4 & 2 \end{pmatrix} + \begin{pmatrix} -3+0 & 0+4 \\ 4+0 & 0+0 \end{pmatrix}$$

$$\begin{pmatrix} 9 & 8 \\ 4 & 2 \end{pmatrix} + \begin{pmatrix} -3 & 4 \\ 4 & 0 \end{pmatrix}$$

~~$$\begin{bmatrix} 9 & 8 \\ 4 & 4 \end{bmatrix}$$~~

$$\begin{bmatrix} 9-3 & 8+4 \\ 4-4 & 2+0 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 12 \\ 0 & 2 \end{bmatrix}$$

Answer

∴ The End:-