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INU Peshawar.

Q:1 Find PQ Values where P is the Point in three-dimensional Space with Coordinates (4, 1, 3) and the point Q with coordinates (1, 2, 4). Find the distance b/w P and Q. Further find the position vector of the point dividing PQ in the ratio 1:3

Solution:

Given that P(4, 1, 3) and Q(1, 2, 4)

$$\vec{OP} = 4i + j + 3k \, , \quad \vec{OQ} = i + 2j + 4k$$

Now $\vec{PQ} = \vec{OQ} - \vec{OP}$

$$\vec{PQ} = (i + 2j + 4k) - (4i + j + 3k)$$

$$\vec{PQ} = i + 2j + 4k - 4i - j - 3k$$

$$\vec{PQ} = i - 4i + 2j - j + 4k - 3k$$

$$\vec{PQ} = -3i + j + k$$

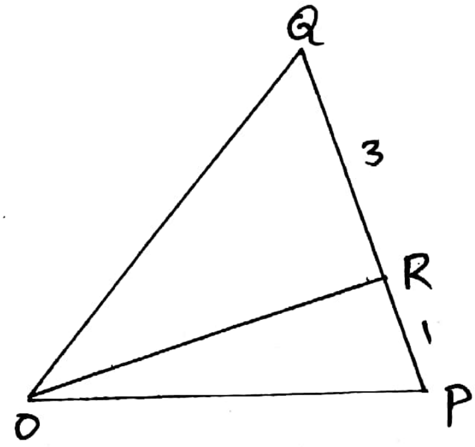
Now the distance of P & Q is

$$|PQ| = \sqrt{(-3)^2 + (1)^2 + (1)^2}$$

$$|PQ| = \sqrt{9 + 1 + 1}$$

$$|PQ| = \sqrt{11}$$

③



Let $\vec{OR}$ be the required position vector dividing $\vec{PQ}$ in the ratio 1 : 3

Using Ratio ~~b~~ Theorem

$$\vec{OR} = \frac{3(\vec{OP}) + 1(\vec{OQ})}{3+1}$$

$$\vec{OR} = \frac{3(4i+j+3k)+1(i+2j+4k)}{4}$$

$$\vec{OR} = \frac{12i+3j+9k+i+2j+4k}{4}$$

$$\vec{OR} = \frac{12i+i+3j+2j+9k+4k}{4}$$

$$\vec{OR} = \frac{13i+5j+13k}{4}$$

$$\vec{OR} = \frac{13}{4}i + \frac{5}{4}j + \frac{13}{4}k$$

is the required answer.

Q:NO 2 Evaluate $\int \frac{4x^3+10x+4}{2x^2+x}\,dx$

Solve

It is an Improper fraction, first we change it into proper fraction and as a sum of polynomial.

Solution:

$$\begin{array}{r|l} & \quad 2x-1 \\ 2x^2+x & \cancel{4x^3}+10x+4 \\ & \pm\cancel{4x^3} \qquad\qquad \pm 2x^2 \\ \hline & -\cancel{2x^2}+10x+4 \\ & \mp\cancel{2x^2}\mp x \\ \hline & \qquad 11x+4 \end{array}$$

$$\frac{4x^3+10x+4}{2x^2+x} = (2x-1) + \frac{11x+4}{2x^2+x}$$

$$\frac{4x^3+10x+4}{2x^2+x} = (2x-1) \qquad \frac{11x+4}{x(2x+1)} \longrightarrow (A)$$

Let $\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{2x+1} \longrightarrow (i)$

Multiplying $x(2x+1)$ on both sides

$$11x+4 = A(2x+1) + Bx \longrightarrow (ii)$$

Put $x = 0$ in equation (ii)

$$11(0) + 4 = A\{2(0)+1\} + B(0)$$

$$4 = A(1) + 0$$

$$\boxed{A = 4}$$

Put $2x + 1 = 0$

$$\Rightarrow 2x = -1$$

$$\Rightarrow x = -\frac{1}{2}$$ in equation (ii)

$$11\left(-\frac{1}{2}\right) + 4 = 0 + B\left(-\frac{1}{2}\right)$$

$$-\frac{11}{2} + 4 = -\frac{1}{2}B$$

$$\frac{-11+8}{2} = -\frac{1}{2}B$$

$$-\frac{3}{2} = -\frac{1}{2}B$$

xing 2 on Both Sides

$$-3 = -B$$

$$\boxed{B = 3}$$

Putting the Values of A and B in equation (i)

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Putting this in eq A

$$\frac{4x^3+10x+4}{2x^2+\cancel{4}x} = (2x-1) + \frac{4}{x} + \frac{3}{2x+1}$$

Taking Integral on Both Sides

$$\int \frac{4x^3+10x+4}{2x^2+x}dx = \int (2x-1)dx + \int \frac{4}{x}dx + \int \frac{3}{2x+1}dx$$

$$= \int 2x\,dx - \int 1\,dx + 4\int \frac{1}{x}dx + 3\int \frac{1}{2x+1}dx$$

$$= \cancel{\int 2x\,dx - \int 1\,dx + 4\int \frac{1}{x}dx + \frac{3}{2}\int \frac{2}{2x+1}dx}$$

$$= 2\int x^1 dx - \int dx + 4\int \frac{1}{x}dx + \frac{3}{2}\int \frac{2}{2x+1}dx$$

$$= \frac{\cancel{2}x^2}{\cancel{2}} - x + 4\ln|x| + \frac{3}{2}\ln|2x+1| + c$$

$$= x^2 - x + 4\ln|x| + \frac{3}{2}\ln|2x+1| + C$$

$$= x^2 + x + \ln|x^4| + \ln|(2x+1)^{3/2}| + c$$

$$= x^2 + x + \ln|(x^4)(2x+1)^{3/2}| + c$$

Ans:

Q NO: (3) Evaluate

(a) $\int_0^2 x^2 e^x \, dx$

Solution:

$$\int \underset{I}{x^2} \cdot \underset{II}{e^x} dx = x^2 \int e^x dx - \int \left(\frac{d}{dx}(x^2) \int e^x dx \right) dx$$

Using by parts Integration

$$\int x^2 e^x dx = x^2 \cdot e^x - \int (2x) e^x dx$$

$$\int x^2 e^x dx = x^2 e^x - 2 \int \underset{I}{x} \cdot \underset{II}{e^x} dx$$

Again by parts

$$\int x^2 e^x dx = x^2 e^x - 2 \left\{ x \int e^x dx - \int \left(\frac{d}{dx}(x) \int e^x dx \right) dx \right\}$$

$$\int x^2 e^x dx = x^2 e^x - 2 \left\{ x \cdot e^x - \int 1 \cdot e^x dx \right\}$$

$$\int x^2 e^x dx = x^2 e^x - 2 \left\{ x e^x - \int e^x dx \right\}$$

$$\int x^2 e^x dx = x^2 e^x - 2 \left\{ x e^x - e^x \right\}$$

$$\int x^2 e^x dx = x^2 e^x - 2x e^x + 2e^x$$

Now applying the limits values

$$\int_0^2 x^2 e^x dx = x^2 e^x \Big|_0^2 - 2xe^x \Big|_0^2 + 2e^x \Big|_0^2$$

$$\int_0^2 x^2 e^x dx = x^2 e^x \Big|_0^2 - 2xe^x \Big|_0^2 + 2e^x \Big|_0^2$$

$$= \left\{2^2 e^2 - 0^2 e^0\right\} - 2\left\{2e^2 - 0.e^0\right\} + 2\left\{e^2 - e^0\right\}$$

$$= (4e^2 - 0) - 2(2e^2 - 0) + 2(e^2 - 1)$$

$$= \cancel{4e^2} - \cancel{4e^2} + 2(e^2 - 1)$$

$$= 2(e^2 - 1)$$

OR

$$= 2\left\{(2.71)^2 - 1\right\}$$

$$= 2(6.3441)$$

$$\boxed{= 12.6882}$$

is the required Solution:

Q:3 part b) (10)

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

Solution:

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

Let $\sqrt{x} = t$

$$\frac{d}{dx} x^{1/2} = \frac{dt}{dx}$$

$$\frac{1}{2} x^{\frac{1}{2}-1} = \frac{dt}{dx}$$

$$\frac{1}{2} x^{-\frac{1}{2}} = \frac{dt}{dx}$$

$$\frac{1}{2\sqrt{x}} = \frac{dt}{dx}$$

$$\frac{1}{\sqrt{x}}\,dx = 2dt$$

$$\int_1^2 \sin\sqrt{x}\,\frac{1}{\sqrt{x}}\,dx = \int_1^2 \sin t\,(2dt)$$

$$= 2\int_1^2 \sin t\,dt$$

$$= 2(-\cos t)\Big|_1^2$$

$= -2\cos\sqrt{x}\Big|_1^2$ Putting the values of t

$$= -2\{\cos\sqrt{2} - \cos\sqrt{1}\}$$

$$= -2\{0.1559 - 0.5403\}$$

$$= -2(-0.3844)$$

$\boxed{= 0.7688}$ is the required Answer.

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(11)

Q No (4) verify that

$$u(x, y, z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Satisfies the three dimensional Laplace's equation.

Solution:

We Will Satisfies that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

As $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$

$$u = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x+0+0)$$

$$\frac{\partial u}{\partial x} = -x(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[1.(x^2+y^2+z^2)^{-3/2} + x. -\frac{3}{\cancel{2}}(x^2+y^2+z^2)^{-5/2}(\cancel{2}x+0+0)\right]$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[(x^2+y^2+z^2)^{-3/2} - 3x^2(x^2+y^2+z^2)^{-5/2}\right]$$

$$\boxed{\frac{\partial^2 u}{\partial x^2} = -(x^2+y^2+z^2)^{-3/2} + 3x^2(x^2+y^2+z^2)^{-5/2}}$$

$$u = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial y} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(0+\cancel{2}y+0)$$

$$\frac{\partial u}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = -\left[1.(x^2+y^2+z^2)^{-3/2} + y.\ -\frac{3}{\cancel{2}}(x^2+y^2+z^2)^{-5/2}(0+\cancel{2}y+0)\right]$$

$$\frac{\partial^2 u}{\partial y^2} = -\left[(x^2+y^2+z^2)^{-3/2} - 3y^2(x^2+y^2+z^2)^{-5/2}\right]$$

$$\boxed{\frac{\partial^2 u}{\partial y^2} = -(x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2}}$$

and $\quad u = (x^2+y^2+z^2)^{-1/2}$

$$\frac{\partial u}{\partial z} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(0+0+\cancel{2}z)$$

$$\frac{\partial u}{\partial z} = -z(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial z^2} = -\left\{ 1.(x^2+y^2+z^2)^{-3/2} + z \cdot -\frac{3}{\cancel{2}}(x^2+y^2+z^2)^{-5/2}(0+0+\cancel{2}z) \right\}$$

$$\frac{\partial^2 u}{\partial z^2} = -\left\{ (x^2+y^2+z^2)^{-3/2} - 3z^2(x^2+y^2+z^2)^{-5/2} \right\}$$

$$\boxed{\frac{\partial^2 u}{\partial z^2} = -(x^2+y^2+z^2)^{-3/2} + 3z^2(x^2+y^2+z^2)^{-5/2}}$$

Now Taking L.H.S

$$L.H.S = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

$$= \left\{ -(x^2+y^2+z^2)^{-3/2} + 3x^2(x^2+y^2+z^2)^{-5/2} \right\}$$
$$+ \left\{ -(x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2} \right\}$$
$$+ \left\{ -(x^2+y^2+z^2)^{-3/2} + 3z^2(x^2+y^2+z^2)^{-5/2} \right\}$$

$$= -(x^2+y^2+z^2)^{-3/2} - (x^2+y^2+z^2)^{-3/2} - (x^2+y^2+z^2)^{-3/2}$$
$$+ 3x^2(x^2+y^2+z^2)^{-5/2} + 3y^2(x^2+y^2+z^2)^{-5/2} + 3z^2(x^2+y^2+z^2)^{-5/2}$$

$$= -3(x^2+y^2+z^2)^{-3/2} + 3(x^2+y^2+z^2)^{-5/2}\left\{x^2+y^2+z^2\right\}$$

$$= -3(x^2+y^2+z^2)^{-3/2} + 3(x^2+y^2+z^2)^{-5/2+1}$$

$$= -3(x^2+y^2+z^2)^{-3/2} + 3(x^2+y^2+z^2)^{-3/2}$$

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$$= 0$$

R.H.S

Hence the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

That is

$$u(x, y, z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Satisfies the Laplace's equation.