Name : Abdurahman

ID : 7892

Subject Numorical Analysis

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Instructor = Shumail Mazh

BE civil engenearing

(1)

Q.No.01= Apply both Eulers Method ~~and~~

Given data

$$\frac{dy}{dx} = 2x; \quad y(0) = 1$$

Ans = Eulers Method :

1 Given data:-

$y = (0) = 1$, $h = 0.1$ $x_0 = 0$

By formula

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(2x_n)$$

1st Iteration :-

$n = 0$

$$y_1 = y_0 + h(2x_0)$$

$$y_1 = 1 + 0.1(2(0))$$

$$y_1 = 1 + 0.1$$

$$\boxed{y_1 = 1.1}$$

$\rightarrow$ $x_{n+1} = x_n + h$

$$x_1 = x_0 + h$$

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$x_1 = 0 + 0.1$

$$\boxed{x_1 = 0}$$

2nd Iteration :-

$n = 1$

$y_2 = y_1 + h(2x_1)$

$y_2 = 1.1 + 0.2\ (2(0.2))$

$$\boxed{y_2 = 1.02}$$

$x_1 \cancel{x+1} + 1 = x_1 + h$

$x_2 = 0.1 + 0.1$

$$\boxed{x_2 = 0.2}$$

3rd Iteration

put $n = 2$

$y_3 + y_2 \neq h\ (2 \times 2)$

$y_3 = 1.02 + 0.2\ (2(0.4))$

$y_3 = 1.06$

(3)

$$x_{n+1} = x_n + h$$

$$x_3 = x_2 + h$$

$$x_3 = 0.2 + 0.1$$

$$\boxed{x_3 = 0.3}$$

(b) By Modified Euler Method

$$\frac{dy}{dx} = 2x$$

Given data

$y_0 = 1,\ x_0, 0,\ h = 0.1$

Formula

$$y^*_{n+1} = y_n + h\ [F(x_n)]$$

$$y^*_{n+1} = y_n + h \cdot (2x_n) \quad -(1)$$

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y^*_{n+1})\right]$$

$$= y_n + \frac{h}{2}[2x_n + 2x_n]$$

$$= y_n + \frac{h}{2}[4x_n]$$

1st iteration	2nd iteration	3rd iteration
$n = 0$	$n = 1$	$n = 2$
$x_{n+1} = x_n + h$	$x_2 = x_1 + h$	$x_3 = x_2 + h$
$x_1 = x_0 + h$	$x_2 = 0.1 + 0.1$	$x_3 = 0.2 + 0.1$
$x_1 = 0 + 0.1$	$x_2 = 0.2$	$\boxed{x_3 = 0.3}$
$\boxed{x_1 = 0.1}$	$y_2 = y_1 + \frac{h}{2}(4x_1)$	$y_3 = y_2 + \frac{h}{2}(4x_2)$
$y_1 = y_0 + \frac{h}{2}(\cancel{4}x_0)$	$y_2 = 1 + \frac{0.1}{2}(4(0.1))$	$= 1.02 + \frac{0.1}{2}(4(0.2))$
$y_1 = 1 + \frac{0.1}{2}(y(0))$	$\boxed{y_2 = 1.02}$	$\boxed{y_3 = 1.06}$
$\boxed{y_1 = 1}$		

Q No 2) use The fourth order Range kutta Method to obtain a solution of

$$\frac{dy}{dx} = x^2 + x - y$$

subject ty = 0 when $x = 0$ for $0 \leq x \leq 0.6$ with $h = 0.2$ work Throughout to four decimal places-

Ans

Given data

$y = 0, x = 0, h = 0.2$

$0 \le x \le 0.6 \qquad y_{n+1} = y_n + k$

1st Iteration :-

$y_1 = y_0 + k, \quad k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = hf(x_n, y_n)$

$k_1 = h(x_0^2 - x_0 - y_0)$

$k_1 = 0.2(0^2 - 0 - 0$

$\boxed{k_1 = 0}$

$k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{h}{2})$

$= 0.2f(x_0 + h/2, y_0 + h/2)$

$= 0.2f(0 + \frac{0.2}{2}, 0 + \frac{0.2}{2})$

(5)

$$k = \frac{1}{6}\left(0 + 2(0.002) + 2(0.0218) + 0.0436\right)$$

$$k = 0.0152$$

$$y_1 = 0 + 0.0152$$

$$\boxed{y_1 = 0.0152}$$

$= 0.2 f(0.1, 0.1)$

$= 0.2 (0.1^{2} + 0.1 - 0.1)$

$$\boxed{k_2 = 0.0020}$$

$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$

$k_3 = hf\left(x_n + \frac{h}{2}, 0 + \frac{0.002}{2}\right)$

$\cancel{= 0.2 f\left(\frac{0 + 0.2}{2}, 0.1\right)}$

$0.2 f(0.1, 0.001)$

$= 0.2 (0.1^{2} + 0.1 - 0.001$

$k_3 = 0.0218$

$$k_4 = hf(x_n + h,\ y_n + k_3)$$

$$= 0.2f(0 + 0.2,\ 0 + \cancel{02}\,0.0218)$$

$$= 0.2f(0.2,\ 0.0218)$$

$$= 0.2(0.2^2 + 0.2 - 0.0218)$$

$$\boxed{k_4 = 0.0436}$$

Q No 03

Q No 3) Given data :-

$a = 0, b = 10, n = 10$

$$h = \frac{b-a}{n} = \frac{10-0}{10} = 01$$

Sol ...:-

x	0	1	2	3	4	5	6	7	8	9	~~10~~ 10
$f(x_0)$	10.1	17.2	24.4	29.2	34.6	41.2	50.9	57.8	60.3	61.2	62.1

using formula

$$f(x)dx = \frac{h}{2}\left[f(x_0) + 2(f(x_1) + f(x_2) + f(x_3) \ldots\ldots f(x_{10}))\right.$$

(b)

$$= \frac{1}{2}\left[10.1 + 2(17.2 + 24.4 \cdot 29.2 + 34.6 + 41.2 + 50.9 + 57.8 + 64.3 + 62.2 + 62.1\right]$$

$$= 412.9 \text{ Ans}$$

(~~4~~) (8)

Q No. 4 $\int_2^3 \ln(x^3+1)\,dx$

use ~~to~~ 10 strips

Sol.. $N = 10$

$h = \frac{3-2}{10} = 0.1$

x	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
$f(x)$	0.693	0.846	1.003	1.162	1.320	1.476	1.628	1.777	1.922	2.062

Now using formula.

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(x_0)+4(f(x_1)+f(x_3)\ldots]+2[f(x_2\ldots)+f x_n\right]$$

$$= \frac{0.1}{3}\left[0.693+4(0.846+1.162+1.476+1.777+\right.$$

$$+2(1.003+]$$

$$1.320+1.628+1.922)+2.062$$

$= 1.184$ Ans.