Question No 1:

Answer:

1) The order of Matrix AB is $m \times n$ because if we consider the order of matrix "A" is $m \times p$ and the order of B is $p \times n$.

2) The number of non-zero rows in an Echelon form is one. The non-zero rows is also called Rank.

3) if B is a singular Matrix then

$$a = 8.$$

4)

Solution: If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

then

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$= -2i - i^2$ $\quad\quad\quad\quad\quad\quad \therefore i^2 = -1$

$= -2(-1) - (-1)$

$= 2 + 1$

$= 3$

5) The matrix A is $\begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is a Scalar Matrix because we know that when diagnal element are same and non-diagnal are zero then its called scalarmatrix

6) Solution:- $\frac{dy}{dx} + 2xy = y$

$\frac{dy}{dx} = y - 2xy$

$\frac{dy}{dx} = y(1 - 2x)$

$\frac{dy}{dx} = y(1 - 2x)$

$\frac{dy}{dx} = (1 - 2x)\,dx$

Taking integration

$$\int \frac{1}{y}\, dy = \int (1-2x)\,dx$$

$$\ln y = \int 1\,dx - \int 2x\,dx$$

$$\ln y = \int 1\,dx - \int 2x\,dx$$

$$\ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + c$$

$$\ln y = x - x^2 + c$$

$$e^{\ln y} = e^{x - x^2 + c}$$

$$\boxed{y = e^{x(1-x)+c}}$$

7) order & degree of differential equation

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \text{ is}$$

order is 1 &

degree is 3

8) order and degree of differential equation

$$\frac{d^2y}{dx} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right) \text{ is}$$

order 2 , Degree 1

9) <u>Solution:</u>

$$2y' + x^2 y = x^2+3 \quad , \quad y(0)=5$$

$$y' + \left(\frac{x^2}{2}\right)y = \frac{x^2}{2}+3$$

$$y' + \left(\frac{x^2}{2}\right)y = \frac{1}{2}\left(x^2+3\right)$$

$$\mu = \frac{x^2}{2}$$

$$e^{\int \frac{x^2}{2}dx} = e^{x^3/6}$$

$$\Rightarrow \quad e^{x^3/6}y' + e^{x^3/6}\left(\frac{x^2}{2}\right)y$$

$$= \frac{1}{2}e^{x^3/6}\left(x^2+3\right)$$

$$y(x) = \frac{e^{x^3/6}x^2 + 3e^{x/6} + c}{2e^{x^3/6}}$$

$$y(0) = \frac{0+3}{9} = 3/9$$

$$\boxed{y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6}}{9e^{x^3/6}} \pm 3/9}$$ Ans

10)
$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Expanding by C1

$$1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - 1\begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1\begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix}$$

$$\Rightarrow 1(bc^2 - cb^2) - (ac^2 + a^2c) + (ab^2 - a^2b)$$

$$= bc^2 - cb^2 - ac^2 - a^2c + ab^2 - a^2b$$

$$= ab^2 - cb^2 + a^2c - a^2b - ac^2 + bc^2$$

$$= a^2c - a^2b + ab^2 - cb^2 + bc^2 - ac^2$$

$$= a^2(c-b) + b^2(a-c) + c^2(b-a)$$ Ans

Question 2:

'A' Solution:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2cb^3 - a^3b^2c$$

Take abc common.

$$abc(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$$

$$\Rightarrow abc[bc(c-b) - ac(c+a) + ab(b-a)]$$

Ans

"B" **Solution:**

$$\begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix}$$

Characteristic eq $\Rightarrow |A-\lambda I| = 0 \rightarrow$ (A)

$$\begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Now take determinent

$$|A-\lambda I| = 0$$

$$\begin{pmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{pmatrix}$$

Expand by R_1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & +2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \rightarrow (B)$$

Again Expand by R_2

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$

$$\Rightarrow 3-\lambda\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} -1\begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$\Rightarrow (3-\lambda)\Big[(3-\lambda)(2-\lambda)-(-1)(-1)\Big] + 1\big((-1)(2-\lambda)-(1)(-1)\big)$$
$$-\big((-1)(-1)-(-1)\big)(3-\lambda)\Big]$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2+1)+(-2+\lambda-1)-(+1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2\,5\lambda+5)+(-3+\lambda)-(4-\lambda)$$

$$= 3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$= -\lambda^3+8\lambda^2-18\lambda+8 \longrightarrow (i)$$

$$\Rightarrow +1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C1.

$$\Rightarrow -1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -1 & 2\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1\left(6-3\lambda-2\lambda+\lambda^2-1\right)+1\left(-2+\lambda-1\right)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$$

$$= -\lambda^2+6\lambda-8 \rightarrow (ii)$$

$$-1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C1

$$-\left\{-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right.$$

$$\Rightarrow -\left\{-(-2+\lambda-1)+2(6-3\lambda-2\lambda+\lambda^2-1)\right)$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$-\lambda^2+6\lambda-8 \longrightarrow (iii)$$

Put (i),(ii),(iii) in (B)

$$(2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$$

$$\Rightarrow -2\lambda^3+16\lambda^2-36\lambda+16\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2+6\lambda-8$$
$$-\lambda^2+16\lambda-8$$

$$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2-36\lambda-8\lambda$$
$$+6\lambda+6\lambda+16-16$$

$$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$$

By synthalic division we get.

$$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$$

$$(\lambda=0)$$

$$\lambda-2=0$$

$$\boxed{\lambda=2}$$

$$\lambda^2-8\lambda+16=0$$

By factorization method

$$\lambda^2-4\lambda-4\lambda+16=0$$

$$\lambda(\lambda-4)-4(\lambda-4)=0$$

$$(\lambda-4)(\lambda-4)$$

$$\lambda,4,\lambda,4$$

$$\lambda_1=0,\ \lambda_2=2,\ \lambda_3=4\ ;\ \lambda_4=4$$

Question # No "3" $(x^2+3y^2)dx - 2xy\,dy = 0$

$x = 2,\ y = 6.$

Solution:

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$(x^2+3y^2)dx = 2xy\,dy$$

Dividing B.S by $2xy\,dx$, we get

$$\frac{dy}{dx} = \frac{x^2+3y}{2xy}$$

$$\frac{dy}{dx} = \frac{x^2}{2xy} + \frac{3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{x}{y} + \frac{3y}{x}\right) \longrightarrow A$$

let $y = vx$

Diff: $dy = v\,dx + x\,dv$

Dividing by dx.

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \longrightarrow (i)$$

Put (i) in A.

$$v + x\frac{dv}{dx} = \frac{1}{2}\left(\frac{x}{xv} + 3\,\frac{vx}{x}\right)$$

$$v + x\frac{dv}{dx} = \frac{1}{2}\left(\frac{1}{v} + 3v\right)$$

Multiplying b.s by "2"

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

Multiplying B.S by $\frac{dx}{dv}$, we get

$$2x\,dv = \frac{1+v^2}{v}\,dx.$$

Multiplying B.S by $\frac{v}{x(1+v^2)}$
we get.

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx.$$

Taking "$\int$" on B.S.

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

Take "e" on B.S

$$e^{\ln|1+v^2|} = e^{\ln(xc)}$$

$$1+v^2 = xc$$

$$1+v^2 = xc$$

Put $v = y/x$

$$1+\left(y/x\right)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3 c \longrightarrow B$$

Put $x=2, y=6$ in eq B.

$$(4)+(36) = 8c$$

$$c = 5$$

Put $c=5$ in B, we get.

So $$x^2+y^2 = 5x^3$$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(-5x-1)$$

Page #14 Taking $\sqrt{\ }$ on B.S , we get

$$y = +x\sqrt{5x-1},$$

$$y = -x\sqrt{5x-1}$$