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Page # 01

Q1 $\frac{d^2z}{dxdy}$ and $\frac{d^2z}{dydx}$ for arcsin$\left(\frac{x}{y}\right)$

$\cancel{z = \sin^{-1}\left(\frac{x}{y}\right)}$ $z = \frac{1}{1-\left(\frac{x}{y}\right)^2} = \frac{1}{1-\frac{x^2}{y^2}} = z = \frac{1}{\frac{y^2-x^2}{y^2}}$

$\frac{dz}{dx} = \frac{d}{dx}\left(\frac{y^2}{y^2-x^2}\right)$ $\qquad \frac{y^2}{y^2-x^2}$

$\frac{(y^2-x^2)\,0-(y^2)(0-2x)}{(y^2-x^2)^2} = \boxed{\frac{2xy^2}{(y^2-x^2)^2}} = \frac{dz}{dx}$

$\frac{\partial^2 z}{\partial x dy} = \frac{\partial}{dy}\left(\frac{2xy^2}{(y^2-x^2)^2}\right) = \frac{(y^2-x^2)^2\, 2x2y - 2xy^2}{((y^2-x^2)^2)^2}$

$\frac{d}{dy}(y^2-x^2)^2$

$(a-b)^2 = a^2+b^2-2ab$

$y^4+x^4-2y^2x^2$

$\frac{d}{dy} = 4y^3+0-2x^2 2y = 4y^3-4x^2y$

$$\boxed{\frac{(y^2-x^2)^2(4xy)-(2xy^2(4y^3-4x^2y))}{((y^2-x^2)^2)^2} = \frac{\partial^2 z}{dxdy}}$$

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page # 02

Q2

Function $f(x,y) = e^x \sin y + e^y \cos x$

Laplace's equation

Sol: $f(x,y) = e^x \sin y + e^y \cos x$

$$\frac{df(x,y)}{dy} = e^x \sin y + e^y \cos x$$

$$\frac{df}{dy} = e^x \{ x \sin y + e^y \cos x \}$$

$$\frac{df(x,y)}{dx} = \sin y e^x (1) + e^y (-\sin x)$$

$$\frac{df(xy)}{dx} = e^x \sin y - e^y \sin x$$

$$\frac{d^2 f(xy)}{dx^2} = e^x (1) \sin y - e^y \cos x (1)$$

$$\frac{\partial^2 f(xy)}{\partial x^2} = e^x \sin y - e^y \cos x \quad \text{--- (1)}$$

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Page #03

$$\frac{\partial f(x,y)}{\partial y} = \frac{\partial}{\partial y}\left(e^x \sin y + e^y \cos x\right)$$

$$\frac{\partial f(x,y)}{\partial y} = e^x (+\cos y)(1) + e^y (1) \cos x$$

$$\frac{\partial f(x,y)}{\partial y} = e^x \cos y + e^y \cos x$$

$$\frac{\partial^2 f(x,y)}{\partial y^2} = e^x(-\sin y)(1) + e^y (1) \cos x$$

$$\frac{\partial^2 f(x,y)}{\partial y^2} = -e^x \sin y + e^y \cos x \quad \text{— (ii)}$$

Laplace eqn $\frac{\partial f(x,y)}{\partial x^2} + \frac{\partial f(x,y)}{\partial y^2} = 0$

$$\cancel{e^x \sin y} - \cancel{e^y \cos x} - \cancel{e^x \sin y} + \cancel{e^y \cos x} = 0$$

$$0 = 0$$

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Q3:: If $f(x,y) = x^3 e^{-y} + y^3 \sec x$,
Find the partial derivative of $f(x,y)$ with respect x and, y.

$$\boxed{\frac{d}{\partial x} f(x) = e^{-y} 3x^2 + y^3 \sec x . \tan x}$$

$\therefore \frac{d}{dx} \sec x = \sec x \tan x$

$$\frac{d}{dy} f(x,y) = x^3 e^{-y} \frac{d}{dy}(-y) + 3y^2$$

$\frac{d}{dx} e^{y} = e^{y} \frac{d}{dy} y$

$x^3 e^{-y} (-1)$

$e^{y} \cdot 1$

$-x^3 e^{-y} + 3y^2 \sec x$

$$\boxed{\frac{d}{dy} f(x,y) = -x^3 e^{-y} + 3y^2 \sec x}$$

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Page # 05

Q4. Vector projection $b = 6\hat{i} + 3\hat{j} + 3\hat{k}$

$a = \hat{i} - 2\hat{j} - 2\hat{k}$

Solution:-

Projection of vector $\vec{b}$ on $\vec{a} = \frac{1}{|\vec{a}|}(\vec{b}.\vec{a})$

$\vec{a} = \hat{i} - 2\hat{j} - 2\hat{k}$

$\vec{b} = 6\hat{i} + 3\hat{j} + 3\hat{k}$

$(\vec{b}.\vec{a}) = (6\times1)\hat{i} + (3\times(-2))\hat{j} + (3\times(-2))\hat{k}$

$(\vec{b}.\vec{a}) = 6 + 6 - 6$

$\boxed{(\vec{b}.\vec{a}) = -6}$

$\vec{a} = \sqrt{(1)^2 - (2)^2 - (2)^2}$

$|\vec{a}| = \sqrt{1 + 4 + 4}$

$\boxed{|\vec{a}| = \sqrt{9}}$

$\boxed{|\vec{a}| = 3}$

Projection of $\vec{b}$ on $\vec{a}$

$= \frac{1}{|\vec{a}|}(\vec{b}.\vec{a})$

$= \frac{1}{\sqrt{9}}(-6)$

$= \frac{1}{3}(-6)$

$= \frac{-6}{3}$

$\boxed{= -2}$

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page #06

Q 6

$f(x,y) = xe^y + \cos(xy)$ at the point $(2,0)$ directional of $a = 3i - 4j$

$$\frac{d}{dx}\left(xe^y + \cos(xy)\right)$$

$$\cancel{(1)e^y - \sin(xy)\left(y + x\frac{dy}{dx}\right)}$$

$$\frac{d}{dx}f(x,y) = e^y(1) \cancel{- \sin xy\left(y + x\frac{dy}{dx}\right)} - y\sin(xy)$$

$$\frac{\partial f(x,y)}{\partial y} = xe^y(1) - x\sin(xy)$$

$$(e^y - y\sin xy)i + (xe^y - x\sin(xy))j$$

$(2,0)$

$$e^0 - 0\sin(2)(0)\, i + 2e^0 - 2\sin(2\times 0)j$$

$$1 - 0 + 2 - 0\, j$$

$1i + 2j$ 3 -4 point

$$(3-1)i \quad (-4-2)j$$

$$\frac{2i - 6j}{\sqrt{2^2 + 6^2}} = \frac{2i - 6j}{\sqrt{4+36}} \quad \frac{2i - 6j}{\sqrt{40}}$$

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Q5 remaining

$$(1i+2j) \cdot \frac{2i-6j}{\sqrt{40}}$$

$$= \frac{2-12}{\sqrt{40}} = \boxed{\frac{-10}{\sqrt{40}}}$$

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page #08

Q6 $f(x,y,z) = x^2+y^2+z^2-14$ at Point $(1,-2,3)$

$f(x,y,z) = x^2+y^2+z^2-14$

$\nabla f(x,y,z) = 2xi+2yj+2zk$ $P(1,-2,3)$

$\nabla f(1,-2,3) = 2i - 4j + 6k$

Tangent plane on Point P_0

$2(x-1)+4(y+2)+6(z-3)=0$

Normal line

$x=1+2t$

$y=-2+4t$

$z=3+6t$

$\frac{9}{3}$

page #A09

Q7 $\int_0^1 \int_0^1 (xy + y^2)\, dx\, dy$

$\int_0^1 \left[\int_0^1 (xy + y^2)\, dy \right] dx$

$\int_0^1 \left[\frac{xy^2}{2} + \frac{y^3}{3} \Big|_{y=0}^{y=1} \right] dx$

$\int_0^1 \left[2x + 3 \right] dx$

$= x^2 + 3 \Big|_{x=0}^{x=1}$

$= (1)^2 + 3$

$= 1 + 3$

$\boxed{= 4}$

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page #10

Q8 area of the region R enclosed by the parabola $y = x^2$ and the line $y = x + 2$

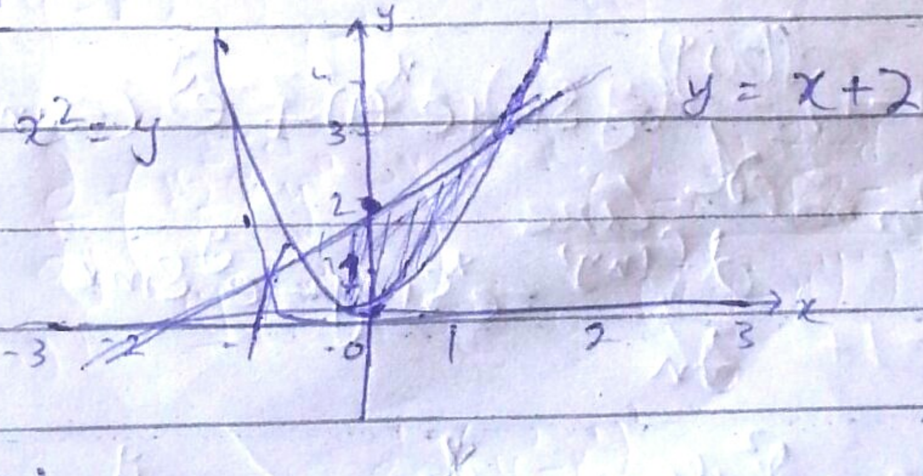
Solve: we $y = x^2$ and $y = x + 2$

$$x^2 = x + 2$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

when $x = 2$, $y = 4$, and $x = -1$ $y = 1$



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$$= \int_{-1}^{2} (x + 2 - x^2)\,dx = \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$$

$$= \left\{\frac{4}{2} + 4 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3}\right\}$$

$$= 8 - \frac{1}{2} - \frac{9}{3} = 5 - \frac{1}{2} = \boxed{\frac{9}{2}}$$