Question

construct the mohr's circle diagram and find the principle stress and maximum in plane shear stress - for the stress state of point c located at the centre of UDL and 1inch below the top fibre of beam cross section shown in figure.

How ever to construct mohr's circle It is necessary to draw the Shear stress and flexure stress variation diagram for maximum Shear force and bending moment respectively. Compare the results obtained from the mohr's circle with the stress transformation equation

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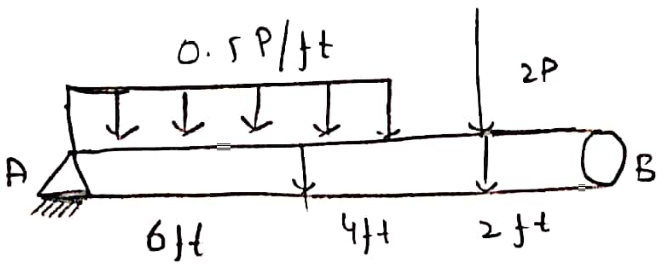
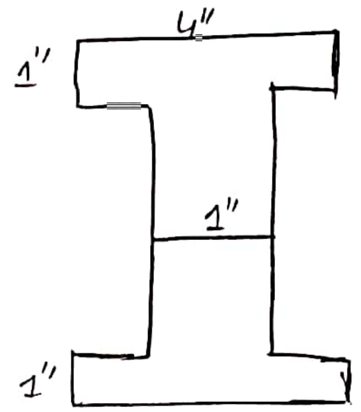
P= 2





P= last 2 digits of ID

that is P= 39

Solution:-

~~we~~ we have to find

→ Reactions

→ Shear force

→ Bending moment diagram

$\Sigma f y = 0$ Upward Positive

$\Rightarrow RA + RB \; (0.5 \times 39 \times 6) - 2(39) = 0$

$RA + RB - 117 - 78 = 0$

$RA + RB = 39$ — (A)

Now $\Sigma MA = 0$ anticlockwise

$(RA \times 12) - (78 \times 10) - (117 \times 3) = 0$

$RA \times 12 - 780 - 351 = 0$

$\dfrac{RA \cancel{12}}{\cancel{12}} = \dfrac{1131}{\cancel{12}}$ $\boxed{RA = 94.25}$

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So put the value of RB in eq (A)

eq (a) => $RB + 94.25 = 39$

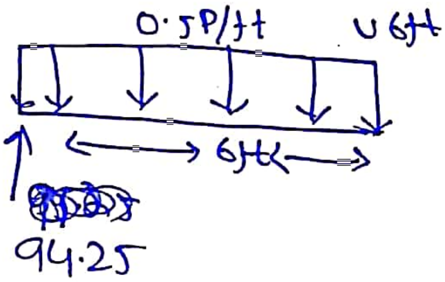
$RB = 94.25 - 39$

$\boxed{RB = 55.25\,lb}$

Shear force :- first where we have to find the shear force at 6ft from left support.

$\Sigma fy = 0$

$\Rightarrow -V_{6ft} + 94.25 - 117 = 0$



$\Rightarrow -V_{6ft} = 101.75$

$\boxed{V_{6ft} = -101.75}$

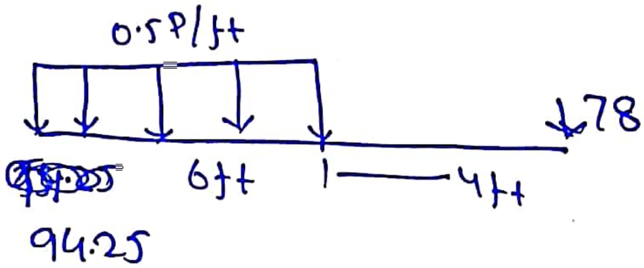
Now we have to find shear stress at 10ft from left support.

$\Sigma fy = 0$

$55.25 - 117 - 78 - V_{10ft} = 0$

$-V_{10ft} = +139.75$

$\boxed{V_{10ft} = -139.75}$



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Bending moment :-

$$\Sigma M_{6ft} = -(94.25 \times 0.5 \times 6)(39 \times 6)(6/3)$$

$$\Sigma M_{6ft} = -282.75 + 468$$

$$\boxed{= 185.25}$$

Now we have to find moment at 3ft

$$\Sigma M_{3ft} = -(94.25 \times 3) + (39 \times 6 \times 3)$$

$$\Rightarrow \Sigma M_{3ft} = -(282.75 + 702)$$

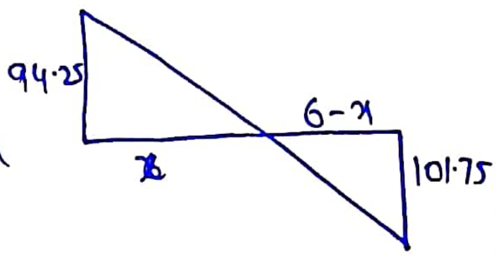
$$\boxed{\Sigma M_{3ft} = 419.25}$$

Now we have to find moment at changing point

$$\frac{94.25}{x} = \frac{101.75}{6-x}$$

$$\Rightarrow$$

$$94.25 \times (6-x) = 101.75x$$



$$\Rightarrow 565.5 - 94.25x = 101.75x$$

$$\Rightarrow 565.5 = 101.75x + 94.25x$$

$$\Rightarrow \frac{565.5}{\cancel{196}} = \frac{\cancel{196}x}{\cancel{196}}$$

$$\boxed{x = 2.88\,ft}$$

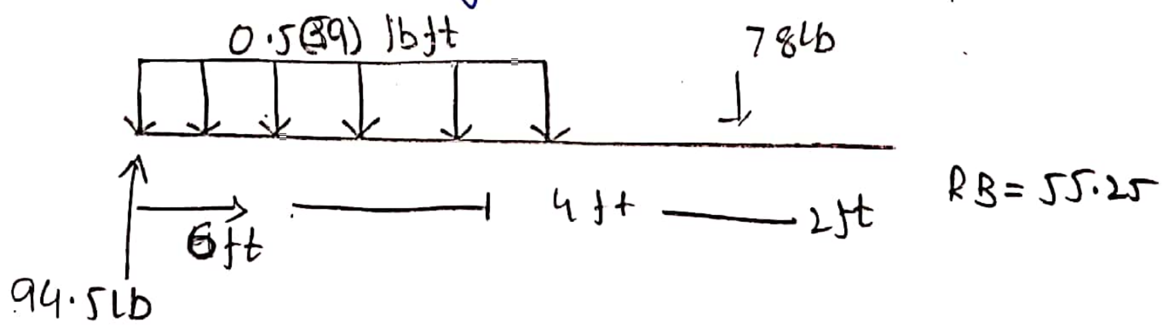
$$\Sigma M_{2.88\,ft} = 0$$

$$\Rightarrow M_{2.88ft} - (94.25 \times 2.88) + \left(39 \times 6 \times \frac{2.88}{2}\right) = 0$$

$$M_{2.88\,ft} = -(271.44) + (47.64) = 0$$

$$\boxed{M_{2.88\,ft} = 319.08\ lbft}$$

Shear force and Bending moment diagram



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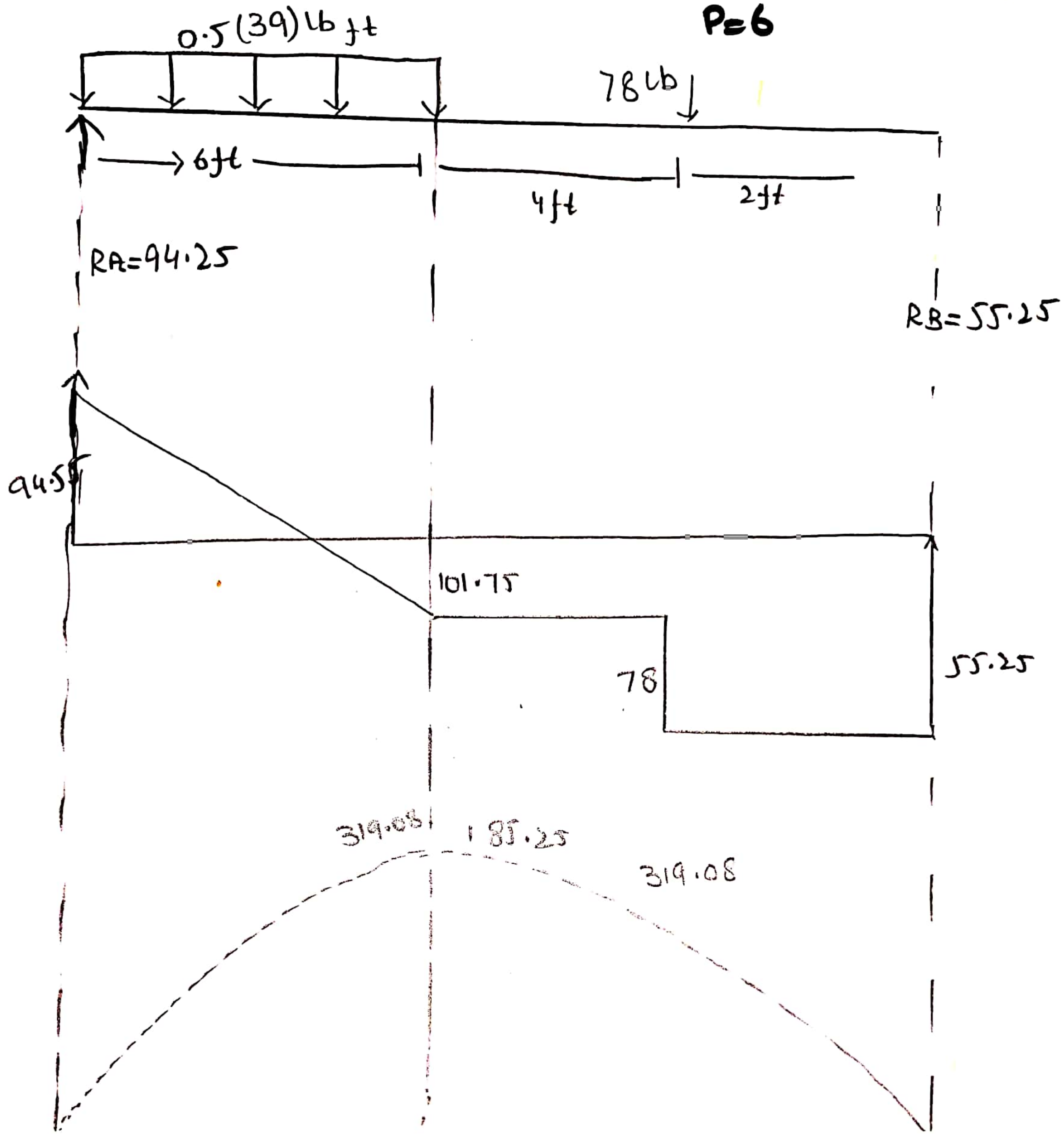
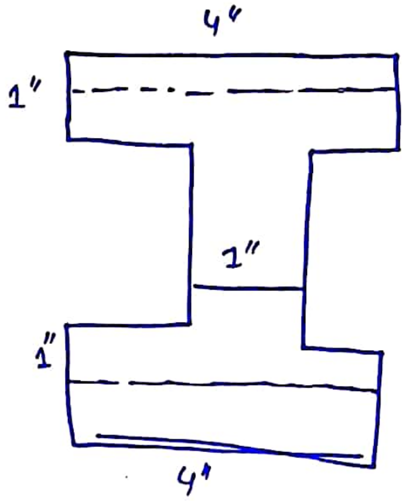
P=6


0.5(39) lb ft
78 lb
6 ft
4 ft
2 ft
RA=94.25
RB=55.25
94.55
101.75
78
55.25
319.08
185.25
319.08

Part B:-

Now we have to find moment of inertia of Beam cross section



$y_1 = 5.5$

$y_2 = 3$

$y_3 = 0.5$

$A_1 = 4 \text{ inch}^2$

$A_2 = 4 \text{ inch}^2$

$A_3 = 4 \text{ inch}^2$

$$\bar{y} = \frac{(A_1 y_1) + (A_2 y_2) + (A_3 y_3)}{A_1 + A_2 + A_3}$$

Now putting values

$$\Rightarrow \bar{y} = \frac{(4 \times 5.5) + (4 \times 3) + (4 \times 0.5)}{4 + 4 + 4}$$

$$\Rightarrow \boxed{\bar{y} = 3''}$$

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Now

$$I_1 = \frac{bh^3}{12}$$

$$I_1 = \frac{4 \times 1^3}{12} \Rightarrow \boxed{I_1 = 0.33 \text{ inch}^4}$$

$$I_2 = \frac{bh^3}{12}$$

$$I_2 = \frac{1 \times 4^3}{12} \Rightarrow \boxed{I_2 = 5.33 \text{ inch}^4}$$

$$I_3 = \frac{bh^3}{12}$$

$$I_3 = \frac{4 \times 1^3}{12} \Rightarrow \boxed{I_3 = 0.33 \text{ inch}^4}$$

Now "d"	Ad^2
$d_1 = \bar{y} - y_1$	$A_1 d_1^2$
$\Rightarrow d_1 = 3 - 5.5 \boxed{= -2.5}$	$= 4 \times (-2.5)^2$
	$\boxed{= 25 \text{ inch}^4}$
$\Rightarrow d_2 = \bar{y} - y_2$	$A_2 d_2^2 =$
$\Rightarrow d_2 = 3 - 3$	$4 \times (0)^2$
$\boxed{d_2 = 0}$	$\boxed{= 0}$
$d_3 = \bar{y} - y_3 \boxed{= 2.5}$	$A_3 d_3^3 = \boxed{25 \text{ inch}^4}$

Now:-

$$I_{1x} = I_1 + A_1 d_1^2$$

$$\Rightarrow I_{1x} = 0.33 + 25$$

$$\boxed{I_{1x} = 25.33 \text{ inch}^4}$$

$$I_{2x} = I_2 + A_2 d_2^2$$

$$I_{2x} = 0 + 5.33$$

$$\boxed{I_{2x} = 5.33 \text{ inch}^4}$$

$$I_{3x} = I_3 + A_3 d_3^2$$

$$I_{3x} = 0.33 + 25$$

$$\boxed{\Rightarrow I_{3x} = 25.33 \text{ inch}^4}$$

Now $I_{xx} = I_{1x} + I_{2x} + I_{3x}$

$$I_{xx} = 25.33 + 5.33 + 25.33$$

$$\boxed{I_{xx} = 55.99 \cong 56 \text{ inch}^4}$$

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Part 'c'

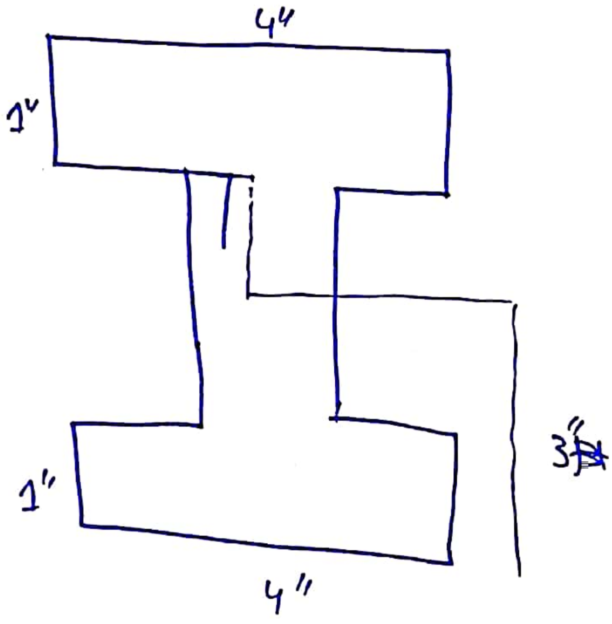
⇒ we have to find
- → Shear Stress
- → flexure Stress
- → variation diagram

Shear stress :- case ⇒ 1

So we have to find the shear Stress at top fibre

we know that

$$\tau = \frac{VQ}{It}$$



As top fibre A=0

$Q = \bar{y} A \quad \because \bar{y} = 3''$

$V = 55.25 \quad I = 56 \quad b = 4$

$$\tau = \frac{55.25 \times 0}{56 \times 4}$$

$$\boxed{\tau = 0}$$

Case # 2

find stress 1 inch below the top fibre but here two cases

$\bar{y} = 2 + \frac{1}{2} = 2.5$

$A = 1 \times 4 = 4$

$Q_A = \bar{y} A$

$\Rightarrow Q_A = 2.5 \times 4 = 10$

$$\Rightarrow \frac{(55.25)(10)}{56 \times 4}$$

$$\Rightarrow \boxed{\tau_A = 2.46 \text{ psi}}$$

$$\tau_B = V Q_B$$

$$\Rightarrow \tau_B = \frac{(55.25)(10)}{56 \times 1}$$

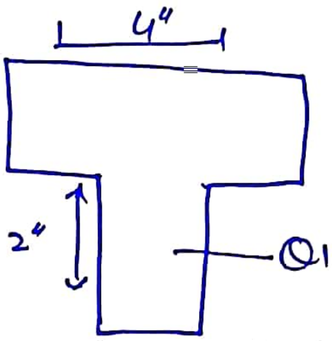
$$\boxed{\Rightarrow \tau_B = 9.86} \text{ psi}$$

Case # 3

Find stress at centroid axis

Now here

$Q = Q_1 + Q$ — (1)



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$Q_1 = \bar{y}_1 A_1 = 2/_2 (1 \times 2) = 2$

$Q_2 = 2.5 \times 4 = 10$

So $Q = 2 + 10$

$Q = 12$

Now $\tau_{max} = \dfrac{VQ}{Ib}$ $b = 1 \text{inch}$

$$\tau_{max} = \frac{55.25 \times 12}{56 \times 1}$$

$$\boxed{\tau_{max} = 11.83 \text{ psi}}$$

Case # 4

Find out shear stress 1inch above the bottom fibre

Now again we have two cases

$$\tau_A = \frac{55.25 \times (2.5 \times 4)}{56 \times 1}$$

$$\boxed{\tau_A = 9.86 \text{ psi}}$$

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$$\tau_B = \frac{55.25 \times (2.5 \times 4)}{56 \times 4}$$

$$\boxed{\tau_B = 2.46 \text{ Psi}}$$

case # 5

Find shear stress at bottom fibre

here $\bar{y} = 0$ so $Q = 4 \times 0 = 0$

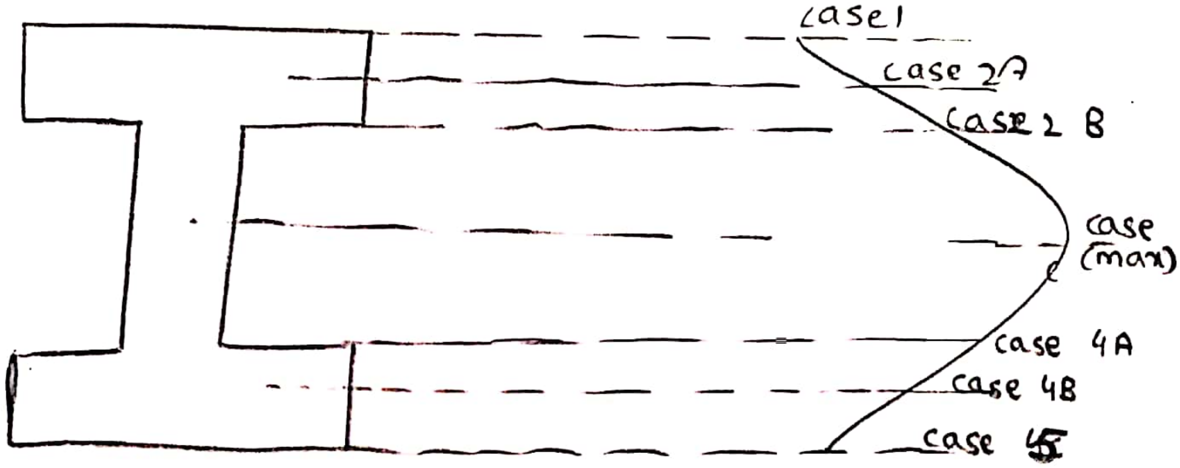
$$\tau = \frac{VQ}{IB}$$

$$\Rightarrow \frac{55.25 \times 0}{56 \times 1} = 0$$

$$\boxed{\tau = 0 \text{ Psi}}$$

—x—x—x—x—x

Shear stress Variation diagram



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Case #6

Now we have to find maximum shear stress at a distance of 6ft from left support of beam along its length.

As we know that shear force at 6ft is

$$V = 101.75 lb$$

$$\tau_{max} = \frac{VQ}{Ib} \Rightarrow \frac{101.75 \times 12}{56 \times 1} =$$

$\boxed{\tau_{max} \Rightarrow 21.80}$ psi

Case #7

Now we have to find shear stress at a distance of 6ft from left support and 1 inch below the top fibre.

$$\tau = \frac{VQ}{Ib}$$

$$\Rightarrow \tau = \frac{101.75 \times 10}{56 \times 4} \Rightarrow \boxed{4.54 \text{ psi}}$$

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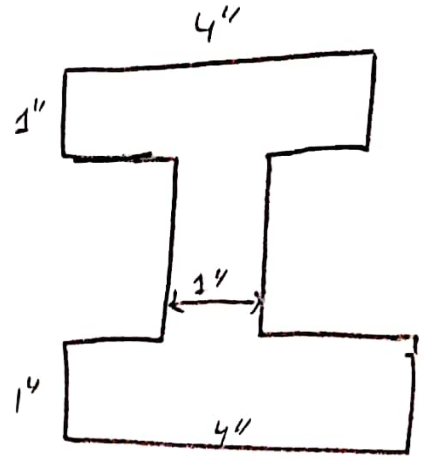
P=15

Flexure stress analysis :-



as we know that

$$6 = \frac{My}{I}$$

The maximum bending moment is 185.25 psi

Case 1 find stress at top fibre

$$6_{tp} = \frac{(185.25)(3)}{56}$$

$$\boxed{= 9.92 \text{ psi}}$$

Case 2 find stress at 1 inch below top fiber

$$= \frac{(185.25)(2)}{56}$$

$$\boxed{= 6.61 \text{ psi}}$$

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Case 3 find flexure stress at centroid axis

$$\sigma_{cent} = \frac{185.25 \times 0}{56} = \boxed{0 \text{ Psi}}$$

Case 4 find flexure stress at 1 inch abow the bottom fibre

$$\sigma = \frac{My}{I}$$

$$\Rightarrow \frac{185.25 \times 2}{56} \boxed{= 6.61 \text{ Psi}}$$

Case 5 find flexure stress at bottom fiber

$$\sigma_{bott} = \frac{My}{I}$$

$$\sigma_{Bott} = \frac{185.25 \times 3}{56}$$

$$\boxed{\sigma_{Bott} = 9.82 \text{ Psi}}$$

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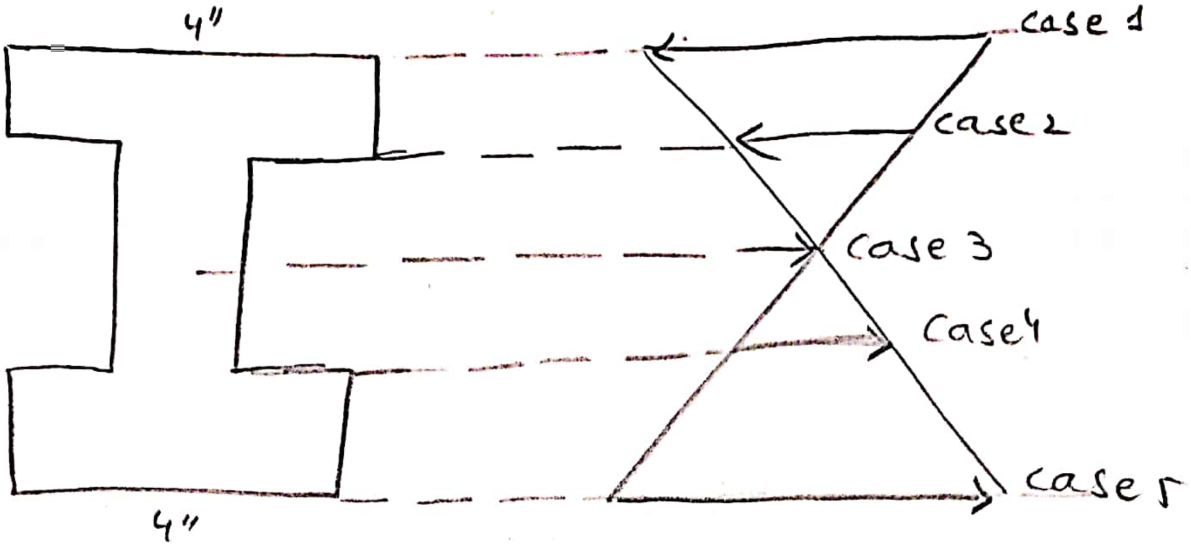
Flexure stress variation diagram



stress state of a point element:-

Now we have to find stress state of a point element located at a distance of 3ft from left support and 1 inch below from the top fibre. As to the condition of stressed elements at point C in this given I section. It require to find all the ~~section~~ stresses at this point.

As in given problem the stresses acting on point C is flexural and Shear stresses. There is no torsional stress force acting on this beam. due to load symmetry along the beam axis (lognitudnal axis)

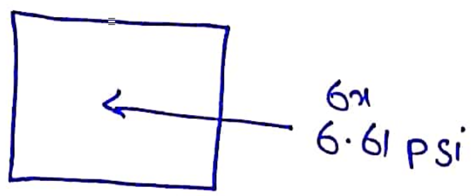
Flexure stress at point c

$\boxed{\sigma = 6.61\ \text{psi}}$ case #2

" shear stress at point c"

$\boxed{\tau = 4.54\ \text{psi}}$ case #7

consider this point "c" is a planer element



6.61 is a compressive because point "c" lies in compression zone of beam cross section

If the point c lies below the centroidal then stress would be tensile.

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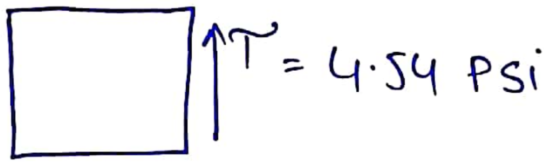
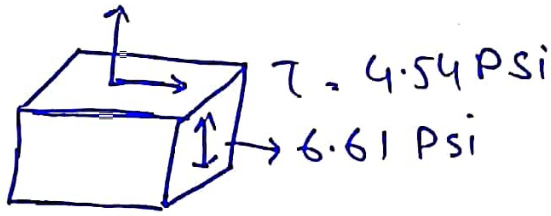
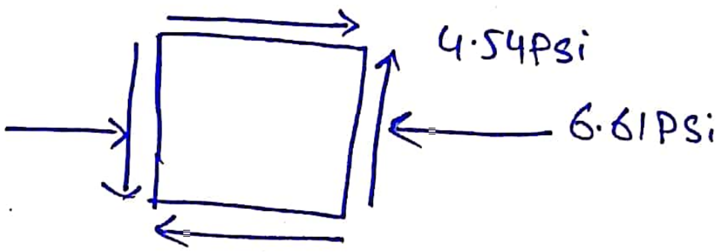




combine stress on 2D element



Part D :— we have to find

→ principle stresses

→ stresses transformation

→ mohr's circle

→ comparison of mohr's circle result with principle stress equation

Principle stresses:- we have to find θp

$$\tan 2\theta p = \frac{\tau}{(6x - 6y)/2}$$

$$\tan 2\theta_p = \frac{4.54}{(-6.61-0)/2}$$

$$\tan 2\theta P = -1.3736$$

$$2\theta P = \tan^{-1}(-1.3736)$$

$$\frac{2\theta P}{2} = \frac{53.54}{2} \quad \boxed{= -26.57}$$

Now for $6x$

$$6x' = \frac{6x+6y}{2} + \frac{6x-6y}{2}\cos\theta + \tau xy \sin 2\theta$$

$$\Rightarrow 6x' = -3.305 + 3.305 \cos 2(26.97) + 4.54 \times \sin 2(26.54)$$

$$\Rightarrow \boxed{6x' = 18.47 \text{ psi}}$$

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$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{(-6.61 + 0)}{2} - \frac{(-6.61 - 0)}{2}\cos 2(-4.552) - (4.54 \times \sin 2(26.54))$$

$$\boxed{\sigma_{y'} = -0.0977 \text{ Psi}}$$ compression

Shear plan

$$\tau_{x'y'} = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_{x'y'} = \frac{(-6.61 - 0)}{2}\sin 2(40.47) + (4.54 \cos 2)\, 40.47)$$

$$\boxed{\Rightarrow \tau_{x'y'} = 3.34}$$

Stress transformation

Now find stress state condition of point c at a clockwise orientation $\theta = -30^\circ$ (assumed)

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

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$$\Rightarrow \sigma_{y'} = \frac{(6.61+0)}{2} - \frac{(-6.61-0)}{2}\cos 2(-30) - 4.54\sin 2(-30)$$

$$\boxed{\sigma_{x'} \Rightarrow -8.268 \text{ psi}}$$ compression

$$\sigma_{y'} = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\Rightarrow \frac{-6.61+0}{2} - \frac{(6.61-0)}{2}\cos 2(-30) - 4.54\sin 2(-30)$$

$$\boxed{\sigma_{y'} = -4.45 \text{ psi}}$$ compression

$$\tau_{x'y'} = -\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\Rightarrow \tau_{x'y'} = \frac{6.61-0}{2}\sin 2(-30) + 4.54)\cos 2(-30)$$

$$\boxed{\Rightarrow \tau_{x'y'} = -6.9436 \text{ psi}}$$

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Mohr's circle :-

coordinates

$$(h,k) = \left(\frac{-6.61}{2}, 0\right)$$

$$(h,k) = (-3.34, 0)$$

$$\boxed{(h,k) = (3.34, 0)}$$

Radius

$$r = \sqrt{\left(\frac{6x-6y}{2}\right)^2 + Txy^2}$$

$$r = \sqrt{\frac{6.61-0}{2} + (4.54)^2}$$

$$\boxed{r = 4.89}$$

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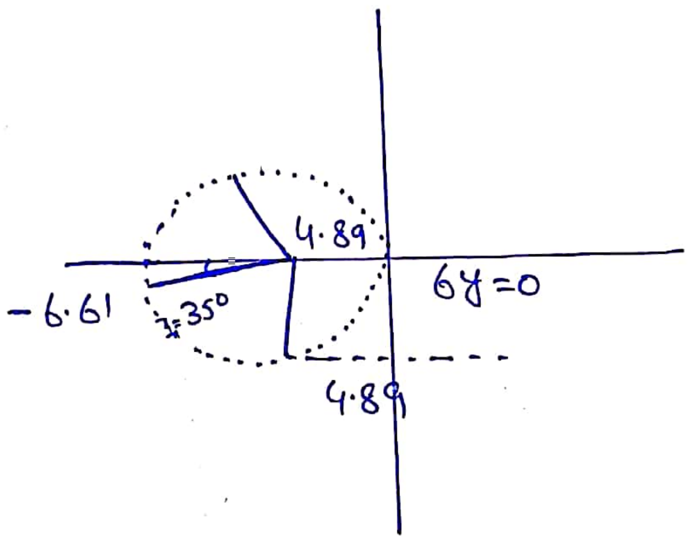
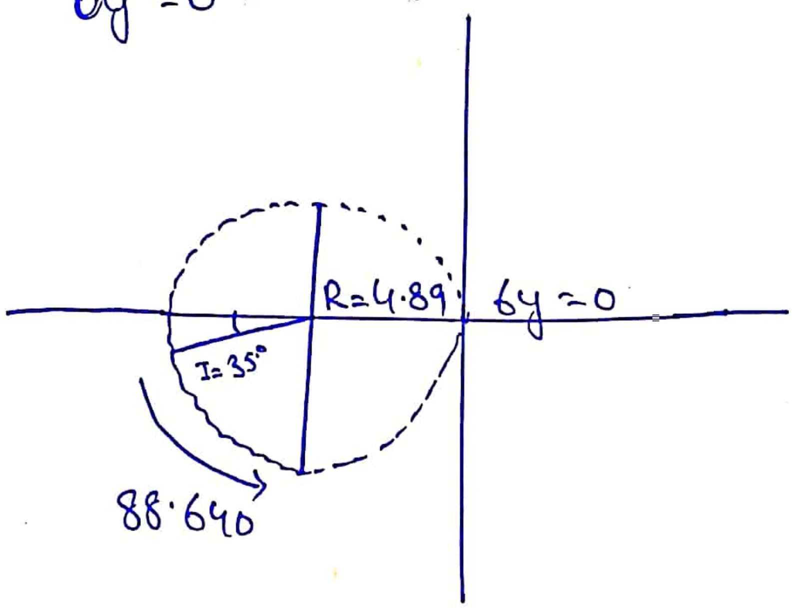
$V = 6.61\,psi$

~~Scal~~. Scale :- $2\,psi = 1\,cm$



for principle stress:-

$\sigma x = -6.61$
$\sigma y' = 0$



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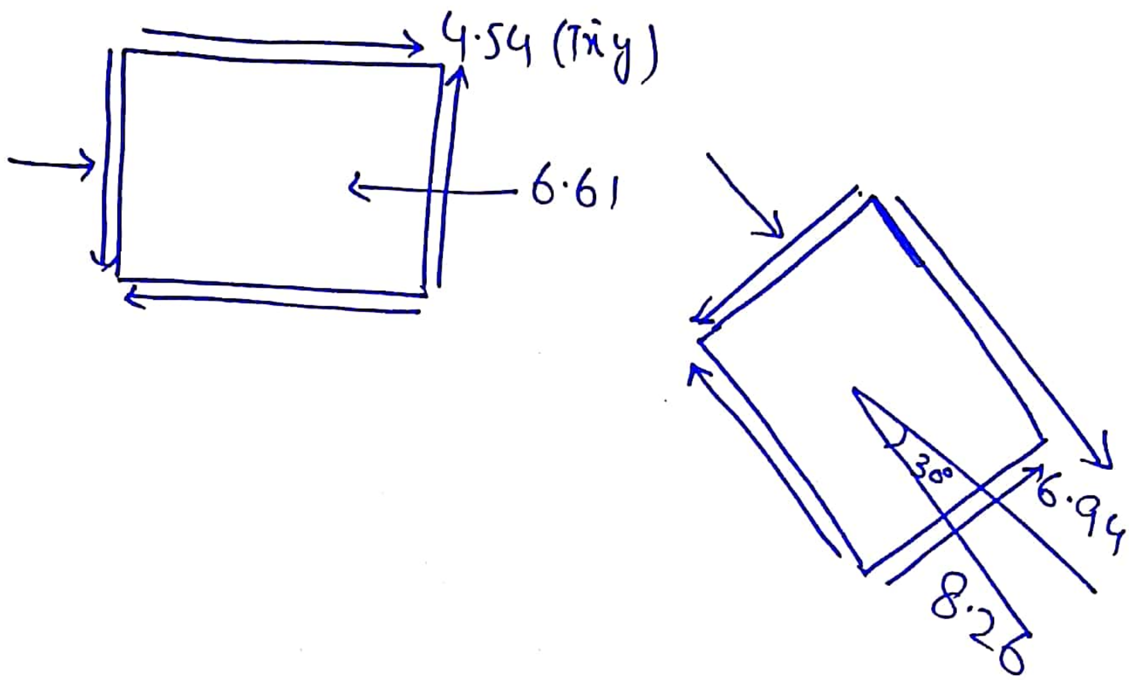
comparison of mohr's circle with shear stress transformation



Principle and plan Shear Stress