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Subject Differential Equation

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Q1

1) The order of Matrix AB is m×n.

2) The number of non zero rows in on Echelon form is "Rank of Matrix"

3) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is Singular Matrix

Then "$a = 8$"

4) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ $|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$

$|A| = -2i - i^2 \Rightarrow = (-2(-1) - (-1))$

$= 2+1 =$ "3 Ans"

5) $A = 9\times9 - 0 = 81 - 0$ "Non singular matrix"

OR The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is "Scaler Matrix" becuss diagnol element are same and non diagnal are zero.

(6) $dy/dx + 2xy = y \Rightarrow dy + 2xy\,dx = y\,dx$ (Non Separable)

7) The order and degree of different equation, $\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$ is

"order = 1"

Degree = 3

8) The order and degree of equation $\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$ is

"order 2"

Degree 1

(x) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ Expend by C1

$1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - 1\begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1\begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix}$

$1(bc^2 - cb^2) - 1(ac^2 + ca^2) + 1(ab^2 - a^2b)$

$bc^2 - cb^2 - ac^2 - a^2c + ab^2 - a^2b$

$a^2c - a^2b + ab^2 - cb^2 - ac^2 - bc^2$

$a^2(c-b) + b^2(a-c) + c^2(b-a)$ Ans

(1x) The differental eauation $2\frac{dy}{dx} + x^2y = 2x+3$ $y = (0) = 5$ is

$2\frac{dy}{dx} + x^2y = 2x+3$

$2y' + x^2 = x^2+3)$ $y(0) = 5$

$y' + \left(\frac{x^2}{2}\right)y = \frac{1}{2}(x^2+3)$

$u = \frac{x^2}{2}$

$e^{\int \frac{x^2}{2}dx} = e^{x^3/6}$

$e^{x^3/6}y' + e^{x^3/6}\left(\frac{x^2}{2}\right)y = \frac{1}{2}e^{x^3/6}(x^2+3)$

$y(x) = \frac{e^{x^3/6}x^2 + 3e^{x^3/6} + c}{2e^{x^3/6}}$

$y(0) = \frac{0+3}{2} = 3/2$

$y(x) = \frac{e^{x^3/6}x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + \frac{3}{2}$ Ans

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(vi) Solution of $\frac{dy}{dx}+2xy=y$ = ,

Solution:-

$$\frac{dy}{dx}+2xy=y$$

$$\frac{dy}{dx}=y-2xy$$

Taking y comon $y(1-2x)$

$$\frac{dy}{y}=(1-2x)\,dx$$

Take integration.

$$\int\frac{1}{y}\,dy=\int 1-2x\,dx$$

$$\ln y=\int 1\,dx-\int 2x\,dx$$

$$\ln y=x-x^2+c$$

$$\ln y \quad x-x^2+c$$

$$e^{\ln y}=e^{x}-x^2+c$$

$$\boxed{y=e^{x(1-x)+c}}$$ Ans

Q#2 (A)

Express the Determinante.

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$ as the product of factor's which are linear a, b, c..

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Solution:-

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$a \begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b \begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c \begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a\,(b^2c^3 - b^3c^2) - b\,[a^2c^3 - a^3c^2) + c\,(a^2b^3 - a^3b^2)$$

$$+ c\,(a^2b^3 - a^3b^2)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2cb^2 - a^3b^2c$$

Common (abc)

$$\Rightarrow abc\,(bc^2 - b^2c - ac^2 + ab - a^2b)$$

$$abc\,[bc\,(c-b) - ac(c+a) + ab\,(b-a)]$$

Q2 (B)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Solution

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Characteristic eqn → $|A-\lambda I| = 0$ —(A)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinate.

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expend by R_1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \quad \text{—(B)}$$

Again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ Expend by R_1

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\left[\left((3-\lambda)(2-\lambda)-(-1)(-1)\right) + \left((-1)(2-\lambda)-(-1)(-1)\right) - 1\left((-1)(-1)-(-1)(3-\lambda)\right)\right]$$

$$= (3-\lambda)(\lambda^2 - 5\lambda + 5) + (-3+\lambda) - (4-\lambda)$$

$$\left(-\lambda^3 + 8\lambda^2 - 18\lambda + 8\right) \text{ — (a)}$$

$$\Rightarrow \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$-1(6-3\lambda - 2\lambda + \lambda^2 - 1) + 1(-2+\lambda-1)$$

$$= -\lambda^2 + 5\lambda - 5 - 3 + \lambda$$

$$\boxed{-\lambda^2 + 6\lambda - 8} \text{ — (b)}$$

$\Rightarrow$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} \quad \text{Expand by } C_1$$

$$= [-1] \left[\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$= \left[(-2+\lambda-1) + 1(6-3\lambda-2\lambda+\lambda^2-1)\right]$$

$\Rightarrow$

$= -(3 - \lambda + \lambda^2 - 5\lambda + 5)$

$= -\lambda^2 + 5\lambda - 5 - 3 + \lambda$

$\boxed{-\lambda^2 + 6\lambda - 8} \longrightarrow$ c

put eq a b and c in B

$(2-\lambda)[-\lambda + 8\lambda^2 - 16\lambda + 8] - \lambda^2 + 6\lambda - 8 - \lambda^2 + 6\lambda - 8$

$= -2\lambda^3 + 16\lambda^2 - 36\lambda + 16 + \lambda^4 - 8\lambda^3 + 18\lambda^2 - 8\lambda - \lambda^2 + 6\lambda - 8$

$- \lambda^2 + 16\lambda - 8$

$= \lambda^4 - 2\lambda^3 - 8\lambda^3 + 16\lambda^2 + 16\lambda^2 - \lambda^2 - \lambda^2 - 36\lambda - 8\lambda + 6\lambda + 16\lambda + 16 - 16$

$\Rightarrow \lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0$

By synthatic division

We get

$\lambda(\lambda-2)(\lambda^2 - 8\lambda + 16) = 0$

$\lambda = 0$

$\lambda - 2 = 0 \Rightarrow \boxed{\lambda = 2}$

$\lambda^2 - 8\lambda + 16$

By Factorization Method.

- $\lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\lambda(\lambda - 4) - 4(\lambda - 4) = 0$

$(\lambda - 4)(\lambda - 4)$

$\lambda = 4 \quad , \quad \lambda = 4$

$\boxed{\lambda_1 = 0, \ \lambda_2 = 2, \ \lambda_3 = 4, \ \lambda_4 = 4}$ Ans

Q#3 $(x^2+3y^2)\,dx - 2xy\,dy = 0$

$x=2$, $y=6$

Solution

$(x^2+3y^2)dx - 2xy + dy = 0$

$\Rightarrow (x^2+3y^2)dx = 2xy\,dy$

Dividing both side by $2xy\,dy$

We get

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^{\cancel{2}}}{2\cancel{x}y} + \frac{3y^{\cancel{2}}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \quad \text{—— (A)}$$

Let $y = vx$

Dift $dy = v\,dx + x\,dv$

$\div$ing by dx

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{—— (1)}$$

Put eq (1) in eq (A)

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{\cancel{x}v} + 3\frac{v\cancel{x}}{\cancel{x}}\right]$$

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiplaying bath side by 2

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

xing bath sides by $\frac{dx}{dv}$
we get

$$2x\,dx = \frac{1+v^2}{v}\,dx$$

xing bath sides by $\frac{v}{x(1+v^2)}$

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Take "$\int$" on bath sides

$$\int\frac{2v}{1+v^2}\,dv = \int\frac{1}{x}\,dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

Take "e" on bath sides.

$$e^{\ln|1+v^2|} = e^{\ln|xc|}$$

$$1+v^2 = xc$$

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put $v = y/x$

$1 + (y/x)^2 = xc$

$x^2 + y^2 = x^3 c$ —— (ii)

Put $x = 2$ $y = 6$ eq (ii)

$4 + 36 = 8c$

$c = \frac{40}{8}$

$\boxed{c = 5}$ Put in (ii)

$x^2 + y^2 = 5x^3$

$y^2 = 5x^3 - x^2$

$y^2 = x^2(5x - 1)$

Taking sauear root B.S

$y = \pm x\sqrt{5x-1}$ Ans

OR

$y = +x\sqrt{5x-1}$, $y = -x\sqrt{5x-1}$