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DEPARTMENT    **SOFTWARE ENGINEERING**

SECTION         B

PAPER           LINEAR ALGEBRA

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Q. Answer No 1:

$$\begin{bmatrix} 1 & 103 & 3 & 0 & 5 \\ 0 & 1 & -103 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & -103 \end{bmatrix}$$

Solution:

$$103 = 2$$

$$-103 = -4$$

$$\begin{bmatrix} 1 & 2 & 3 & 0 & 5 \\ 0 & 1 & -4 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Multiplying Row three by 4  
and then add to  
Row two.

$$\begin{bmatrix} 1 & 2 & 3 & 0 & 5 \\ 0 & 1 & -4+4 & 0 & 7-24 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \quad R_2 + 4R_3$$

P# 2

$$\begin{bmatrix} 1 & 2 & 3 & 0 & 5 \\ 0 & 1 & 0 & 0 & -17 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Now multiplying Row two  
by "-2" and then  
add to Row one.

$$\begin{bmatrix} 1 & 2-2 & 3 & 0 & 5+34 \\ 0 & 1 & 0 & 0 & -17 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \quad R_1 \leftarrow 2R_2$$

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 39 \\ 0 & 1 & 0 & 0 & -17 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Now multiplying Row three  
by "-3" and then  
add to Row one.

P # 3

$$\left[ \begin{array}{ccccc|c} 1 & 0 & 3-3 & 0 & 39+18 & \\ 0 & 1 & 0 & 0 & -17 & \\ 0 & 0 & 1 & 0 & -6 & \\ 0 & 0 & 0 & 1 & 2 & \end{array} \right] \quad R_1 - 3R_3$$

$$\left[ \begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 57 & \\ 0 & 1 & 0 & 0 & -17 & \\ 0 & 0 & 1 & 0 & -6 & \\ 0 & 0 & 0 & 1 & 2 & \end{array} \right]$$

So this is the final

~~near~~ Augmented matrix

$$n_1 = 57 \quad n_2 = -17 \quad n_3 = -6 \quad n_4 = 2$$

Verification:

$$n_1 + 2n_2 + 3n_3 = 5$$

putting values.

$$57 + 2(-17) + 3(-6) = 5$$

$$57 - 34 - 18 = 5$$

$$57 - 52 = 5$$

$$5 = 5$$

→ true

Now

P# 4

$$2x_2 - 4x_3 = 7$$

$$-17 - 4(-6) = 7$$

$$-17 + 24 = 7$$

$$7 = 7 \rightarrow \text{true}$$

∴

Answer No 2:

Part A:

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Solution:

P # 5

first into second :

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

multiply Row two by  
"-2" and then add  
to Row three

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2-2 & -5+8 & -1+4 \end{bmatrix} \quad R_3 - 2R_2$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

So this is matrix two.

P# 6

second matrix into first.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

multiply Row two by "2"  
and then add to  
Row one.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0+2 & -8 & 4-5 \end{bmatrix} \quad R_3 + 2R_2$$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

So this is the first  
matrix.

P# 7

Answer No 2

Part B:

$$(a) \begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$$

Solution:

it is in echelon form because it satisfies all the following conditions:

(1) All the entries in a column below a leading entry are zero.

(2) Each leading entry of a row is in a column to the right of the leading entry of the above row.

(3) To satisfy the 3rd condition there is no



zero-row which should be  
below the all non-zero  
rows.

$$(b) \begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

it is in reduced  
echelon form because it  
is already in echelon  
form and satisfy the  
further two conditions.

(1) All the leading entries  
in non-zero rows are  
1.

(2) Each leading 1 is  
the only non-zero  
entry in its column.

$$(c) \begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

P. # 9

Answer C:

it is in echelon form because it satisfies all the following conditions-

(1) All the entries in a column below a leading entry are zero.

(2) Each leading entry of a row is in column to the right of the leading entry of the row above.

(3) To satisfy the 3rd condition there is no zero-row which should be below the all non-zero rows.

$$(d) \begin{pmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

P# 10

it is neither in  
echelon form nor in  
reduced echelon form  
because it doesn't  
satisfy the following  
condition.

(1) All the zero rows  
are below the non-zero  
rows.



Answer No 3

Part A

The difference between  
echelon form and reduced  
echelon form are given  
below.

Echelon form :

A rectangular  
matrix is in echelon  
(or row echelon form) if  
it has the following.

P # 11

three properties.

(1) All non-zero rows of  
above any zeros. are  
all rows of

(2) Each leading entry of  
a row is in a  
column to the right  
of the leading entry  
of the row above  
it.

(3) All entries in a  
column below a leading  
entry are zeros.

## Reduced Echelon Form:

if a matrix in  
echelon form satisfies  
the following additional  
conditions, then it is  
in reduced echelon  
form (or reduced row  
echelon form).

# P # 12

(1) The leading entry in each non-zero row is 1.

(2) Each leading 1 is the only non-zero entry in its column.

A

Answer No 3:

Part B:

$$\begin{bmatrix} 1 & 102 & 8 \\ 2 & 8 & -1 \\ -103 & 0 & 0 \\ 1 & -4 & 10 \text{ first last} \end{bmatrix}$$

Solution:

# P# 13

$$ID_0 = 16284$$

$$ID_2 = 8$$

$$-ID_3 = -2$$

$$ID_{\text{first}} - \text{last} = 14$$

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ -2 & 0 & 0 \\ 1 & -4 & 14 \end{bmatrix}$$

Multiply Row four by  
"2" and subtract Row  
two from Row four.

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ -2 & 0 & 0 \\ 0 & -16 & 29 \end{bmatrix} \quad 2R_4 - R_2$$

Now add Row ~~two~~ three  
to Row three.

P #14

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ 0 & 8 & -1 \\ 0 & -16 & 29 \end{bmatrix} \quad R_3 + R_2$$

Now multiply Row three  
by two "2" and  
then add to Row 4.

$$\begin{bmatrix} 1 & 6 & 8 \\ 2 & 8 & -1 \\ 0 & 8 & -1 \\ 0 & 0 & 27 \end{bmatrix} \quad R_{04} + 2R_3$$

Now subtract Row two  
from Row three.

$$\begin{bmatrix} 1 & 8 & 8 \\ 2 & 8 & -1 \\ -2 & 0 & 0 \\ 0 & 0 & 27 \end{bmatrix} \quad R_3 - R_2$$

Now add Row three  
to Row two.

P # 15

$$\begin{bmatrix} 1 & 8 & 8 \\ 0 & 8 & -1 \\ -2 & 0 & 0 \\ 0 & 0 & 27 \end{bmatrix}$$

Now multiply Row ~~two~~ two  
by  $(-3)$  and then subtract  
from 4 times of  
Row one.

$$\begin{bmatrix} 1 & 0 & 35 \\ 0 & 8 & -1 \\ -2 & 0 & 0 \\ 0 & 0 & 27 \end{bmatrix} \quad 4R_1 - 3R_2$$

Now multiply Row one  
by "2" and then  
add to Row three.

$$\begin{bmatrix} 1 & 0 & 35 \\ 0 & 8 & -1 \\ 0 & 0 & 70 \\ 0 & 0 & 27 \end{bmatrix} \quad R_3 + 2R_1$$



# P# Last.

Now multiply Row three by  
"2" and then add  
to 5 times of Row "4".

$$\begin{bmatrix} 1 & 0 & 25 \\ 0 & 8 & -1 \\ 0 & 0 & 70 \\ 0 & 0 & -5 \end{bmatrix} \quad 5R_4 - 2R_3$$

Now Multiply Row four  
by "14" and then add  
to Row three.

$$\begin{bmatrix} 1 & 0 & 25 \\ 0 & 8 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -5 \end{bmatrix} \quad R_3 + 14R_4$$

Now interchange  $R_3$  into  $R_4$ .

$$\begin{bmatrix} 1 & 0 & 25 \\ 0 & 8 & -1 \\ 0 & 0 & -5 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 \leftrightarrow R_4$$

this is required echelon  
form

(End)