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Subject : Theory of structure II

Exam : Summer 2020

Semester : 8th B-tech

Instructor : Engr. Humaira Arshad

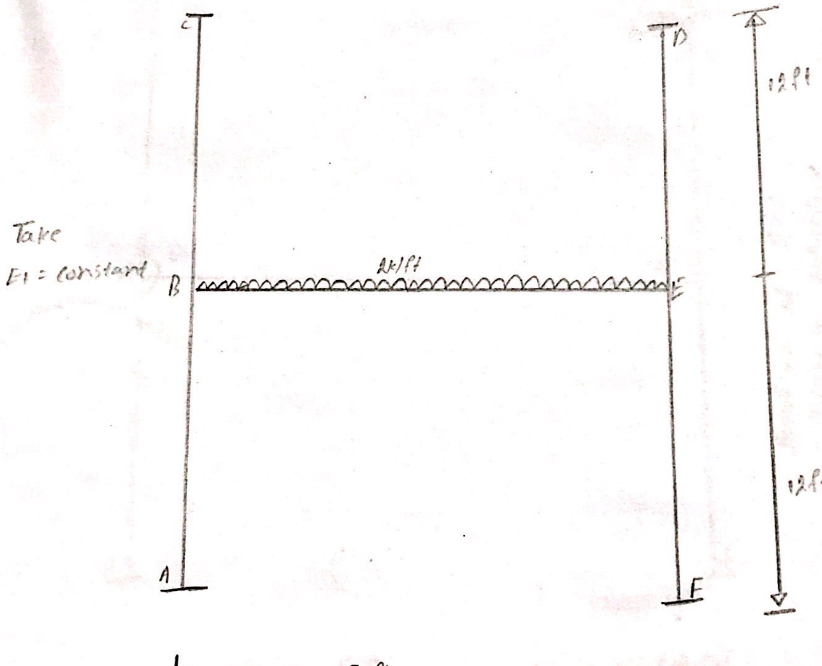
(page # 01)

Q's: Analyze the given frame using stiffness method.

Take:

EI = constant

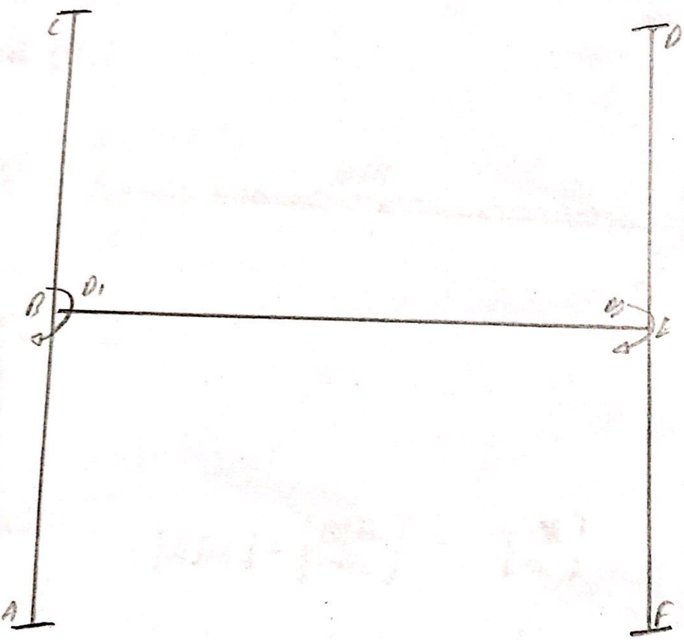
$k \cdot I = 2$ degree (neglecting the axil effect).



$k \cdot I = 2$ degree (neglecting the axial effects)

Solution: (#02)

* Step #01: Selection of redundant joint displacements and assign coordinates at those locations. Also compute AD values.

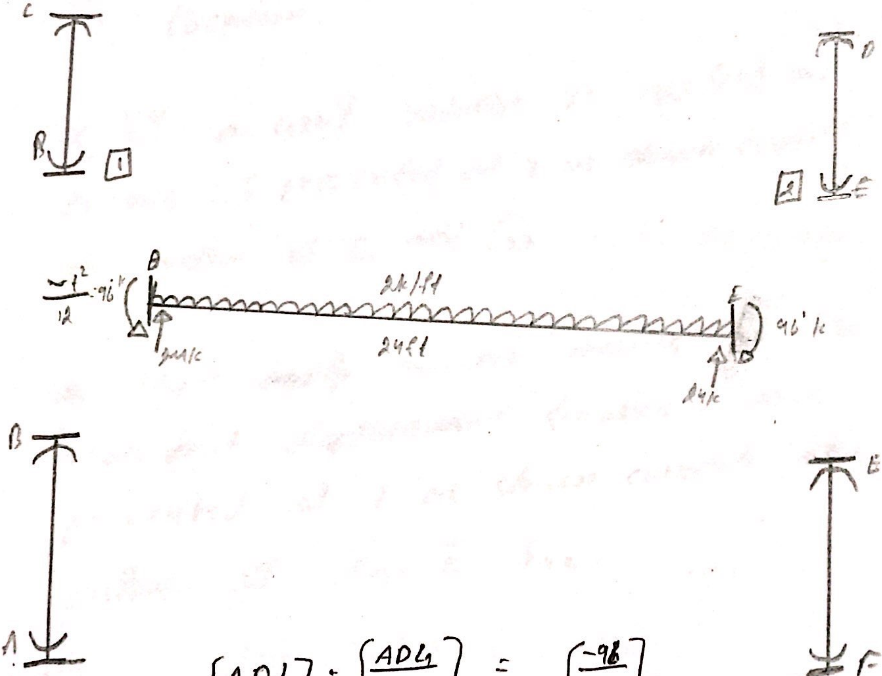


Rotation at B & C is taken as redundant joint displacement.

$$[D]_{2,1} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \qquad [AD]_{2,1} = \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(#03)

step#02 : Compute ADL matrix (fixed end actions)



$$[ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} = \begin{bmatrix} -96 \\ 96 \end{bmatrix}$$

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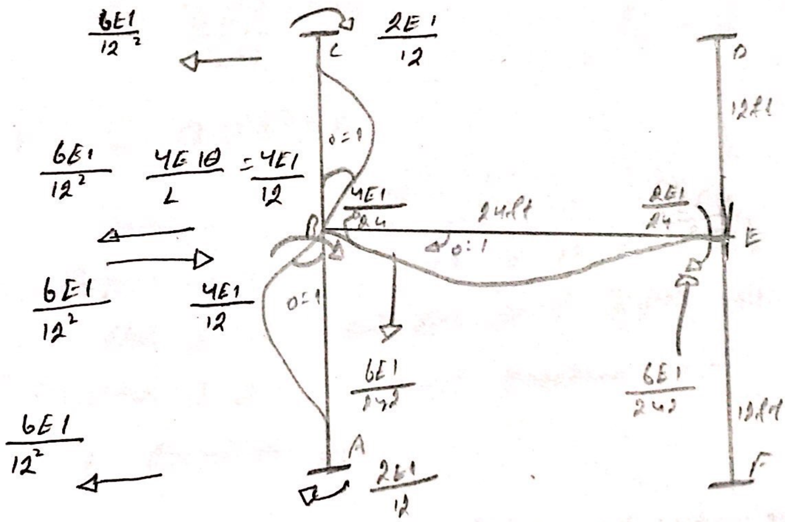
step#03: Primary structure acted upon by a unit value of D & computation of stiffness coefficients "S" values in the BKDS corresponding to the redundant joint displacement location

* Ist a unit rotation is applied at location 1 & prevented at 2 as shown compute the value of S_{11} and S_{21}

* Then apply a unit rotation at the redundant displacement location 2 and prevented at 1 as shown. compute the values of S_{12} & S_{22}

(#05)

i. 1^{st} $D_1 = 1$ & $D_2 = 0$ & compute the value of S_{11} and S_{21}



step # 03 (i) contd... (#06)

$$S_{11} = \frac{4EI}{12} + \frac{4EI}{12} + \frac{4EI}{24}$$

$$S_{11} = 0.833EI$$

$$S_{21} = \frac{2EI}{24}$$

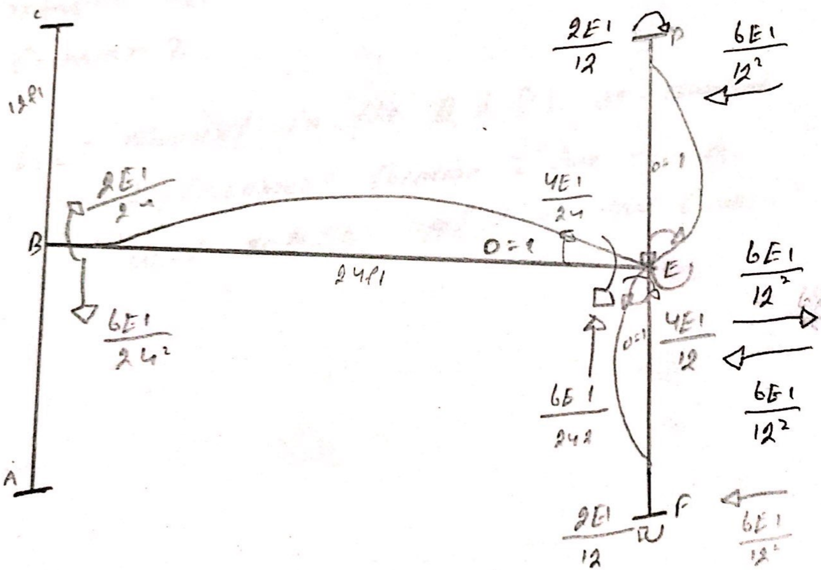
$$S_{21} = 0.0833\,EI$$

S_{11} = Action (sum of moments in this case) in the BKDS at redundant displacement location 1 due to unit rotation at that location.

S_{21} = Moment in the BDS at redundant displacement location 2 due to a unit rotation applied at location 1.

(#07)

ii. Now $D_2 = 1$ & $D_1 = 0$ a shown, computer the values of S_{12} & S_{22}.



Step # 03 : Contt... (#08)

$$S_{12} = \frac{2EI}{24}$$

$$S_{22} = \frac{4EI}{24} + \frac{4EI}{12} + \frac{4EI}{12}$$

$$S_{12} = 0.083EI$$

$$S_{22} = 0.833EI$$

S_{12} = Moment in the BKDS at redundant displacement location 1 due to unit rotation applied at redundant displacement location 2.

S_{22} = Moment in the BKDS at redundant displacement location 2 due to a unit rotation applied at that location.

step # 03 : contd.... (# 09)

$S_{11} = 0.833EI$ $S_{12} = 0.0833EI$

$S_{12} = 0.0833EI$ $S_{22} = 0.833EI$

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

$$[S] = EI \begin{bmatrix} 0.833 & 0.0833 \\ 0.0833 & 0.833 \end{bmatrix}$$

stiffness coefficient matrix.

Step #04: Apply equilibrium condition at the location of the redundant joint displacement to write equilibrium equations and solve for unknown joint displacement

$$AD_1 = ADL_1 + S_{11} D_1 + S_{12} D_2$$

$$AD_2 = ADL_2 + S_{12} D_1 + S_{22} D_2$$

$$\begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} + \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$[AD]_{2\times1} = [ADL]_{2\times1} + [S]_{2\times2} \cdot [D]_{2\times1}$$

$$[D] = [S]^{-1} \cdot [AD - ADL]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix}^{-1} \begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix}$$

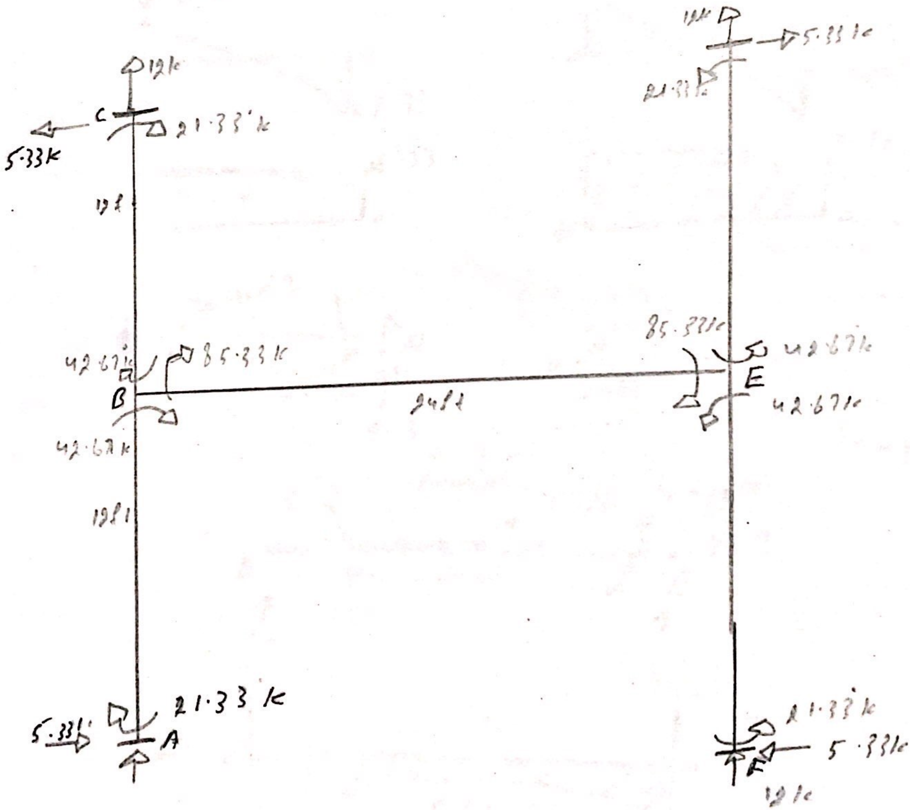
$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.833 & 0.0833 \\ 0.0833 & 0.833 \end{bmatrix}^{-1} \begin{bmatrix} 0-(-96) \\ 0-96 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 128 \\ -128 \end{bmatrix} \frac{1}{EI}$$

-ive sign shows that our assumed redundant joint displacement direction is wrong.

(# 11)

Step # 05 : Computer the member end action as you will get.



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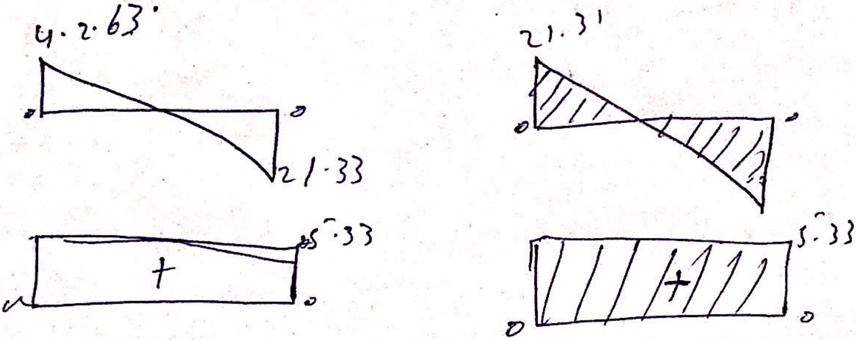
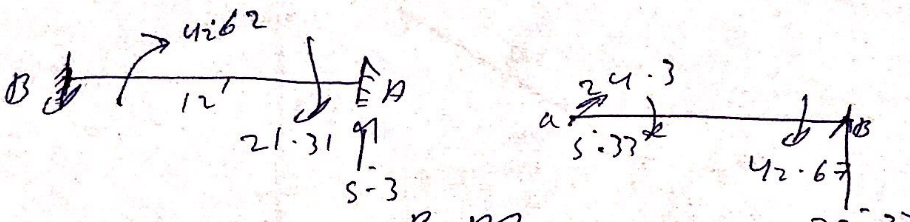
(# 12)

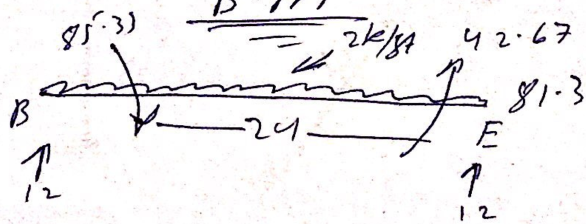
step #6 : Draw shear force & bending moment diagrams

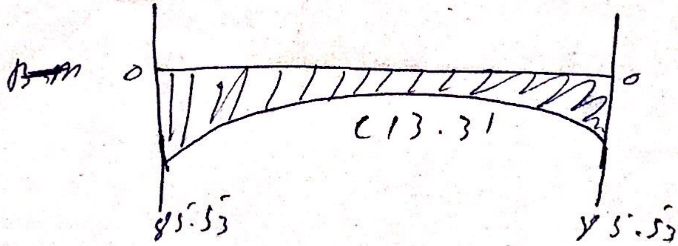
* shear force





B.M





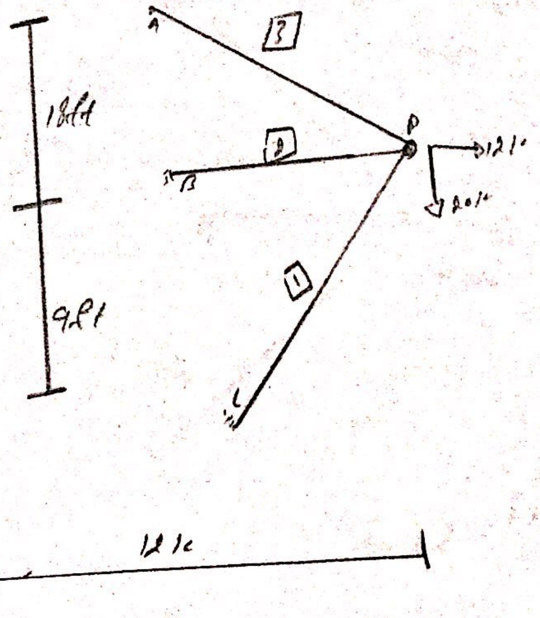
Q3: Analyze the given pin jointed frame using stiffnes method.

Take EA = constant

$L1 = 15 ft$

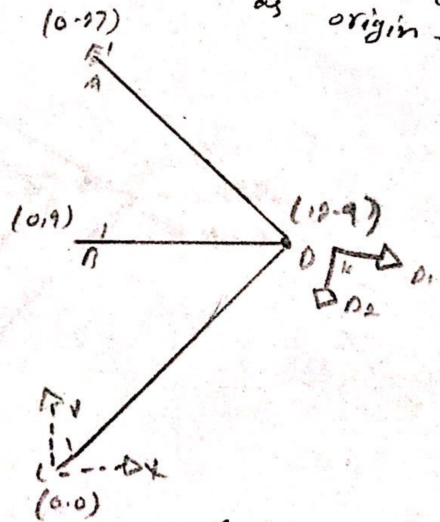
$L_2 = 12 ft$

$L_3 = 21.63 ft.$



Step # 02 : Identify the unknown joint displacement and computes the values of [AD] matrix.

Chose one point as an coordinates to each joint w.r.t the chosen origin Here point C is taken as origin.

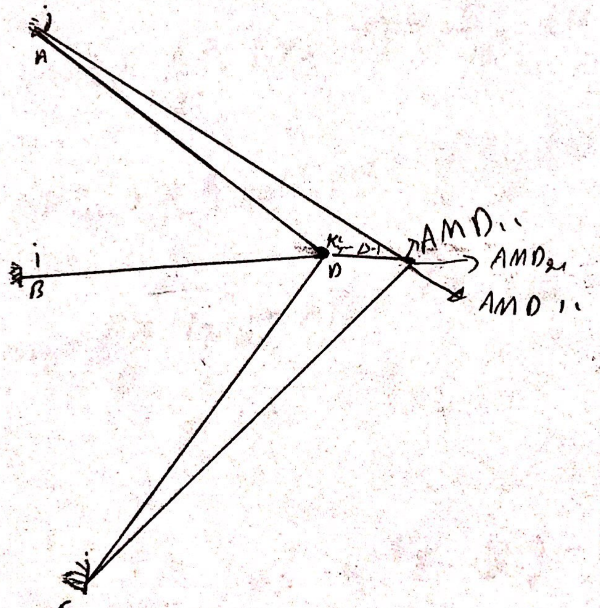


$$\{D\}_{2\cdot 1} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[AD]_{2\cdot 1} = \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 10 \end{bmatrix}$$

Step # 02 : Computation of AMD and stiffness matrices.

i. When $D_1 = 1$ & $D_2 = 0$.



compute the values of AMD & then stiffness coefficients.

S_{11} will be the sum of all horizontal components of AMD. S_{21} will be the sum of all vertical components of AMD.

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step #02: Computation of AMD and stiffness matrices.

1 when $D_1 = 1$ and $D_2 = 0$

$$AMD_{11} = \frac{EA}{L^2}(x_k - x) = \frac{EA}{15^2}(12-0) = 0.0533EA$$

$$AMD_{21} = \frac{EA}{L^2}(x_k - x) = \frac{EA}{12^2}(12-0) = 0.0833EA$$

$$AMD_{31} = \frac{EA}{L^2}(x_k - x) = \frac{EA}{21.63^2}(12-0) = 0.0256EA$$

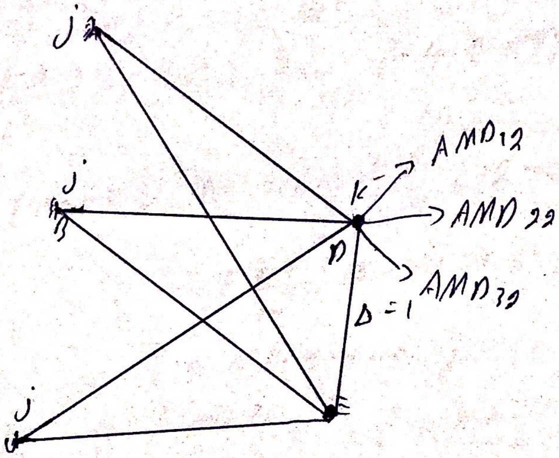
$$S_{11} = \frac{EA}{L^3}(x_k - x_j)^2 = \frac{EA}{15^3}(12-0)^2 + \frac{EA}{12^3}(12-0)^2 + \frac{EA}{21.63^3}(12-0)^2$$

$$= 0.1402EA$$

$$S_{21} = \frac{EA}{L^3}(x_k - x_j)(y_k - y_j) = \frac{EA}{15^3}(12-0)(-9-0) + \frac{EA}{12^3}(12-0)(-9-(-9)) + \frac{EA}{21.633^3}(12-0)(-9-(-27)) = -0.0107EA.$$

Step # 02: Computation of AMD and Stiffness matrix

ii. when $D_2 = 1$ & $D_1 = 0$.



Compute the values of AMD & then stiffness coefficients

S_{12} will be the sum of all horizontal components & S_{22} will be the sum of all vertical components of AMD.

(18)

Step # 02: Computation of AMD and stiffness matrices.

(ii) when $D_2 = 1$ & $D_1 = 0$.

$$AMD_{12} = \frac{EA}{L^2}(y_k - y_j) = \frac{EA}{15^2}(-9+0) = -0.04EA$$

$$AMD_{22} = \frac{EA}{L^2}(y_k - y) = \frac{EA}{12^2}(-9-(-9)) = 0$$

$$AMD_{32} = \frac{EA}{L^2}(y_k - y_j) = \frac{EA}{21.63^2}(-9-(-27)) = 0.0385EA$$

$$S_{12} = \frac{EA}{L^3}(x_k - x_j)(y_k - y_j) = \frac{EA}{15^3}(12-0)(-9-0)$$
$$+ \frac{EA}{12^3}(12-0)(-9-(-9)) + \frac{EA}{21.63^3}$$
$$(12-0)(-9-(-27)) = -0.0107EA$$

$$S_{22} = \frac{EA}{L^3}(y_k - y_j)^2 = \frac{EA}{15^3}(-9-0)^2 + \frac{EA}{12^3}(-9+9)^2$$
$$+ \frac{EA}{21.63}(-9-27)^2 = 0.056EA.$$

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Step # 02: computation of AMD and stiffness matrices

AMD matrix will be.

$AMD_{11} = 0.0533 EA$ $\quad$ $AMD_{21} = 0.0833 EA$

$AMD_{12} = -0.04 EA$ $\quad$ $AMD_{22} = 0$

$AMD_{31} = 0.0256 EA$

$AMD_{32} = 0.0385 EA$

$$[AMD] = EA \begin{bmatrix} 0.0533 & 0.04 \\ 0.0833 & 0 \\ 0.0256 & 0.0385 \end{bmatrix}$$

stiffness matrix will be

$S_{11} = 0.1402 EA$ $\quad$ $S_{21} - 0.0107 EA$ $\quad$ $S_{12} = -0.0107 EA$

$S_{22} = 0.056 EA$

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \qquad [S] = EA \begin{bmatrix} 0.1402 & -0.0107 \\ -0.0107 & 0.056 \end{bmatrix}$$

step #03: Apply equilibrium condition at location of the redundant joint displacement to write equilibrium equations and solve for unknown joint displacement.

$$AD_1 = S_{11} D_1 + S_{12} D_2$$

$$AD_2 = S_{21} D_1 + S_{22} D_2$$

$$\begin{bmatrix} AD1 \\ AD2 \end{bmatrix} = \begin{bmatrix} S11 & S12 \\ S21 & S22 \end{bmatrix} \begin{bmatrix} D1 \\ D2 \end{bmatrix}$$

$$[AD]_{2\times 1} = [S]_{2\times 2} \cdot [D]_{2\times 1}$$

$$\{D\} = [S]^{-1} \cdot [AD].$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EA} \begin{bmatrix} 0.1402 & -0.0107 \\ -0.0107 & 0.056 \end{bmatrix}^{-1} \begin{bmatrix} 12 \\ 20 \end{bmatrix}$$

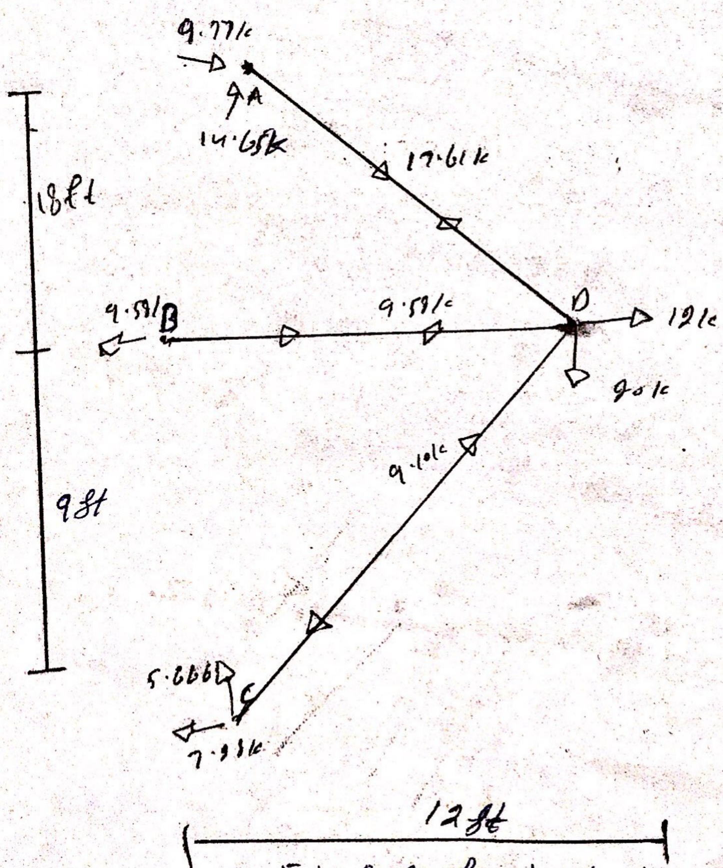
$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EA} \begin{bmatrix} 115.065 \\ 380.83 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \end{bmatrix} = \begin{bmatrix} AMD_{11} & AMD_{12} \\ AMD_{21} & AMD_{22} \\ AMD_{31} & AMD_{32} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \end{bmatrix} = EA \begin{bmatrix} 0.0533 & -0.04 \\ 0.0833 & 0 \\ 0.0256 & 0.0385 \end{bmatrix}$$

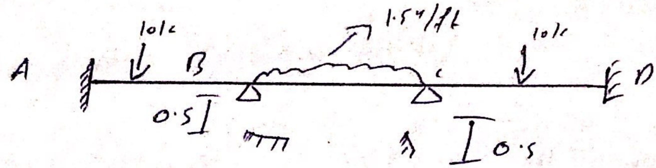
$$\begin{bmatrix} 115.065 \\ 380.83 \end{bmatrix} \frac{1}{EA} = \begin{bmatrix} -9.10k \\ 9.58k \\ 17.61k \end{bmatrix}.$$

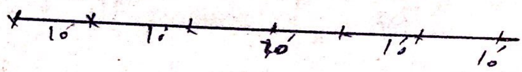
Stiffness Method for Truss Analysis.



Final Analyzed structure.

Q #3 page # 23





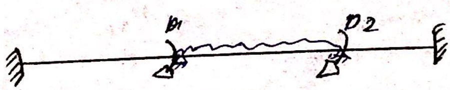
$E = 30{,}000\ ksi$ $I = 200\ in^4$

$EI = 41666.7\ k\,ft^2$

$\Delta = 1/2'' = 1/24'$

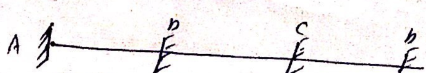
Step # 01

selection of redundant joint displacements



$$[D] = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[AD] = \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



Basic kinematic Determinate structure.

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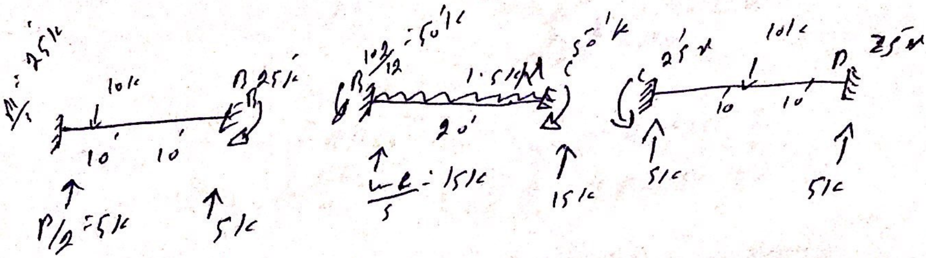
(#24)

step # 2

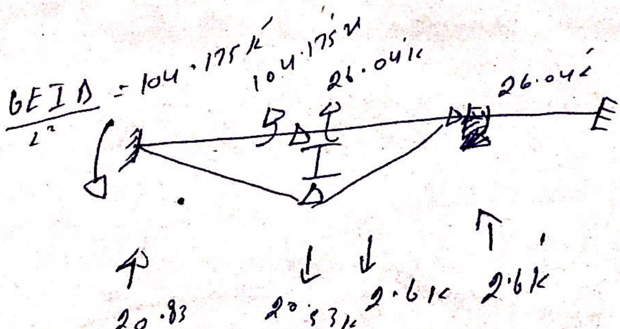
Compute ADL matrix

(i) Due to direct loading (ADL')



$$[ADL'] = \begin{bmatrix} ADL_1' \\ ADL_2' \end{bmatrix} = \begin{bmatrix} -25 \\ 25 \end{bmatrix}$$

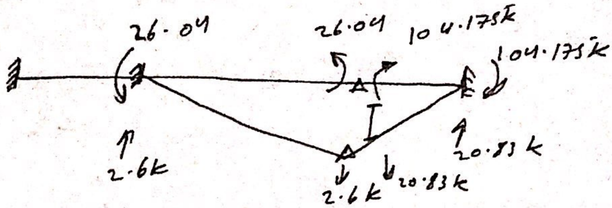
(ii) Due to settlement at joint B



$$[ADL''] = \begin{bmatrix} ADL_1'' \\ ADL_2'' \end{bmatrix} = \begin{bmatrix} -78.135 \\ 26.04 \end{bmatrix}$$

(25)

(ii) Due to settlement at joint C



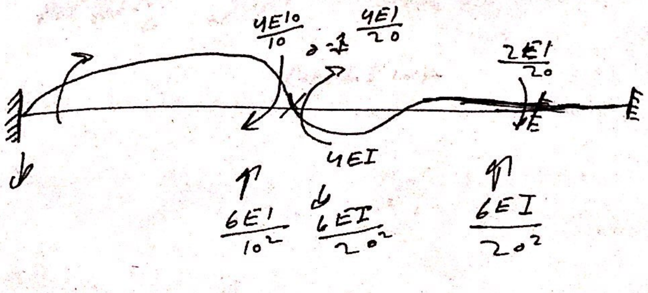
$$[ADC'''] = \begin{bmatrix} ADL_1''' \\ ADL_2''' \end{bmatrix} = \begin{bmatrix} -26.04 \\ 78.135 \end{bmatrix}$$

$$[ADL] = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} = \begin{bmatrix} ADL_1' + ADL_1'' + ADL_1''' \\ ADL_2' + ADL_2'' + ADL_2''' \end{bmatrix}$$

$$= \begin{bmatrix} -129.175 \\ +129.175 \end{bmatrix}$$

step#3 stiffness coefficients matrix [S]

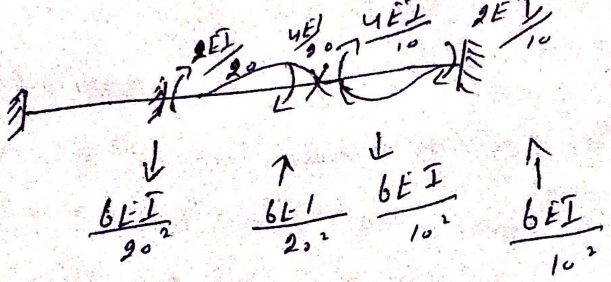
(i) $D_1 = 1$; $D_2 = 0$



$$S_{11} = \frac{4EI}{10} + \frac{4EI}{20} = \underline{3EI} = 0.6EI$$

$$S_{21} = \frac{2EI}{20} = 0.1EI$$

(ii) $D_2 = 1, \; D_1 = 0$



$S_{12} = 0.1EI$

$S_{22} = 0.6EI$

$$(S) = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = EI \begin{bmatrix} 0.6 & 0.1 \\ 0.1 & 0.6 \end{bmatrix}$$

step # 4: Applying joint equilibrium condition

$$\{AD\} = \{ADL\} + [S]\{D\}$$

or

$$\{D\} = [S]^{-1}\{AD - ADL\}.$$

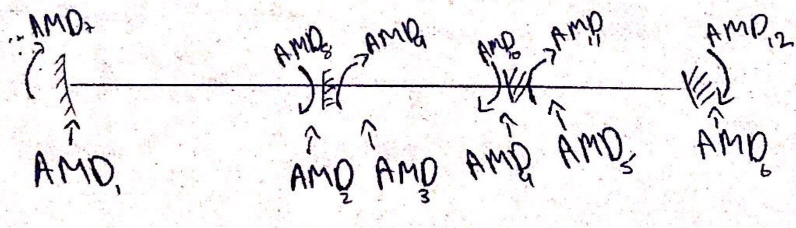
$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \rightarrow \frac{1}{EI}\begin{bmatrix} 1.7143 & -02852 \\ -0.2857 & 1.7143 \end{bmatrix} = \begin{bmatrix} 129.175 \\ -129.175 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 258.35 \\ -258.35 \end{bmatrix}$$

STep A 5(a) complete and values

(i) First apply unit rotaion at redundant location 1 and compute the member end action → we will take these values from step #3 (i).

(ii). Then apply unit rotation at redundant location 2 and compute the member and actions → we will take these values from step #3 (ii).



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$$AMD = \begin{bmatrix} AMD_{11} & AMD_{12} \\ AMD_{21} & AMD_{22} \\ 31 & AMD_{32} \\ 41 & 42 \\ 51 & 52 \\ 61 & 62 \\ 71 & 72 \\ 81 & 82 \\ 91 & 92 \\ 101 & 102 \\ 11\cdot1 & 11\cdot2 \\ 12\cdot1 & 12\cdot2 \end{bmatrix} = EI \begin{bmatrix} -0.06 & 0 \\ 0.06 & 0 \\ -0.015 & -0.015 \\ 0.015 & 0.015 \\ 0 & -0.06 \\ 0 & 0.06 \\ 0.2 & 0 \\ 0.4 & 0 \\ 0.2 & 0 \\ 0.1 & 0.2 \\ 0 & 0.4 \\ 0 & 0.2 \end{bmatrix}$$

(6) Compute AML Values

These are the member end actions when the actual load is applied on the structure. we will take from step # 2.

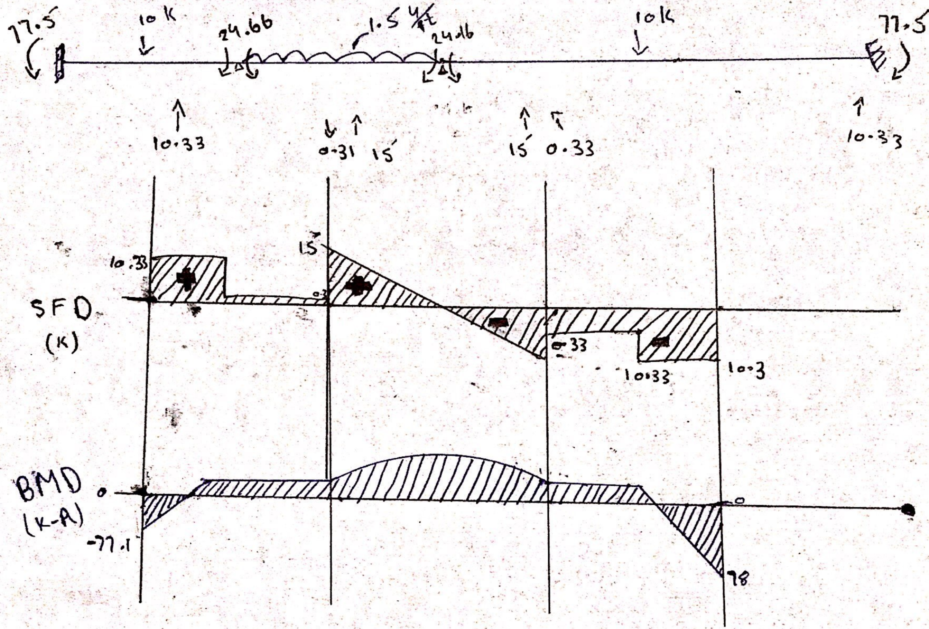
$$AML = \begin{bmatrix} AML_1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{bmatrix} = \begin{bmatrix} 25.83 \\ -15.83 \\ 15 \\ 15 \\ -15.83 \\ 25.83 \\ -129.175 \\ -79.125 \\ -50 \\ 50 \\ 79.175 \\ 129.175 \end{bmatrix}$$

Now member end action are calculated as

$$[AM] = [AML] + [AMD]\,[D]$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \\ AM_5 \\ AM_6 \\ AM_7 \\ AM_8 \\ AM_9 \\ AM_{10} \\ AM_{11} \\ AM_{12} \end{bmatrix} = \begin{bmatrix} 25.83 \\ -15.83 \\ 15 \\ 15 \\ -15.83 \\ 25.83 \\ -129.175 \\ -79.175 \\ -50 \\ 50 \\ +79.125 \\ +129.175 \end{bmatrix} + \begin{bmatrix} -0.06 & 0 \\ 0.06 & 0 \\ -0.015 & -0.015 \\ 0.015 & 0.015 \\ 0 & -0.06 \\ 0 & 0.06 \\ 0.2 & 0 \\ 0.4 & 0 \\ 0.2 & 0.1 \\ 0.1 & 0.2 \\ 0 & 0.4 \\ 0 & 0.2 \end{bmatrix} \begin{bmatrix} 258.35 \\ -258.35 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{bmatrix} = \begin{bmatrix} 10.33 \\ -0.33 \\ 15 \\ 15 \\ -0.33 \\ 10.33 \\ -77.5 \\ 24.16 \\ -24.16 \\ 24.16 \\ -24.16 \\ 77.5 \end{bmatrix}$$


77.5
10 K
24.66
1.5 k/ft
24.66
10k
77.5
10.33
0.31
15
15
0.33
10.33
SFD
(K)
10.33
15
0.33
10.33
10.3
BMD
(K-ft)
0
-77.1
0
78

The End.