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ASSIGNMENT :- 1 / Q-I

The E-field pattern of an ~~at~~ antenna, independent of φ varies as follows:

$$E = \begin{cases} 1 & 0^\circ \le \theta \le 45^\circ \\ 0 & 45^\circ < \theta \le 90^\circ \\ 1/2 & 90^\circ < \theta \le 180^\circ \end{cases}$$

(a) What is the directivity of this antenna?

(b) What is the radiation resistance of the antenna at 200m from it if the field is equal to 10 V/m (rms) for $\theta = 0^\circ$ at that distance & the terminal current is 5A (rms)?

(a) $U = \dfrac{r^2 E^2}{2\eta} = \dfrac{r^2 |E|^2}{\eta}$, $U_{max} = \dfrac{r^2}{\eta} = \dfrac{1}{120\pi}$

Contd)

$$P_{rad} = \frac{r^2}{\eta} \int_0^{2\pi} d\phi \left[\int_0^{45^\circ} \sin\theta \, d\theta + \int_{90^\circ}^{180^\circ} \frac{1}{4} \sin\theta \, d\theta \right]$$

$$= \frac{r^2}{\eta} [2\pi] \left[-\cos\theta \Big|_0^{45^\circ} + \frac{1}{4} (-\cos\theta) \Big|_{90^\circ}^{180^\circ} \right]$$

$$= \frac{2 r^2 \pi}{\eta} \left[-\cos 45^\circ + \cos 0^\circ - \frac{1}{4} \cos 180^\circ + \frac{1}{4} \cos 90^\circ \right]$$

$$P_{rad} = 0.54289 \, \frac{2\pi r^2}{\eta}$$

$$D = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi \, r^2/\eta}{0.54289 (2\pi) r^2/\eta} = 3.684$$

(b) When the field is equal to 10V/m, for $\theta = 0°$

$$\Rightarrow E = \begin{cases} 10\,V/m & 0 < \theta \le 45° \\ 0 & 45° < \theta \le 90° \\ 1/2 \times 10\,V/m & 90° < \theta \le 180° \end{cases}$$

$$P_{rad} = \frac{r^2}{\eta}\left[\int_0^{2\pi}\left\{\int_0^{45°} |E|^2 \sin\theta\, d\theta + \int_{90°}^{180°} |E|^2 \sin\theta\, d\theta\right\} d\phi\right]$$

$$P_{rad} = r^2 (0.54289)\left(\frac{2\pi}{\eta}\right)|10|^2 = 36,193$$

$$P_{rad} = \frac{1}{2}|I|\,R_r = |I_{rms}|^2 R_r$$

$$\Rightarrow R_r = \frac{36,193}{|I_{rms}|^2} = \frac{36,193}{25} = 1,447.72$$

ASSIGNMENT :- 2/Q.2

Transmitting and receiving antennas operating at 1GHz with gains (over isotropic) of 20 and 15dB, respectively, are separated by a distance of 1Km. Find the maximum power delivered to the load when the input power is 150W. Assume that the:

(a) antennas are polarization-matched.

(b) transmitting antenna is circularly polarized (either right or ~~lefet~~ left hand) and the receiving antenna is linearly polarized.

$$\frac{P_r}{P_t} = |\hat{P}_t \cdot \hat{P}_r|^2 \left(\frac{\lambda}{4\pi R}\right)^2 G_{ot} \cdot G_{or}$$

$G_{ot} = 20\,dB \Rightarrow G_{ot}$ (Power ratio) $= 10^2 = 100$

$G_{or} = 15\,dB \Rightarrow G_{or}$ (Power ratio) $= 10^{1.5} = 31.623$

$f = 1\,GHz \Rightarrow \lambda = 0.3$ meters.

$R = 1 \times 10^3$ meters

(a) For $|\hat{P}_t \cdot \hat{P}_r|^2 = 1$

$$P_r = \left(\frac{0.3}{4\pi \times 10^3}\right)^2 (100)(31.623)(150 \times 10^{-3}) = 270.344\ \mu\,\text{Watts}.$$

(b) When transmitting antennas is circularly polarized and receiving antenna is linearly polarized, the PLF is equal to $|\hat{P}_t \cdot \hat{P}_r|^2 = \left|\left(\frac{\hat{a}_x \mp j\hat{a}_y}{\sqrt{2}}\right) \cdot a_x\right|^2 = 1/2.$

Thus

$$P_r = \tfrac{1}{2}(270.344 \times 10^{-6}) = 135.172 \times 10^{-6} = 135.172\ \mu\,\text{Watts}$$

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ASSIGNMENT : 3 / Q.3

Three isotropic sources, with spacing d between them are placed along the z-axis. The excitation coefficient of each outside element is unity while that of the center element is 2. For a spacing of $d = \lambda/4$ between the element, find the:

(a) array factor

(b) angle (in degrees) where the nulls of the pattern occur ($0° \le \theta \le 180°$)

(c) angle (in degrees) where the maxima of the pattern occur ($0° \le \theta \le 180°$)

(a) $$E_t = E_1 + E_2 + E_3 = 2E_0 \frac{e^{-jkr}}{r} + E_0 \frac{e^{-jkr_1}}{r_1} + E_0 \frac{e^{-jkr_2}}{r_2}$$

where the center element is placed at the origin. For far-field observations.

$$\left.\begin{aligned} r_1 &\simeq r - d\cos\theta \\ r_2 &\simeq r + d\cos\theta \end{aligned}\right\} \text{ for phase variations}$$

$r_1 \simeq r_2 \simeq r$ for amplitude variations.

and

$$E_t = E_0 \frac{e^{-jkr}}{r}\left\{2 + e^{jkd\cos\theta} + e^{-jkd\cos\theta}\right\}$$

$$\simeq E_0 \frac{e^{-jkr}}{r}\left\{2\left[1 + \frac{1}{2}\left(e^{jkd\cos\theta} + e^{-jkd\cos\theta}\right)\right]\right\}$$

$$= E_0 \frac{e^{-jkr}}{r}\left\{2\left[1 + \cos(kd\cos\theta)\right]\right\}$$

Thus the array factor is equal to

$$AF(\theta) = 2\left[1 + \cos(kd\cos\theta)\right] = 4\cos^2\left(\frac{kd}{2}\cos\theta\right)$$

which in normalized form can also be written as

$$AF(\theta)_n = 1 + \cos(kd\cos\theta) = 2\cos^2\left(\frac{kd}{2}\cos\theta\right)$$

(b) The nulls of the pattern can be found using either of the above forms for the array factor. For example.

<u>one form</u>

$$AF(\theta) = 1 + \cos(kd\cos\theta_n) = 0$$

$$\cos(kd\cos\theta_n) = -1$$

$$kd\cos\theta_n = \cos^{-1}(-1) = n\pi, \; n = \pm 1, \pm 3 \ldots$$

$$\theta_n = \cos^{-1}(n\lambda/2d), \; n = \pm 1, \pm 3, \pm 5 \ldots$$

<u>the other form</u>

$$2\cos^2\left(\frac{kd}{2}\cos\theta_n\right) = 0$$

$$\frac{kd}{2}\cos\theta_n = \cos^{-1}(0) = \frac{n\pi}{2}, \; n = \pm 1, \pm$$

$$\theta_n = \cos^{-1}(n\pi(2d)), \; n = \pm 1, \pm 3.$$

which are of identical forms. Therefore, both form yield the same results. Thus for $d = \lambda/4$

$\theta_n = \cos^{-1}\left(\frac{n\lambda}{2d}\right)_{d=\lambda/4} = \cos^{-1}(2n)$, $n = \pm 1, \pm 3 \ldots \Rightarrow$ No null exists

(c) Similarly the maxima of the pattern can be found using either of the two forms for the array factor. For example.

One form

$AF(\theta) = 1 + \cos(kd\cos\theta_m) = 2$

$\cos(kd\cos\theta_m) = 1$

$kd\cos\theta_m = \cos^{-1}(1) = 2m\pi$, $m = 0, \pm 1 \ldots$

$\theta_m = \cos^{-1}\left(\frac{m\lambda}{d}\right)$, $m = 0, \pm 1, \pm 2 \ldots$

other form

$AF(\theta) = 2\cos^2\left(\frac{kd}{2}\cos\theta_m\right) = 2$.

$\cos\left(\frac{kd}{2}\cos\theta_m\right) = \pm 1$

$\frac{kd}{2}\cos\theta_m = \cos^{-1}(\pm 1) = m\pi$, $m = \pm 1, \pm$

$\theta_m = \cos^{-1}\left(\frac{m\lambda}{d}\right)$, $m = 0, \pm 1$

Which are of identical form. Therefore; both yield the same results.

Thus for $d = \lambda/4$

$\theta_m = \cos^{-1}(4m)$, $m = 0, \pm 1, \pm 2; \longrightarrow$
- $m = 0 : \theta_0 = \cos^{-1}(0) = 90°$
- $m = \pm 1 : \theta_1 = \cos^{-1}(4) \Rightarrow$ Does not exist
- The same is true for the values of m (i.e $m = \pm 2, \pm 3 \ldots$

Therefore, the only maxima occurs at $90°$.

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