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ID 7655

Subject Differential equation

Submitted to Ma'am Shumaila

Submitted by Syed USAMA

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Date 20/8/2020

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Q1 Solve initial value problem

$$dy/dt = e^{y-t} \sec(y)(1+t^2)$$

$$y(0) = 0$$

Salution:-

$$\frac{dy}{df} = e^{y-t} \sec(y)(1+t^2)$$

$y(0) = 0$ So $x = 0$ $y=0$

$$dy = e.e \sec(y)(1+t^2)\,dt$$

$$\frac{1}{e^y \cdot \sec(y)}\,dy = (1+t^2)e^{-t}\,dt$$

As $\cos(y) = \frac{1}{\sec(y)}$

$$\int e^{-y} \cos y\,dy = \int (1+t^2)e^{-t}\,dt$$

using parts integration by parts

$$e^{-y}\int \cos y\,dy - \int\left(\int \cos \frac{d}{dy} e^{-y}\right) =$$

$$(1+t^2)\int e^{-t} - \int\left(\int e^{-t} \frac{d}{dt}\right) = (1+t^2)$$

eq (1)

L. H. S

$$e^{-y}\int \cos y\,dx - \int\left(\int \cos y \frac{d}{dy} e^{-y}\right)$$

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$e^{-y}\sin y - \int(\sin y \cdot e^{-y}(-1))$

$e^{-y}\sin y + \int(\sin y \; e^{-y})$

$e^{-y}\sin y + \int(e^{-y}\sin y)$

Again using integration by parts

$e^{-y}\sin y + e^{-y}(\cos y) - \int(\int \sin y \frac{d}{dy}e^{-y}$

$\cancel{e^{-y}\sin y + e^{-y}\int(\cos y)\frac{e^{-y}}{-1}}$

$e^{-y}\sin y^{-y}(-\cos y) - \int(-\cos y \frac{e^{-y}}{-1})$

$e^{-y}\sin y - e^{-y}\cos y - \int(\cos y e^{-y})$

since $\int(\cos y e^{-y})$ - LHS

since it is again same to the 1st one so LHS will become

$L.H.S = e^{-y}(\sin y - \cos y) - L.H.S$

$\frac{2LHS}{2} = \frac{e^{-y}(\sin y - \cos y)}{2}$

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Now for taking R.H.S

$$\cancel{\int (1+t^2)\, e^{-t} \frac{d}{dt}(1+t^2)}$$

$$\Rightarrow \int (1+t^2)\, e^{-t}\, dt$$

$$\Rightarrow (1+t^2)\int e^{-t} - \int\left(\int e^{-t} \, d/dt\, (1+t^2)\right)$$

$$\Rightarrow -(1+t^2)\, e^{-t} + \int (2t) e^{-t}$$

again using integeration by parts

$$\Rightarrow -(1+t^2) e^{-t} + \left(2t \int e^{-t} - \int\left(\int e^{-t}\, d/dt\, 2t\right)\right)$$

$$\Rightarrow -(1+t^2)\, e^{-t} + \left(-2t\, e^{-t} - \int(-e^{-t}\, 2)\right)$$

$$\Rightarrow -(1+t^2)\, e^{-t} + \left(-2te^{-t} + \int(2e^{-t})\right)$$

$$\Rightarrow -(1+t^2)\, e^{-t} + \left(-2te^{-t} - 2e^{-t}\right) + c$$

$$\Rightarrow -(1+t^2)\, e^{-t} - 2t\, e^{-t} - 2e^{-t} + c$$

$$\Rightarrow -(t^2+2t+3)\, e^{-t} + c = \text{R.H.S}$$

$$\frac{e^{-y}(\sin y - \cos y)}{2} = -(t^2+2t+3)e^{-t} + c$$

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We know that

$t = 0 \qquad y = 0$

put it above

$\Rightarrow \frac{1}{2}(0-1) = -3 + c$

$c = 5/2$

put value of c

$\frac{e^{-y}}{2}(\sin y - \cos y) = (x^2+2t+3)e^{-t} + 5/2$

Ans

Q2 $(\sqrt{x+y}+\sqrt{x-y})\,dx-(\sqrt{x+y}-\sqrt{x-y})\,dy = 0$

Sol

$$\frac{dy}{dx} = \frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}} \quad -1$$

This is a Homogeneous differential eq in x and y to solve this

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Then eq(i) becomes

$$v + x\frac{dv}{dx} = \frac{\sqrt{x+vx}}{\sqrt{x+vx}} + \frac{\sqrt{x-vx}}{\sqrt{x-vx}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v}}{\sqrt{1+v}} + \frac{\sqrt{1-v}}{\sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}} \times \frac{\sqrt{1+v}}{\sqrt{1-v}} \frac{+\sqrt{1+v}}{+\sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{1+\cancel{v}+1-\cancel{v}+2\sqrt{1-v^2}}{2v}$$

$$v + x\frac{dv}{dx} = \frac{2\left(1+\sqrt{1-v^2}\right)}{2v}$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}-v^2}{v}$$

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$$x\frac{dv}{dy} = \frac{(\sqrt{1-v^2})(1+\sqrt{1-v^2})}{v}$$

$$\frac{v\,dv}{\sqrt{1-v^2}(1+\sqrt{1-v^2})} = \frac{dx}{x}$$

taking integrals on b.s

$$\int\frac{v\,dv}{\sqrt{1-v^2}(1+\sqrt{1-v^2})} = \int\frac{dx}{x}$$

Put $1+\sqrt{1-v^2} = t$

$$\Rightarrow \frac{1}{2}(1-v^2)^{-1/2}(-2v)\,dv = dt$$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\int\frac{-dt}{t} = \int\frac{dx}{x}$$

$$-\ln t = \ln x + \ln c$$

$$-\ln t = \ln x + \ln c$$

$$-\ln(1+\sqrt{1-v^2}) = \ln cx$$

$$\ln(1+\sqrt{1-v^2}) = \ln(cx)^{-1}$$

$$1+\sqrt{1-v^2} = (cx)^{-1}$$

$$1+\sqrt{1-\frac{y^2}{x^2}} = (cx)^{-1}$$

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Q3 Solve

$$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$$

Sol

$$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$$

$$f(D)y = F(x)$$

As it is non homogenous linear equation so solution will be

$$y = y_c + y_p \quad \text{— (i)}$$

complementary solution y_c

$$D^4 - D^2 = 0 \implies D^2(D^2+1) = 0$$

either $D^2 = 0 \implies D = 0$

$D^2 + 1 = 0 \qquad D^2 = -1 \qquad \therefore \sqrt{-1} = i$

$D = \sqrt{-1} \quad , \quad D = i$ or $D = 0 + i$

roots are real and complex

$$y_c = c_1 e^{0x} + c e^{0x}(\cos x\, c_2 + c_3 \sin x)$$

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$y_c = C_1 + C_2 \cos x + C_3 \sin x$

$y_p = \frac{1}{f(D)} F(x)$

$y_p = \frac{1}{D^4+D^2}\left(3x^2 - 4\sin x - 2\cos x\right)$

$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$

$f(D) = D^4 + D^2$

at $D=0 \rightarrow f(D) = 0$

So $f'(D) = 4D^3 + 2D$

Now also $D=0 \Rightarrow f(D)=0$

again differentiating

$F''(D) = 12D+2$

So $f''(0) = 12(0)+2 = 2$

So replacing $\frac{1}{f(D)}$ with $\frac{x^2}{F''(D)}$

$\Rightarrow y_p = \frac{x^2\, 3x^2}{12D+2} + \frac{x^2}{12D+2} 4\sin x - \frac{x^2}{12D+2} 2\cos x$

Putting $D=0$

$y_p = \frac{3x^4}{2} + 2x\sin x - x^2\cos x$

So equation 1 becomes

$y = C_1 + (C_2 - x^2)\cos x + (C_3 + 2x^2)\sin x + \frac{3}{2}x^4$

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