Assignment
Date:20/4/2020

Course Code: MTH 102   **Course Title:** Calculus and analytic geometry
Prerequisite: **Instructor:** HIMAYATULLAH
Module: 3   **Program:** BEE   **Total Marks:** 30

Q1.	(a)	. **Identify** $\lim\limits_{h \to 0} \dfrac{\sqrt{2+h} - \sqrt{2}}{h}$	Marks 5 CLO1 C1
	(b)	Find the first order derivatives of the function $y = \left(x + \dfrac{1}{x}\right)\left(x - \dfrac{1}{x} + 1\right)$	Marks 5 CLO1 C1
Q2	(a)	. A dynamite blast blows up a heavy rock with launch velocity of 160m/sec reaches a hight of $s = 160t - 16t^2$ ft after t sec, (i) How high does the rock go (ii) Find the velocity and speed of the rock when it is 256 ft above the ground on the way up and down (iii) find the acceleration of the rock at time 5sec	Marks 10 CLO2 C2
Q3	(a)	Does the curve $y = x^4 - 2x^2 + 2$ have nay horizontal tangent if so where ?	Marks 10 CLO1 C1

NAME:-
　　SAAD BIN TARIQ
ID
　　5534

TEACHER:-
　　SIR HIMMAYAT ULLAH
DEPARTMENT:-
　　BELE)

SUBJECT　　CALCULUS & ANALYTICAL GEOMATRY　Pg#01

Q.NO:- 1
Part(A)　Identify $\lim\limits_{h \to 0} \dfrac{\sqrt{2+h}-\sqrt{2}}{h}$

Sol:-

$$\lim_{h \to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$$

$$\lim_{h \to 0} \frac{\sqrt{2+0}-\sqrt{2}}{h}$$

Now Multiply And Divide by

$$\sqrt{2+h}+\sqrt{2}$$

$$\lim_{h \to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h} \times \frac{\sqrt{2+h}+\sqrt{2}}{\sqrt{2+h}+\sqrt{2}}$$

$$\lim_{h \to 0} \frac{(\sqrt{2+h})^2 - (\sqrt{2})^2}{h(\sqrt{2+h}+\sqrt{2})}$$

$$\lim_{h \to 0} \frac{2+h-2}{h(\sqrt{2+h}+\sqrt{2})}$$

$$\lim_{h \to 0} \frac{h}{h(\sqrt{2+h})+\sqrt{2}}$$

$\underset{h \to 0}{\text{Limit}} \quad \dfrac{1}{\sqrt{2+h}+\sqrt{2}}$

$\underset{h \to 0}{\text{Limit}} = \dfrac{1}{\sqrt{2+0}+\sqrt{2}}$

$= \dfrac{1}{\sqrt{2}+\sqrt{2}}$

$\boxed{= \dfrac{1}{2\sqrt{2}}}$ ANS

QNo1 Find the first order Derivatives of the function

Part(B)

$$y = \left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}+1\right)$$

Sol:-

$$y = \left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}+1\right)$$

Taking Derivative

$$\frac{dy}{dx} = (x+x^{-1})\frac{d}{dx}(x-x^{-1}+1) + (x-x^{-1}+1)\frac{d}{dx}(x+x^{-1})$$

$$= (x+x^{-1})(1+x^{-2}) + (x-x^{-1}+1-x^{-2})\left(x+\frac{1}{x}\right)\left(1+\frac{1}{x^2}\right)$$

$$+ \left(x-\frac{1}{x}\right)\left(1-\frac{1}{x^2}\right)$$

$$= x + x\frac{1}{x^2} + \frac{1}{x} + \frac{1}{x^3} + x - x\frac{1}{x^2} - \frac{1}{x} + \frac{1}{x^3} + 1 - \frac{1}{x^2}$$
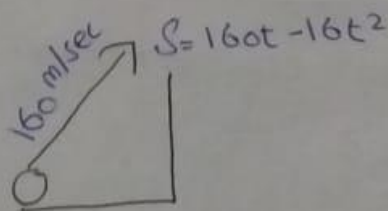
$$2x + 1 - \frac{1}{x^2} + \frac{1}{x^2} \quad \underline{ANS}$$

QNO 2

A dynamite blast blows up a heavy rock. With launch velocity of 160 m/sec reaches a hieght of S=160 $S = 160t - 16t^2$ ft after t sec?

i) How high does the rock go.

ii) Find the velocity and speed of the rock when it is 256 ft above the ground on the way up and down.

iii) Find the acceleration of rock at time 5sec.

SOLUTION:-

$S = 160t - 16t^2$ (160 m/sec)

i) $S = 160t - 16t^2$

$V_2 = 160 - 32 \times 2$

$= 160 - 64$

$\boxed{V_2 = 96 \, m/sec}$

ii) $\dfrac{ds}{dt} = V$

$S = 256 = 160t - 16t^2$

$16t^2 - 160t - 256 = 0$

$16(t^2 - 10t + 16) = 0$

$t^2 - 10t + 16 = 0$

$t^2 - 8t - 2t + 16 = 0$

$t(t-8) - 2(t-8) = 0$

$(t-2)(t-8) = 0$

$$\boxed{t = 2, 8}$$

ii) Find the acceleration of the rock at time 5 sec?

Sol:-

$S'(t) = 160 - 50t$

$S''(t) = -50\, ft/sec^2$

QNo-3 Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangent if so where

Sol:

$$y = x^4 - 2x^2 + 2$$

Taking Derivative

$$\frac{dy}{dx} = \frac{d}{dx}[x^4 - 2x^2 + 2]$$

$$= \frac{d}{dx}(x^4) - 2\frac{d}{dx}(x^2) + \frac{d}{dx}(2)$$

$$\frac{dy}{dx} = 4x^3 - 4x + 0$$

$$\frac{dy}{dx} = 4x^3 - 4x$$

If the tangent is horizontal then $\frac{dy}{dx} = 0$

So,

$$4x^3 - 4x = 0$$

$$4x(x^2 - 1) = 0$$

$$4x = 0 \quad , \quad x^2 - 1 = 0$$

$x = 0, x^2 = 1$
$\Rightarrow x = \pm \sqrt{1}$
$x = \pm \sqrt{1}$
$x = \pm 1$

So

$$x = 0, 1, -1$$

The corresponding Point in $y = x^4 - 2x^2 + 2$ are given below.

For $x = 0$ $y = 0 - 0 + 2$

$\Rightarrow y = 2$

For $x = 1$

$y = 1 - 2(1) + 2$

$\Rightarrow y = 1$

For $x = -1$

$y = -1 - 2 + 2$.

$y = 1$

Hence $(0, 2)$ $(1, 1)$ and $(-1, 1)$ Ans:-