

Midterm Paper (Summer)

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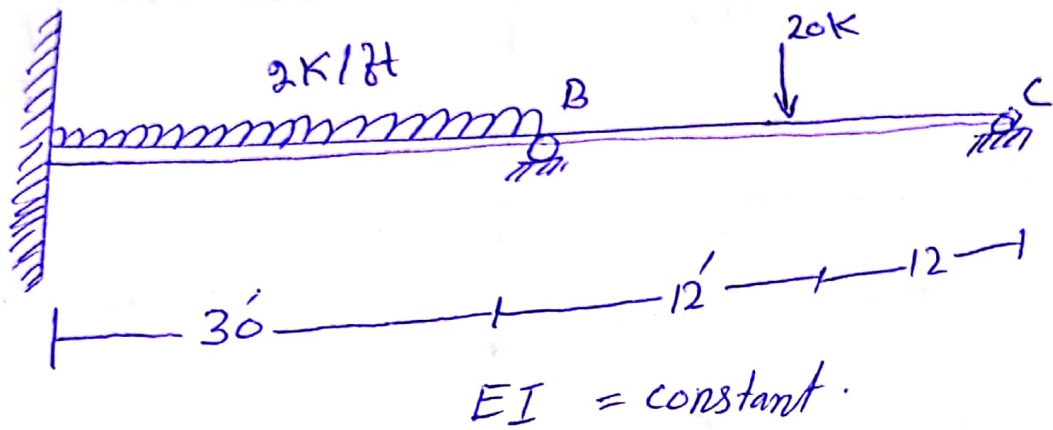
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Section # B

~~At~~ Subject # Structure 2

Teacher Name # Engr - Adeed
Khan

Q No 01

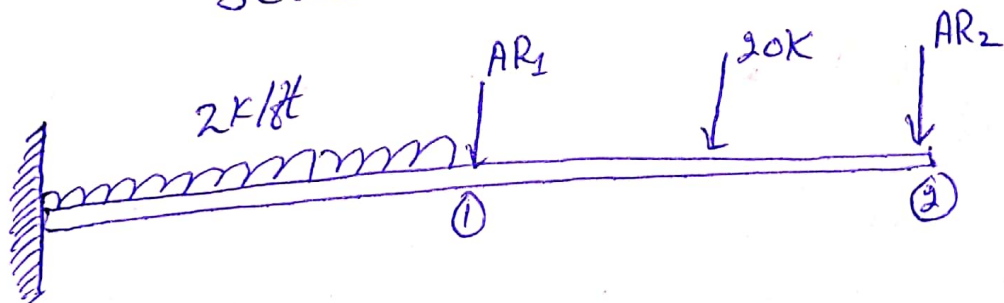


Solution

Structural Indeterminacy = 2^0

Step # 1

Select Redundant Actions

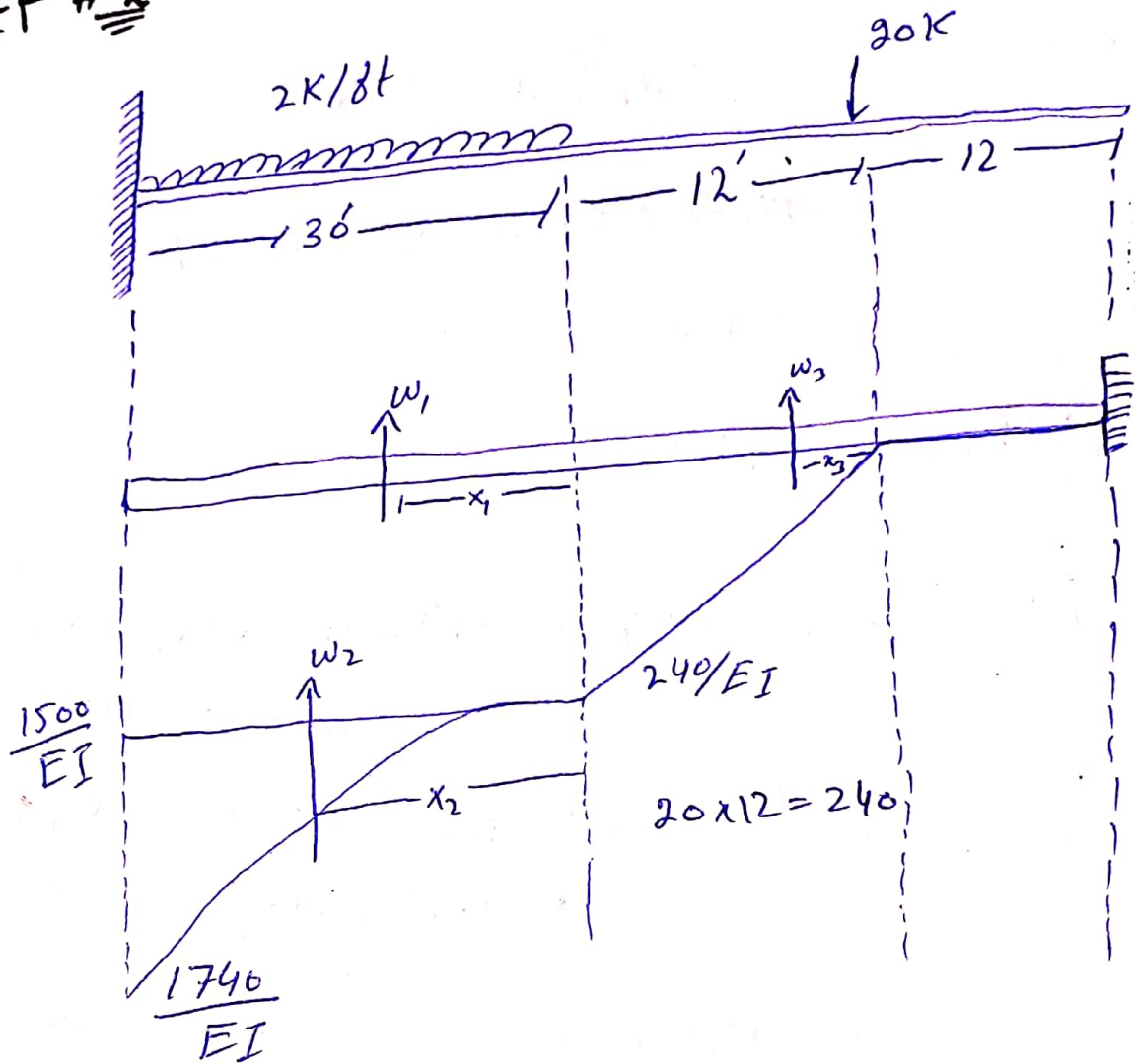


$$\begin{bmatrix} DRS_1 \\ DRS_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$[DRS] = [DRL] + [F] \times [AR]$$

Step #2



$$20 \times (12 + 30) + 2 \times 30 \times 15 = 1740$$

$$w_1 = 1500 \times 30 = 45000$$

$$w_2 = \frac{1}{3} \times 30 \times 240 = 2400$$

$$w_3 = \frac{1}{2} \times 12 \times 240 = 1440$$

$$x_1 = \frac{b}{2} = \frac{30}{2} = 15'$$

$$x_2 = \frac{3}{n+2} = \frac{3}{2+2} \times 30 = 22.5'$$

$$x_3 = \frac{2}{3} \times L = \frac{2}{3} \times 12' = 8'$$

Now Finding DRL

$$\begin{aligned} DRL_1 &= w_1(x_1) + w_2(x_2) \\ &= 45000(15) + 2400(22.5) \\ &= 675000 + 54000 \end{aligned}$$

$$\boxed{DRL_1 = 729000}$$

$$\begin{aligned} DRL_2 &= w_1(x_1 + 24) + w_2(x_2 + 24) + \\ &\quad w_3(x_3 + 12) \\ &= 45000(15 + 24) + 2400(22.5 + 24) + \\ &\quad 1440(8 + 12) \\ &= 1755000 + 111600 + 28800 \end{aligned}$$

$$\boxed{DRL_2 = 1895400/EI}$$

So

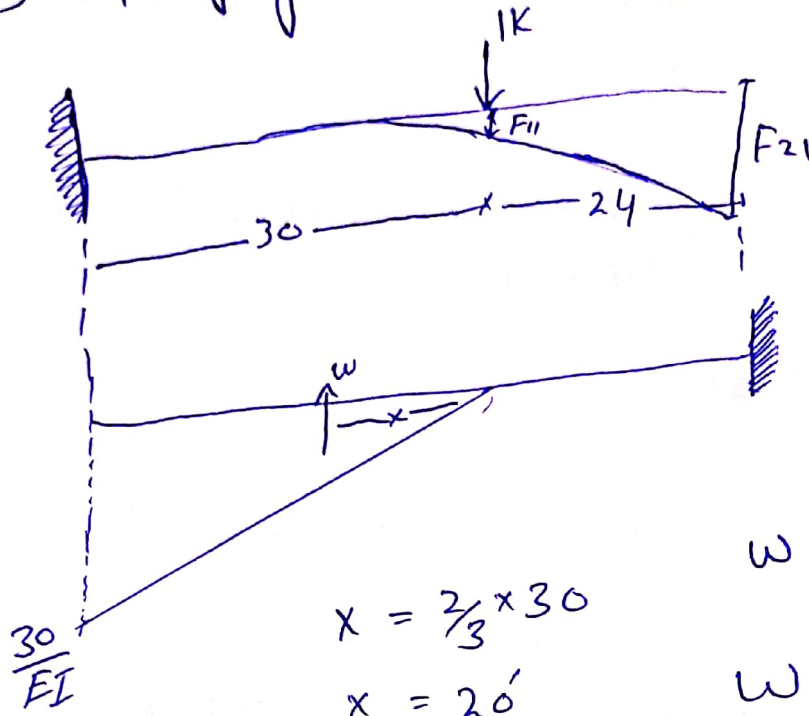
$$DRL = \frac{1}{EI} \begin{bmatrix} 729000 \\ 1895400 \end{bmatrix}$$

Step # 3

Flexibility matrix

$$(F)_{2 \times 2} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

a) Applying unit load on AR₁



$$x = \frac{2}{3} \times 30$$

$$x = 20'$$

$$w = \frac{1}{2} \left(\frac{30}{EI} \times 30 \right)$$

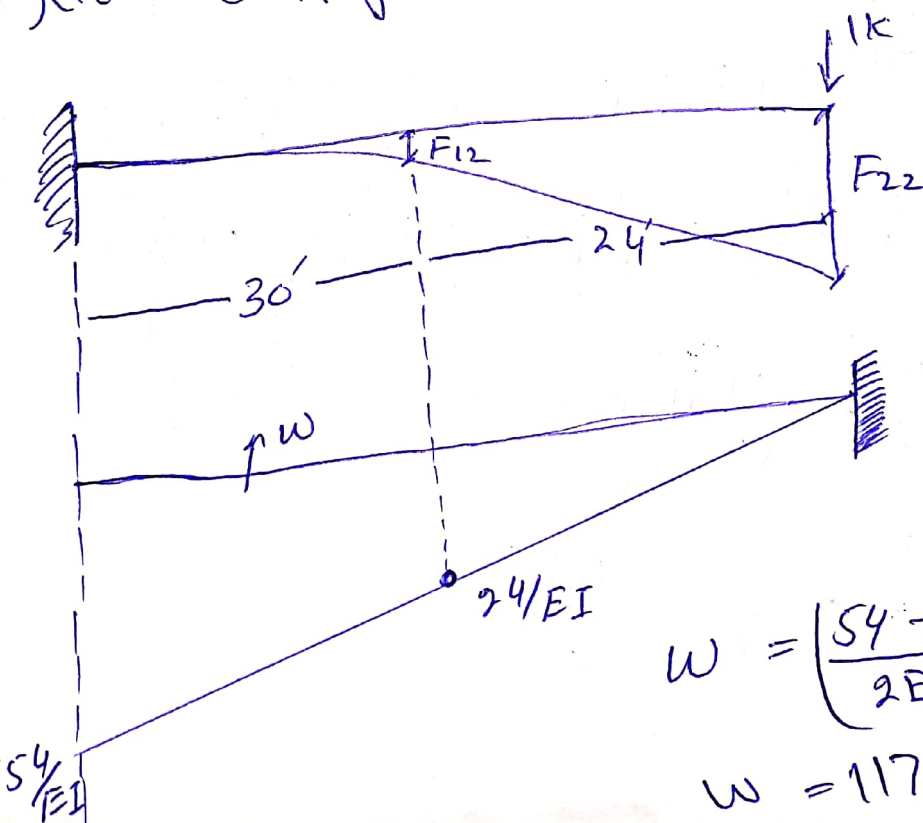
$$w = 450/EI$$

So

$$F_{11} = \frac{450}{EI} (20) = 9000/EI$$

$$F_{21} = \frac{450}{EI} (20+24) = 19800/EI$$

b) Now Apply unit load on AR₂



$$w = \left(\frac{54+24}{2EI} \right) \times 30$$

$$w = 1170/EI$$

Now the distance

$$x = \frac{4}{3} \left[\frac{b + 2(a)}{a + b} \right]$$

$$x = \frac{30}{3} \left[\frac{24 + 2(54)}{54 + 24} \right] = 16.92'$$

$$\boxed{x = 16.92'}$$

$$F_{12} = \frac{1170}{EI} \times 16.92 = \frac{19796.4}{EI}$$

$$\Rightarrow F_{22} = \frac{1170}{EI} \times (16.92 + 24) = \frac{47876.4}{EI}$$

Hence

$$F_{2 \times 2} = \begin{bmatrix} 9000 & \frac{19796.4}{EI} \\ 19800 & \frac{47876.4}{EI} \end{bmatrix} \frac{1}{EI}$$

Step #4

compute the value of AR

$$[DR_s] = [DRL] + [F] \times [AR]$$

$$[AR] = [DR_s - DRL] \times [F]^{-1}$$

$$[F]^{-1} = \frac{1}{F} \times \text{Adj } F$$

$$[F] = \frac{1}{\begin{vmatrix} 9000 & 19796.4 \\ 19800 & 47876.4 \end{vmatrix}} \times \text{Adj} \begin{pmatrix} 9000 & 19796.4 \\ 19800 & 47876.4 \end{pmatrix}$$

$$[F] = (9000 \times 47876.4 - 19796.4 \times 19800)$$

$$= (430887600 - 391968720)$$

$$\Rightarrow [F] = 38918880 \quad \text{Adj } A = \begin{pmatrix} 47876.4 & -19796.4 \\ -19800 & 9000 \end{pmatrix}$$

$$\begin{pmatrix} AR_1 \\ AR_2 \end{pmatrix} = \begin{pmatrix} 0.729000 \\ 0 - 1895400 \end{pmatrix} \frac{1}{EI} \times \frac{1}{38918880} \begin{pmatrix} 47876.4 & -19796.4 \\ -19800 & 9000 \end{pmatrix}$$

$$= \begin{pmatrix} -729000 \\ -1895400 \end{pmatrix} \frac{1}{EI} \times \begin{pmatrix} 47876.4 & -19796.4 \\ -19800 & 9000 \\ \hline 38918880 \end{pmatrix}$$

$$\begin{pmatrix} AR_1 \\ AR_2 \end{pmatrix} = \begin{pmatrix} 66.193 \\ -67.505 \end{pmatrix}$$

Q No 02

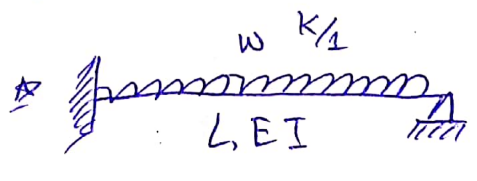
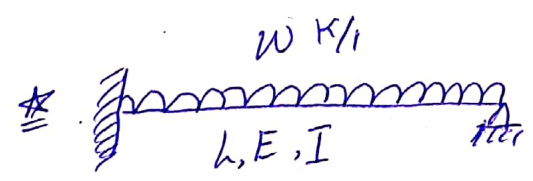
Ans

Force Method

Displacement method

- * $D_s < D_k$
- * Forces are redundant or unknowns
- * Force Found by compatibility equations of displacement
- * no of redundant = D_s
- * Starts with equilibrium of force.
- * Not suitable for computer

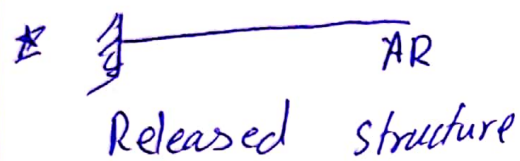
- * $D_s > D_k$
- * ~~S~~ Displacement are redundant or unknown.
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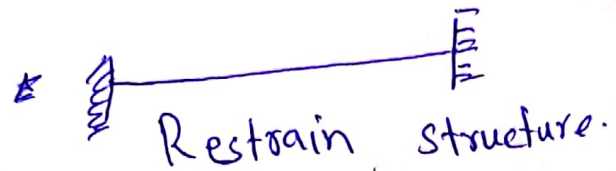
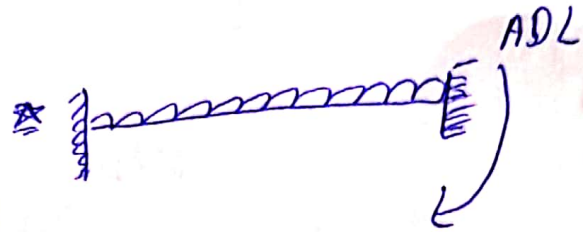
* S.I

* K-I \rightarrow displacement of joint

$$\star DR_s = DR_L + AR \star F$$



$$\star AD = AD_L + S \star D$$



(b) For analysis of structure of matrix approach both the force method or displacement method can be used depends upon situation.

① when the Degree of static Indeterminacy (D_s) is less than the degree of Kinematic Indeterminacy (D_k) i.e. $D_s < D_k$ than it is suggested to use force method of Analysis

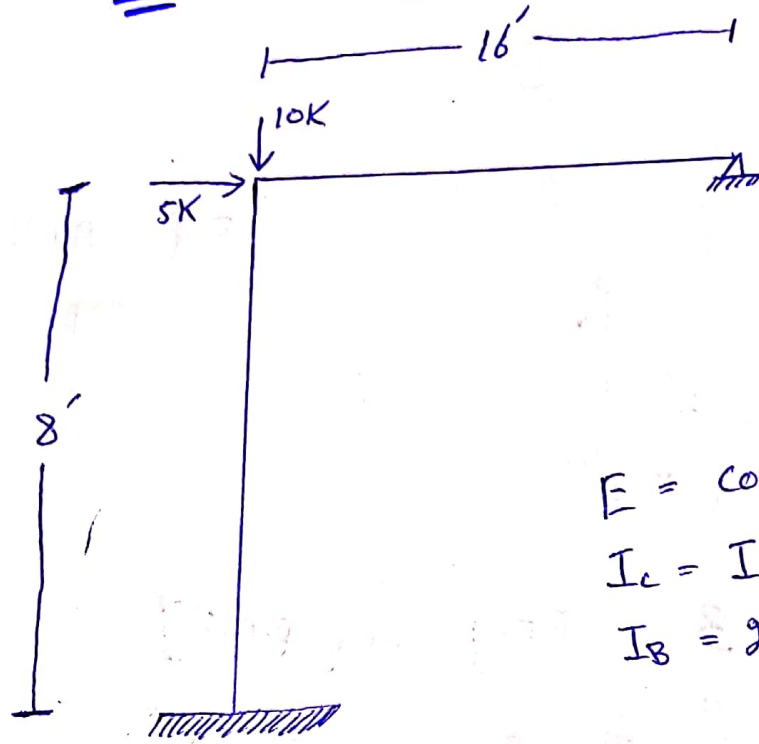
② when the Degree of static Indeterminacy (D_s) is more than the degree of Kinematic Indeterminacy (D_k) i.e. $D_k < D_s$ than it is suggested to use displacement method of Analysis.

OR

When comparing the flexibility and stiffness method. it is seen that flexibility method requires the solution of equation of compatibility for unknown force whereas the stiffness method require the solution of equilibrium for unknown displacement.

Q oN o3

Sol 2



$$E = \text{constant}$$

$$I_c = I$$

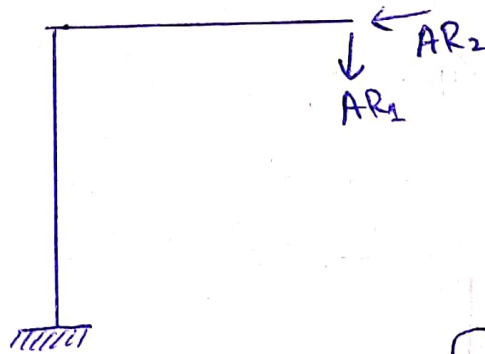
$$I_B = 2I$$

Total statical indeterminacy

$$\Rightarrow R - 3 = 5 - 3 = 2^{\circ}$$

Step #01

Identify Redundant Actions



$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix},$$

$$\begin{bmatrix} DRS_1 \\ DRS_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Step #02

Compute value of $\{DRL\}$

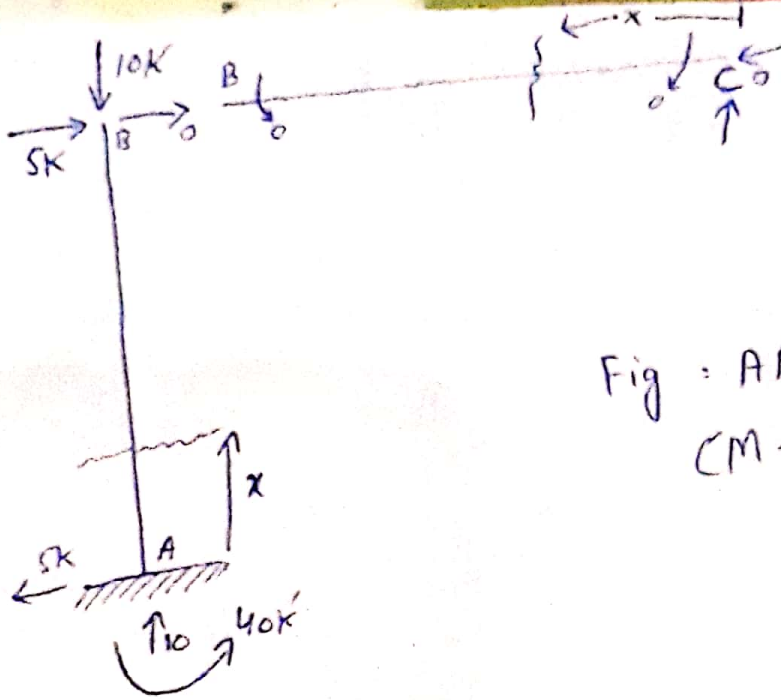


Fig : AML values
(M-values)

Step # 3 [F] or [AMR]

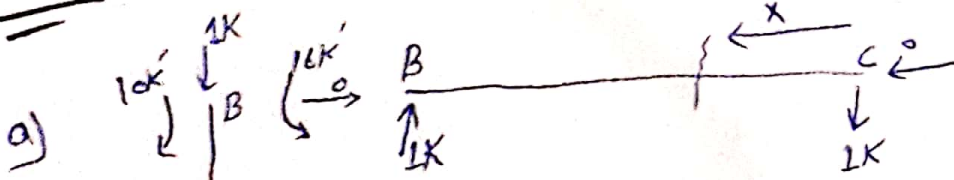


Fig : AMR-values
(M₁ values)

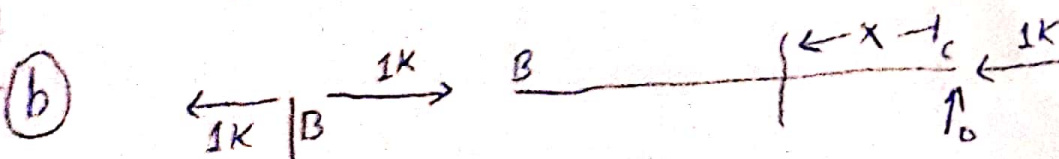


Fig : AMR values
(M₂ values)

Member	AB	BC
origin	A	C
Limits	0-8	0-16
I	I	2I
M	$5x - 40$	0
m_1	-16	x → Take x-section on m_1 Fig from the origin
m_2	$8-x$	0

origin → Select origin should be select the support

• M → Take x-section from origin (AML Fig and Find moment)

⇒ For Finding value of DRL

$$DRL_1 = \int_0^8 \frac{M_{AB} \cdot M_1(LAB)}{EI} + \int_0^{16} \frac{M_{BC} \cdot M_2(BC)}{EI}$$

$$= \int_0^8 \frac{(5x-40)(-16) dx}{EI} + \int_0^{16} \frac{0 \cdot x}{E(2I)} dx$$

$$DRL_1 = \frac{2560}{EI}$$

$$DRL_2 = \int_0^8 \frac{(5x-40)(8-x) dx}{EI} + \int_0^{16} \frac{0 \cdot 0}{E(2I)} dx$$

$$DRL_2 = \frac{-853.33}{EI}$$

⇒ Compute Flexibility matrix:

$$F_{2 \times 2} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

$$\begin{aligned} \Rightarrow F_{11} &= \int_0^8 \frac{m_1^2(AB)}{EI} + \int_0^{16} \frac{m_2^2(BC)}{EI} \\ &= \int_0^8 \frac{(-16)^2}{EI} dx + \int_0^{16} \frac{x^2}{E(2I)} dx \end{aligned}$$

$$F_{11} = \frac{2730.67}{EI}$$

$$\begin{aligned} F_{12} = F_{21} &= \int_0^8 \frac{m_1(AB) \cdot m_2(AB)}{EI} + \int_0^{16} \frac{m_1(BC) \cdot m_2(BC)}{2EI} \\ &= \int_0^8 \frac{(-16)(8-x)}{EI} dx + \int_0^{16} \frac{(x)(0)}{2EI} dx \end{aligned}$$

$$F_{12} = F_{21} = \frac{-512}{EI}$$

$$\begin{aligned} F_{22} &= \int_0^8 (m_2)_{AB}^2 dx + \int_0^{16} (m_2)_{BC}^2 dx \\ &= \int_0^8 \frac{(8-x)^2}{EI} dx + \int_0^{16} \frac{0^2}{2EI} dx \end{aligned}$$

$$F_{22} = 170.67$$

As we know

$$[DRS] = [DRL] + [AR] \times [F]$$

$$\Rightarrow [AR] = \frac{[DRS] - [DRL]}{[F]}$$

$$\Rightarrow [AR] = [F]^{-1} \times [DRS - DRL]$$

$$= \begin{bmatrix} 2730.67 & -512 \\ -512 & 170.67 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 - 2560 \\ 0 + 853.33 \end{bmatrix}$$

$$\begin{bmatrix} AR_1 \\ AR_2 \end{bmatrix} = \begin{bmatrix} -0.000005 \\ 4.997 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$