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Calculas and analytical Geometry.

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BSSE.

Q1 a) Sol:-

$$= \frac{d}{dx}\left[\frac{3x^4-2x^3+5}{x^3+1}\right]$$

$$= \frac{\frac{d}{dx}(3x^4-2x^3+5)\cdot(x^3+1) - (3x^4-2x^3+5)\cdot\frac{d}{dx}[x^3+1]}{(x^3+1)^2}$$

$$= \frac{\left(3\cdot\frac{d}{dx}[x^4] - 2\frac{d}{dx}[x^3] + \frac{d}{dx}[5]\right)(x^3+1) - 3x^4-2x^3+5)\left(\frac{d}{dx}x^3 + \frac{d}{dx}[1]\right)}{(x^3+1)^2}$$

$$= \frac{(3\cdot 4x^3 - 2\cdot 3x^2+0)(x^3+1) - (3x^4-2x^3+5)(3x^2+0)}{(x^3+1)^2}$$

$$= \frac{(x^3+1)(12x^3-6x^2) - 3x^2(3x^4-2x^3+5)}{(x^3+1)^2}$$

$$= \frac{12x^3-6x^2}{x^3+1} - \frac{3x^2(3x^4-2x^3+5)}{(x^3+1)^2}$$

$$= \frac{3x^2(x^4+4x-7)}{(x^3+1)^2}$$

Q1 B Sol;-

$$\frac{d}{dx}\left(\frac{(x^3+1)^2}{x^3-1}\right)$$

$$= \frac{\frac{d}{dx}\left[(x^3+1)^2\right]\cdot(x^3-1)-(x^3+1)^2\cdot\frac{d}{dx}\left[x^3-1\right]}{(x^3-1)^2}$$

$$= \frac{2(x^3+1)\cdot\frac{d}{dx}\left[x^3+1\right]\cdot(x^3-1)-(x^3+1)^2\left(\frac{d}{dx}\left[x^3\right]+\frac{d}{dx}\left[-1\right]\right)}{(x^3-1)^2}$$

$$= \frac{2(x^3+1)\left(\frac{d}{dx}\left[x^3\right]+\frac{d}{dx}\left[1\right]\right)(x^3-1)-(x^3+1)^2(3x^2+0)}{(x^3-1)^2}$$

$$= \frac{2(x^3+1)(3x^2+0)(x^3-1)-3x^2(x^3+1)^2}{(x^3-1)^2}.$$

$$= \frac{3x^2(x^3-3)(x^3+1)}{(x^3-1)^2}.$$

Ans.

Q2 A. Sol:-

$$\int \frac{1}{\sqrt{x^5}}\,dx$$

$$= \int \frac{1}{x^2\sqrt{x}}\,dx$$

$$= \int \frac{1}{x^2 \times x^{\frac{1}{2}}}\,dx$$

$$= \int \frac{1}{x^{\frac{5}{2}}}\,dx$$

now evaulate the integrals..

$$= -\frac{2}{3\cancel{x}\sqrt{x}}$$

add constant of integration $C \in R$

$$= \frac{2}{3x\sqrt{x}} + C, \; C \in R$$

Ans.

Q 2 B Sol:-

$$\int \frac{1}{(8x+7)^8}\,dx.$$

using substitution.

$$= \int \frac{1}{8t^8}\,dt$$

$$= \frac{1}{8} \times \int \frac{1}{t^8}\,dt$$

evaulate the intgrals.

$$= \frac{1}{8} \times \left(-\frac{1}{7t^7}\right)$$

$$= \frac{1}{8} \times \left(-\frac{1}{7(8x+7)^7}\right)$$

$$= \frac{1}{56(8x+7)^7}$$

add the constant $C \in R$.

$$= \frac{1}{56(8x+7)^7} + C, \quad C \in R.$$

Ans.

Q3 A:- Sol:-

$$\int \frac{-x+9}{2x^2-8x+6}\,dx$$

Factorize the expression.

$$= \int \frac{-x+9}{2(x-1)\times(x-3)}\,dx.$$

using intrgral property.

$$= \frac{1}{2} \times \int \frac{-x+9}{(x-1)\times(x-3)}\,dx.$$

using partial fraction decomposition.

$$= \frac{1}{2} \times \int -\frac{4}{x-1} + \frac{3}{x-3}\,dx$$

$$= \frac{1}{2} \times \left(-\int \frac{4}{x-1}\,dx + \int \frac{3}{x-3}\,dx\right)$$

$$= \frac{1}{2} \times \left(-4\ln(|x-1|) \cancel{x} + \int \frac{3}{x-3}\,dx\right)$$

$$= \frac{1}{2} \times \left(-4\ln(|x-1|) + 3\ln(|x-3|)\right)$$

$$= -2\ln(|x-1|) + \frac{3}{2} \times \ln(|x-3|)$$

$$= -2\ln(|x-1) + \frac{3}{2} \times \ln(|x-3|) + C, \quad C \in R.$$

Ans.

Q 3 B Sol:-

$$\int \frac{4x^2+8x}{x^{2+1}\times(x^2+2x+3)}$$

$$= \int \frac{x\times(4x+8)}{x^{2+1}\times(x^2+2x+3} dx.$$

$$= \int \frac{\cancel{x}\times(4x+8)}{x^{\cancel{3}2}\times(x^2+2x+3} dx.$$

$$= \frac{4x+8}{x^2\times(x^2+2x+3} dx.$$

By partial fraction decomposition.

$$= \int -\frac{4}{9x} + \frac{8}{3x^2} + \frac{4x-16}{9(x^2+2x+3)} dx.$$

$$= -\int \frac{4}{9x} dx + \int \frac{8}{3x^2} dx + \int \frac{4x-16}{9(x^2+2x+3)} dx$$

$$= -\frac{4}{9}\times \ln(|x|) + \int \frac{8}{3x^2} dx + \int \frac{4x-16}{9(x^2+2x+3} dx.$$

$$= -\frac{4}{9}\times \ln(|x|) - \frac{8}{3x} + \int \frac{4x-16}{9(x^2+2x+3} dx.$$

$$= -\frac{4}{9}\times \ln(|x|) - \frac{8}{3x} + \frac{2}{9}\times \ln(|x^2+2x+3|) - \frac{10\sqrt{2}\arctan\left(\frac{\sqrt{2}x+\sqrt{2}}{2}\right)}{9} + C, C\in R$$

Ans.

Q4 a) Sol:-

$$X+\begin{pmatrix}3 & -1\\ 2 & 2\end{pmatrix}=\begin{pmatrix}5 & 1\\ -3 & 1\end{pmatrix}$$

$$\Rightarrow X+\begin{pmatrix}3 & -1\\ 2 & 2\end{pmatrix}-\begin{pmatrix}3 & -1\\ 2 & 2\end{pmatrix}=\begin{pmatrix}5 & 1\\ -3 & 1\end{pmatrix}-\begin{pmatrix}3 & -1\\ 2 & 2\end{pmatrix}$$

so.

$$X=\begin{pmatrix}5 & 1\\ -3 & 1\end{pmatrix}-\begin{pmatrix}3 & -1\\ 2 & 2\end{pmatrix}.$$

$$X=\begin{pmatrix}2 & 2\\ -5 & -1\end{pmatrix}$$

Ans

Q4 B Sol:-

$$X+\begin{bmatrix}-1 & 0\\ 0 & 2\end{bmatrix}=\begin{bmatrix}2 & 6\\ 1 & 5\end{bmatrix}+\begin{bmatrix}-4 & -8\\ -2 & 0\end{bmatrix}$$

$$X=\begin{bmatrix}2 & 6\\ 1 & 5\end{bmatrix}+\begin{bmatrix}-4 & -8\\ -2 & 0\end{bmatrix}-\begin{bmatrix}-1 & 0\\ 0 & 2\end{bmatrix}$$

$$X=\begin{bmatrix}2 & 6\\ 1 & 5\end{bmatrix}+\begin{bmatrix}-4 & -8\\ -2 & 0\end{bmatrix}-\begin{bmatrix}-1 & 0\\ 0 & 2\end{bmatrix}=\begin{bmatrix}-1 & -2\\ -1 & 3\end{bmatrix}$$

$$X=\begin{bmatrix}-1 & -2\\ -1 & 3\end{bmatrix}$$

Ans.

Q4 c) Sol:-

$$X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$X + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$X + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$X + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

Ans

Q5 a) Sol:-

If $A=\begin{bmatrix}1 & 4\\ 2 & 1\end{bmatrix}$, $B\begin{bmatrix}-3 & 2\\ 4 & 0\end{bmatrix}$, $C=\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}$

then find A^2+BC.

so first.

$$A^2=\begin{pmatrix}1 & 4\\ 2 & 1\end{pmatrix}\times\begin{pmatrix}1 & 4\\ 2 & 1\end{pmatrix}$$

$$=\begin{bmatrix}9 & 8\\ 4 & 9\end{bmatrix}$$

now B.C

$$B.C=\begin{pmatrix}-3 & 2\\ 4 & 0\end{pmatrix}\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}$$

$$=\begin{pmatrix}(-3).1+2.0 & (-3).0+2.2\\ 4.1+0.0 & 4.0+0.2\end{pmatrix}$$

$$=\begin{pmatrix}-3 & 4\\ 4 & 0\end{pmatrix}$$

Now

$$A^2+BC=\begin{pmatrix}9 & 8\\ 4 & 9\end{pmatrix}\begin{bmatrix}-3 & 4\\ 4 & 0\end{bmatrix}$$

$$=\begin{bmatrix}6 & 12\\ 8 & 9\end{bmatrix}$$

Ans.

Finished.