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Section = B

Subject = Calculus

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Question NO 1:

=> Find PQ where P is the point in three dimensional space with coordinates (4, 1, 3) and Q with coordinates (1, 2, 4). Find the distance b/w P and Q. Futher find the position vector of point dividing PQ in ratio 1:3.

Solution:

Coordinates of P = (4, 1, 3)

$$OP = 4i + 1j + 3k$$

or $OQ = \vec{OQ} - \vec{OP}$

$$= (i + 2j + 4k) - (4i + 1j + 3k)$$

$$\boxed{= -3i + 1j + 1k \quad \text{—— (i)}}$$

Now Distance b/w P & Q = |PQ|

$$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{9 + 1 + 1}$$

$$\boxed{= \sqrt{11}} \quad - (2)$$

Let M be the point which is divided PQ in ratio 1:3 Then ~~the~~ by The ratio theorm position vector of M = $\vec{OM}$

$$= \frac{3(4i + 1j + 3k) + (1)(i + 2j + 4k)}{1+3}$$

$$= \frac{12i + 3j + 9k + i + 2j + 4k}{4}$$

$$\boxed{= \frac{13i + 5j + 13k}{4}} \quad - (3)$$

Hence eq 1, 2, 3 are required

Solution -

Question No 2 :

Evaluate

$$\int \frac{4x^3+10x+4}{2x^2+x}$$

Solution :

$$\begin{array}{r|l} & 2x-1 \\ \hline 2x^2+x & \cancel{4x^3}+10x+4 \\ & \underline{\pm\cancel{4x^3} \pm 2x^2} \\ & -\cancel{2x^2}+10x+4 \\ & \underline{\mp\cancel{2x^2} \mp x} \\ & 11x+4 \end{array}$$

So $2x-1+\dfrac{11x+4}{2x^2+x} = \dfrac{4x^3+10x+4}{2x^2+x}$

$$\Rightarrow \int \frac{4x^3+10x+4}{2x^2+x} = \int 2x-1+\int \frac{11x+4}{2x^2+x} \quad -\text{ 1 }$$

$$= 2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x}\,dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x + 4}{x(2x+1)} dx \quad — (2)$$

Now Find

$$\int \frac{11x+4}{x(2x+1)} dx = ?$$

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \quad — (A)$$

xing and ÷ $x(2x+1)$ b.H.S

$$\frac{11x+4}{\cancel{x(2x+1)}} = \frac{A(2x+1) + Bx}{\cancel{x(2x+1)}}$$

$$11+4 = A(2x+1) + Bx \quad — (3)$$

Put $x=0$ in eq (3)

$$\boxed{4 = A}$$

Now Put $x = -1/2$ in eq (3)

$$11\left(-\tfrac{1}{2}\right)+4=B\left(-\tfrac{1}{2}\right)$$

$$-\frac{11}{2}+4=-\frac{B}{2}$$

$$\frac{-11+8}{\cancel{2}}=\frac{-B}{\cancel{2}}$$

$$-3=-B \Rightarrow \boxed{B=3}$$

Put the value of A and B in A

$$\frac{11x+4}{x(2x+1)}=\frac{4}{x}+\frac{3}{2x+1}$$

Taking integral on b.H.S

$$\int \frac{11x+4}{x(2x+1)}\,dx=\int \frac{4}{x}\,dx+\int \frac{3}{2x+1}\,dx$$

$$=4\int \frac{1}{x}\,dx+3\int \frac{1}{2x+1}\,dx$$

$$=4\ln|x|+\tfrac{3}{2}\ln|2x+1|$$

Puting these value in 2

$$= x^2 - x + 4\ln|x| + \frac{3}{2}\ln|2x+1|$$

Now put these in 1

$$\int \frac{4x^3 + 10x + 4}{2x^2 + x}\,dx = x^2 - x + 4\ln|x| + \frac{3}{2}\ln|2x+1| + C$$

Question No 3 :

a) $\int_0^2 x^2 e^x \, dx$

Solution : $\int_0^2 x^2 e^x \, dx$

Now first find Integration

$= \int x^2 e^x \, dx$

$= x^2 \int e^x \, dx - \int \left(e^x dx \frac{d}{dx} x^2 \right) dx$

$= x^2 e^x - \int e^x (2x) \, dx$

$= x^2 e^x - 2 \left[x \int e^x dx - \int \left(e^x dx \frac{d}{dx} x \right) dx \right]$

$= x^2 e^x - 2 \left[x e^x - \int e^x dx \right]$

$= x^2 e^x - 2x e^x + 2 e^x$

Now put limits

$= \left| x^2 e^x - 2x e^x + 2e^x \right|_0^2$

$$= \left(2^2 e^2 - 2(2)e^2 + 2e^2 - (0 - 0 + 2e^0)\right)$$

$$= \left(\cancel{4e^2} - \cancel{4e^2} + 2e^2 - 2\right)$$

$$\boxed{= 2e^2 - 2 \text{ Ans}}$$

b) $\displaystyle\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$

Solution:

First find Integration

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx = ? \quad \text{— (1)}$$

Let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$2\,dy = \frac{1}{\sqrt{x}}\, dx$$

$$\int \sin(y)(2\,dy) = 2\int \sin(y)\, dy$$

$= 2(-\cos y)$

$= -2\cos y$

Put $y = \sqrt{x}$

$= -2\cos\sqrt{x}$

Put limits

$= -2\left|\cos\sqrt{x}\right| = -2\left(\cos\sqrt{2} - \cos 1\right)$

$\boxed{= -2\cos\sqrt{2} + 2\cos(1) \text{ Ans}}$

Question No 4

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Solution

The laplace eq in 3d is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad (A)$$

So $u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$

$$u(x,y,z) = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$\frac{\partial u}{\partial x} = -x(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[x(-3/2)(x^2+y^2+z^2)^{-5/2}(2x) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{3/2} \quad (1)$$

Now

$$\frac{\partial u}{\partial y} = \frac{-1}{2}\left(x^2+y^2+z^2\right)^{-3/2}(2y)$$

$$\frac{\partial u}{\partial y} = -y\left(x^2+y^2+z^2\right)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = -\left[y\left(-3/2\right)\left(x^2+y^2+z^2\right)^{-5/2}\right](2y) + \left(x^2+y^2+z^2\right)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = 3y^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2} \quad -(2)$$

$$\frac{\partial u}{\partial z} = -\frac{1}{2}\left(x^2+y^2+z^2\right)^{-3/2}(2z)$$

$$\frac{\partial u}{\partial z} = z\left(x^2+y^2+z^2\right)^{-3/2}$$

$$\frac{\partial^2 u}{\partial z^2} = 3z^2\left(x^2+y^2+z^2\right) - \left(x^2+y^2+z^2\right)^{-3/2} \quad -(3)$$

Now putting eq (1), (2), (3) in (A)

$$3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2} -$$

$$(x^2+y^2+z^2)^{-3/2} + 3z^2(x^2+y^2+z^2)^{-3/2}$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2 - (x^2+y^2+z^2) + 3y^2 - (x^2+y^2+z^2) + 3z^2 - (x^2+y^2+z^2)\right]$$

$$(x^2+y^2+z^2)^{-5/2}\left[3x^2 - x^2 - z^2 + 3y^2 - y^2 - z^2 + 3z^2 - x^2 - y^2 - z^2\right]$$

$$= (x^2+y^2+z^2)^{-5/2} \quad (0) = 0$$

So the given $u(x,y,z)$ is solution of laplace eq.