

Q 1

(A)

Sol

Let

 x and $y =$ the numbers. $Z =$ Sum of their cubes

$$K = x + y$$

$$y = K - Z$$

$$Z = x^3 + y^3$$

$$Z = x^3 + (K - Z)^3$$

$$\frac{dz}{dx} = 3x^2 + 3(K - Z)^2(-1) = 0$$

$$x^2 - (K^2 - 2Kx + x^2) = 0$$

$$x = \frac{1}{2}K$$

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Maths 2

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$$y = K - \frac{1}{2}K$$

$$y = \frac{1}{2}K$$

$$x = \left(\frac{1}{2}K\right)^3 + \left(\frac{1}{2}K\right)^3$$

$$x = \frac{1}{4}K^3$$

Aus

Q1 (B)

SolLet x and $y = z$ the numbers

$$x + y = 2 \rightarrow \text{Eq (1)}$$

$$1 + y' = 0$$

$$y' = -1$$

$$z = x^2 + y^2 \rightarrow \text{eq} \rightarrow (2)$$

$$\frac{dz}{dx} = 3x^2 + 2yy' = 0$$

$$3x^2 + 2y(-1) = 0$$

$$y = \frac{3}{2}x^2$$

From eq \rightarrow (1)

$$x + \frac{3}{2}x^2 = 2$$

$$2x + 3x^2 = 4$$

Maths 2

Q1
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Q1 (B)

$$3x^2 + 2x - 4 = 0$$

$$x = 0.8685 \text{ \& } -1.5352$$

Use

$$x = 0.8685$$

$$y = \frac{3}{2} (0.8685^2)$$

$$y = 1.1315$$

$$z = 0.8685^3 + 1.1315^2$$

$$z = 1.9354$$

Ans

Sol = The linear approximation is given by the equation.

$$f(x) \cong L(x) = f(a) + f'(a)(x-a).$$

We just need to plug in the known values and calculate the value of $f(3.5)$: ?

$$\begin{aligned} L(x) &= f(3) + f'(3)(x-3) = 12 - 2(x-3) \\ &= 18 - 2x \end{aligned}$$

Then

$$f(3.5) \cong 18 - 2 \times 3.5 = 11$$

Ans

Q2 ABSol

Let $f(x) = \sqrt[3]{x}$. The linear approximation at the point $a = 8$ is given by

$$f(x) \approx L(x) = f(8) + f'(8)(x-8)$$

Find the derivative

$$f'(x) = (\sqrt[3]{x})' = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$$

Compute the value of the derivative at $a = 8$:

$$f'(8) = \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{12}$$

Substituting this, we get the function $L(x)$ in the form.

$$f(x) \approx L(x) = 2 + \frac{1}{12}(x-8) = \frac{x}{12} + \frac{4}{3}.$$

Hence

$$\sqrt[3]{9} \approx L(9) = \frac{9}{12} + \frac{4}{3} = \frac{9+16}{12}$$

$$= \frac{25}{12}$$

Ans

Q3

$$2xy - 9x^2 + (2y + x^2 + 1) \frac{dy}{dx} = 0$$

Sol

$$(2y + x^2 + 1) \frac{dy}{dx} + (2xy - 9x^2) = 0$$

We can rewrite this as

$$(2y + x^2 + 1) dy + (2xy - 9x^2) dx = 0$$

Check for exactness:

$$\text{let } M = 2xy - 9x^2$$

$$\text{let } N = 2y + x^2 + 1$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\partial(2xy - 9x^2) / \partial y = \partial(2y + x^2 + 1) / \partial x$$

$$2x = 2x$$

Thus it is indeed Exact. To further solve,

$$\partial F / \partial x = M = 2xy - 9x^2$$

$$\int \partial F = \int (2xy - 9x^2) \partial x$$

$$F = (x^2)y - 3x^3 + g(y)$$

To get "g"(y), we differentiate it partially with respect to y:

$$\partial F / \partial y = \partial / \partial y ((x^2)y - 3x^3 + g(y)) = N$$

$$\partial F / \partial y = x^2 + g'(y) = N$$

Q3

$$x^2 + g'(y) = 2y + x^2 + 1$$

$$g'(y) = 2y + 1$$

Integrating.

$$g(y) = y^2 + y + C$$

Therefore,

$$F = (x^2)y - 3x^3 + y^2 + y + C$$

Ans