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Cource title : DSP

Q1(a): consider the following analog signal

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t.$$

Solution:

① Minimum sampling rate

$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t.$

$f = 50Hz, f_2 = 100, f_3 = 100Hz.$

So

f is max (generator than f_1)

$f_1 = 50Hz$ is minimum sampling rate to avoid aliasing.

As from equation

$x_a(t) = 3\cos \frac{\cancel{100}\pi t}{\cancel{100}} + 4\sin \frac{\cancel{200}}{\cancel{100}}\pi t$

Then The equation be come

$3\cos\pi t + 4\sin 2\pi t.$

(PTO)

So The effect of this sampling rate on the newly generated ~~discrete~~ discrete time is that there will be no Aliasing phenomenon mean there will be present components in the reconstru of the signal and we can reconsider the orignal signal.

(c) Hence for ideal interpolation we can construct the orignal signal & also frequency component, $f_1 = 50Hz$, $f_2 = 100Hz$ $y_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$.
The Orignal Signal is different from it which will constroted become we use sampling frequency. This distortion of the orignal analog signal was caused by the aliasing effect.

Q1(b):

Consides a discrete time signal which is Giving by $x(n) = \begin{bmatrix} 0.5^n, & n > 0 \\ 0, & n < 0 \end{bmatrix}$

This is signal is samped inthe rate $fs = 2\,Hz$.

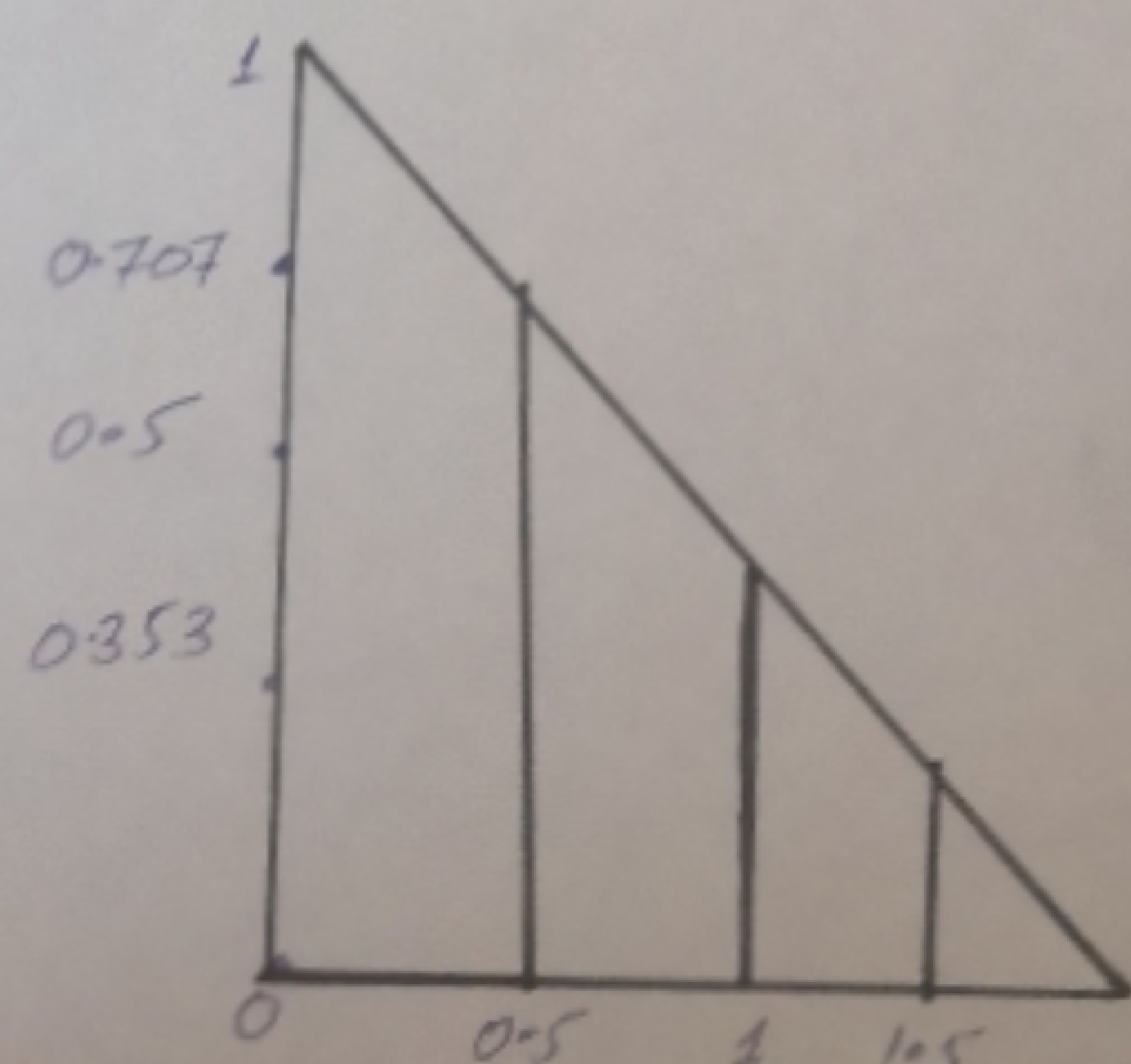
Solution:

$fs = 2\,Hz$

$fs = \frac{1}{T} = T = \frac{1}{fs} = \frac{1}{2} = 0.5\,sec$

(i) Draw the samplied signal

$x(n)$	0.5^n
0	1
0.5	0.7071
1	0.5
1.5	0.353



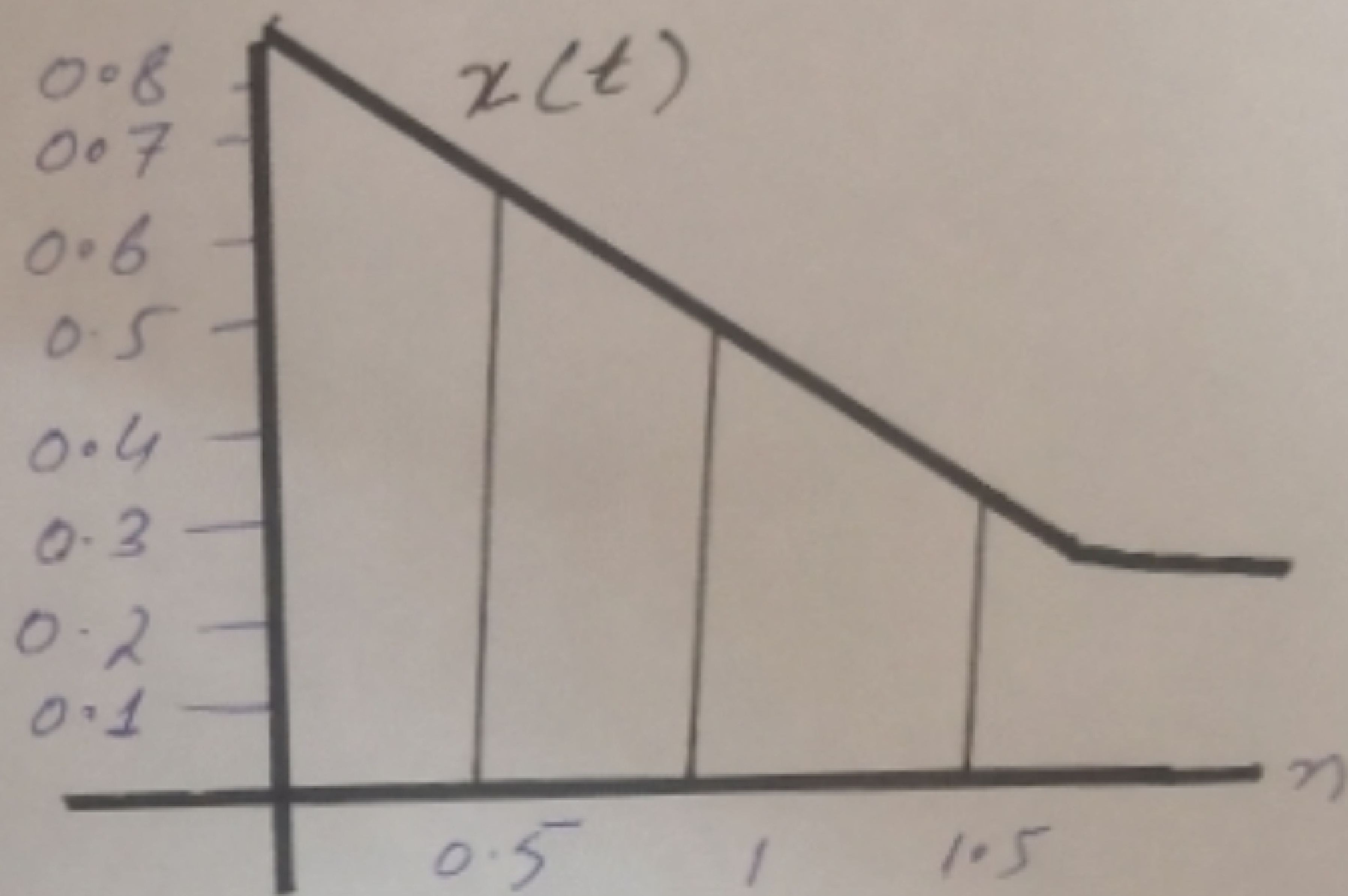
(ii) Quantization level

$L = 2^n$

$n = bits = 3$

Resolution = $\frac{x_{max} - Y_{min}}{L}$

$= \frac{1-0}{8} = 0.125$

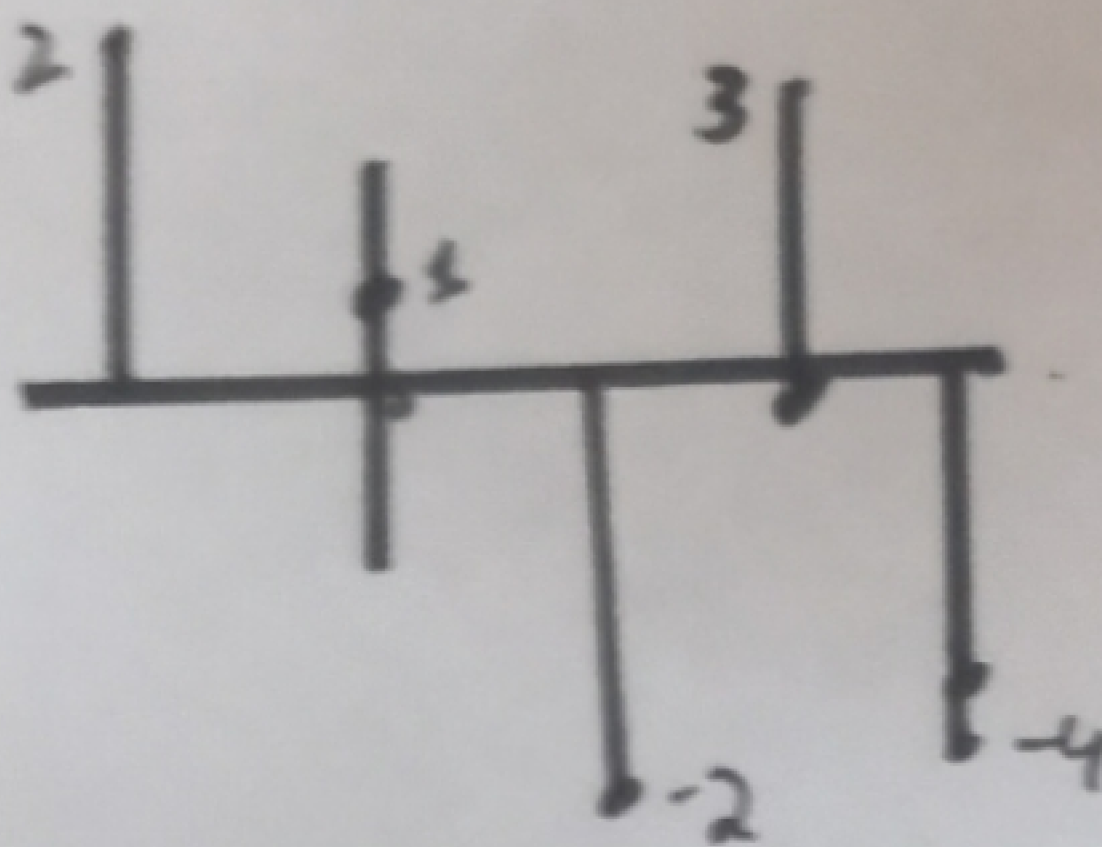


(iii) Tabular form

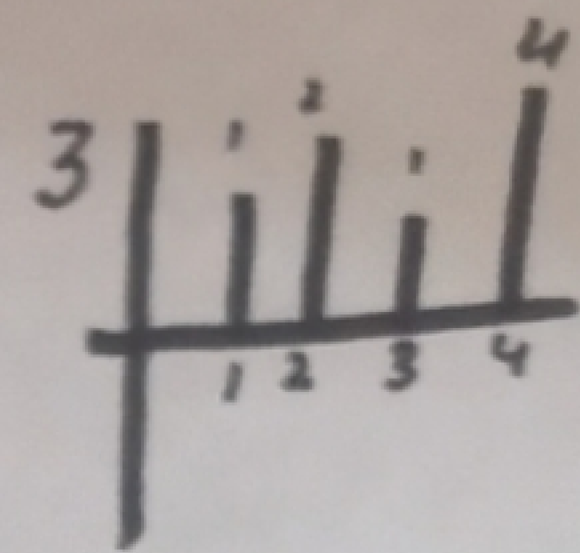
n	x(n)	xq(n) Trucation	Xq(n) Rounding	x[q(n)]-x[n]
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	0.046
2	0.707	0.7	0.7	-0.07
3	0.6035	0.6	0.6	-0.035
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	-0.0265
6	0.353	0.3	0.4	0.047
7	0.1765	0.1	0.2	0.0235

Q2(a)

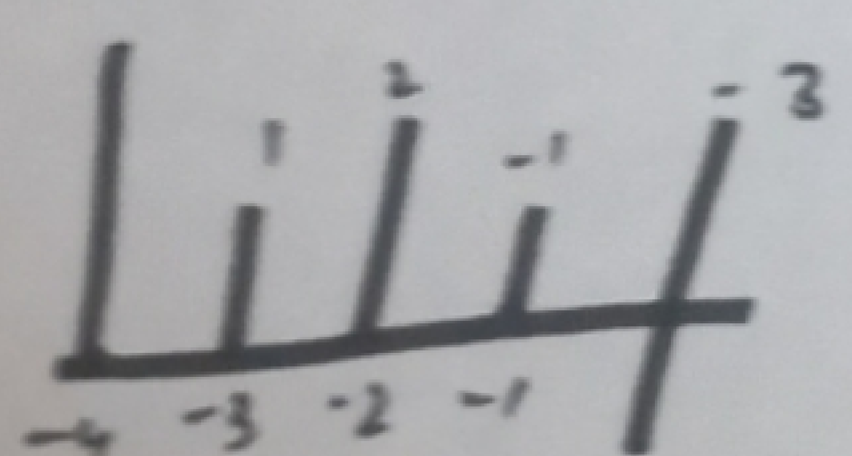
Solution:



H(k)



h(-k) Folded Signal



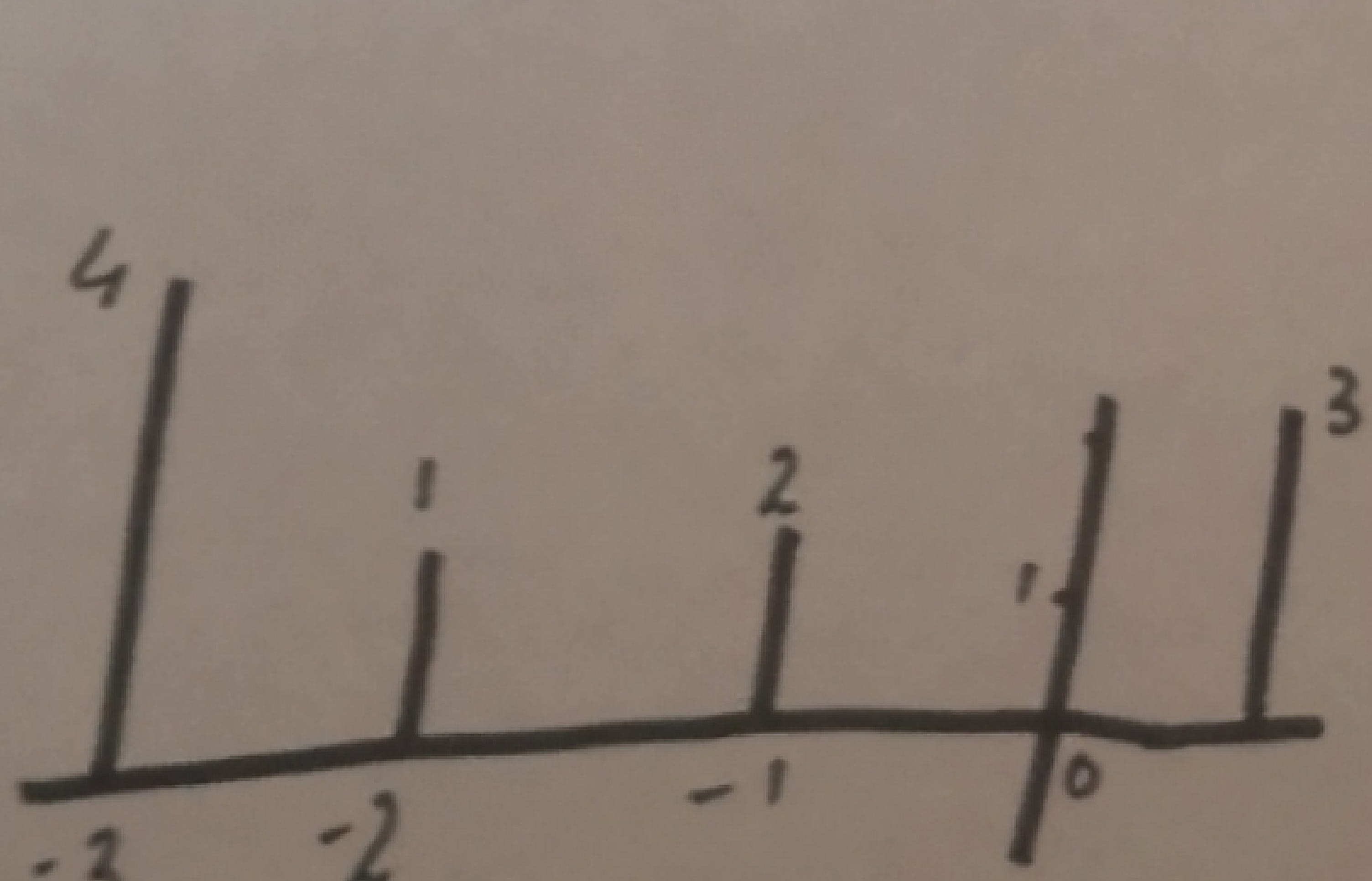
$$Y[0] = \sum_{k=1}^{0} x(-1)h(-1) + x(0)h(0)$$

$$Y(0) = (2)(1) + (1)(3)$$

$$= 2+3 = 5$$

for $n = 1$

$h(1-k)$



$$Y(1) = \sum_{x=-1} x(n)\, n(1-k)$$

$$= x(-1)h(-1) + x(0)h(0) + 0\; x(1)h(1)$$

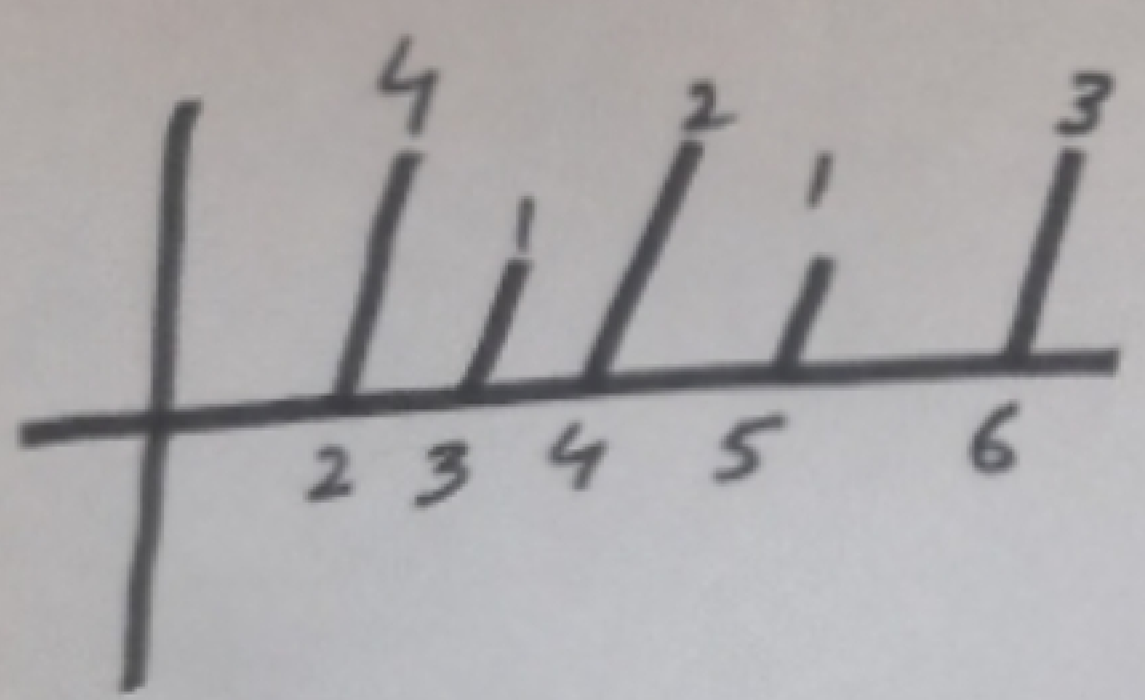
$$+ (x)h(1) + x(2)x(2) + 0(x)(3)h(3)$$

$$Y(2) = (2)(4) + (1)(1) + (-2)(2) + (3)(1) + (-4)(3)$$

$$= 8 + 1 - 4 + 3 - 12 = -4$$

$n = 3$

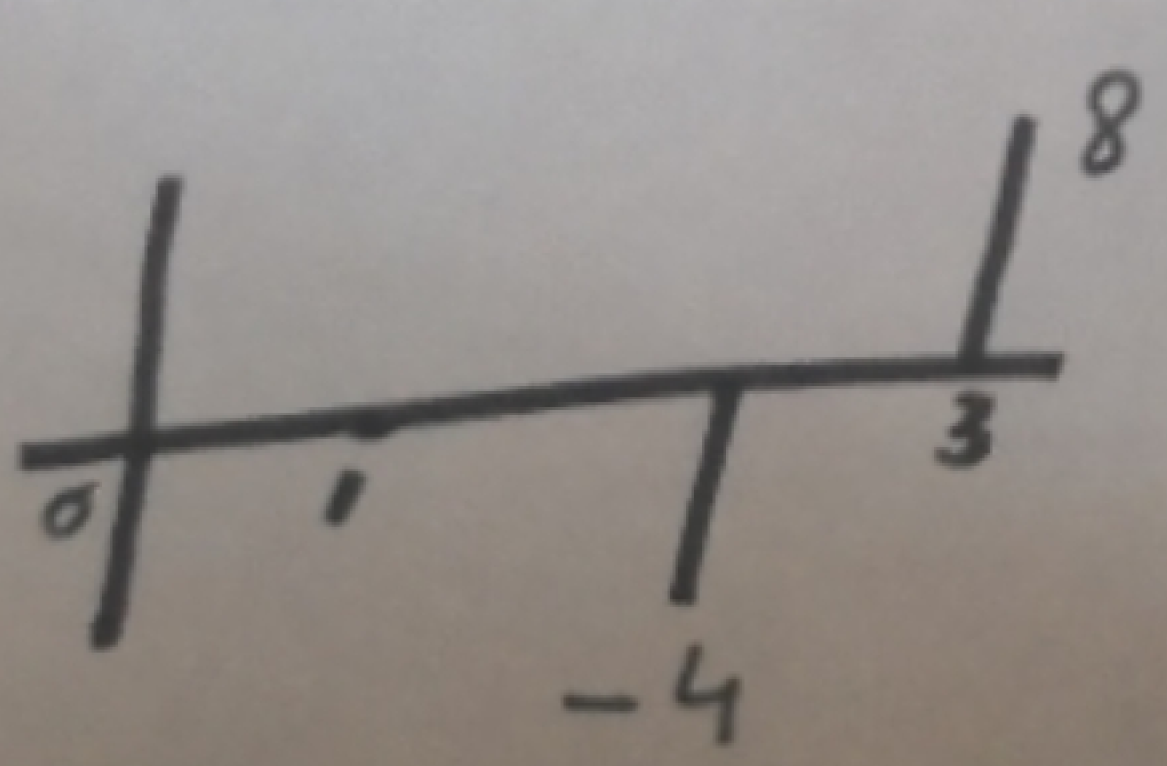
$n(3-k)$



$$Y(3) = \sum_{k=2} x(n)h(n-k)$$

$$Y(3) = x(2)h(2) + x(3)h(3)$$

$$(3)(4) + (-4)(1) = 12 - 4 = 8$$



Q2(b)

$$x(n) = \begin{cases} \alpha^{n+1} & , -3 \le n \le 5 \\ 0 & \text{else where} \end{cases}$$

$$h[n] = \begin{cases} 2^n, & 0 \le n \le 4 \\ 0, & \text{else where} \end{cases}$$

Solution:

$x(n) = \{\alpha^{-2}, \alpha^{-1}, \alpha, \alpha^1, \alpha^3, \alpha^4, \alpha^5, \alpha^6]$

and

$h(n) = \{1, 2, 4, 8, 16\}$

$y(n) = \sum_{k=0}^{4} h(k) + (n-k)$

There fore

$y(-2) - \alpha^{-2}$

$y(-1) = x(-\alpha) + x(1) = \alpha^{-2} + \alpha^{-1}$

$y(0) = h(x) x(-2) + h[x] x[-a) + h(x) x(\alpha)$

$= 9 - \alpha^{-2} + 9 - \alpha^{-2} + 9 - 9 + \alpha^{-3} + 9$

$y(9) = \alpha^{-2} + \alpha^{-3} + 1 + h(9) x(4-3)$

$= \alpha^{-2} + \alpha^{-1} 9 + h(3)(x(1) - x^{-2} + x^{-1} + 1 + 2\alpha^1$

$y(2) = \alpha^{-2} + \alpha^{-1} + 1 + 2\alpha^{1} + h(2)\,x(2)$

$= \alpha^{-2} + \alpha^{-3} + 1 + 2\alpha^{1} + 4\alpha^{2}$

$y(3) = \alpha^{-2} + \alpha^{-1} + 1 + 2\alpha^{1} + 4\alpha^{2} + 6\alpha^{3}.$

$y(4) = \alpha^{-2} + \alpha^{-4} + 1 + 2\alpha^{1} + 4\alpha^{2} + 8\alpha^{3}\, h(4)\cdot x(4)$

$= \alpha^{-2} + \alpha^{-3} + 1 + 2\alpha^{1} + 4\alpha^{2} + 8\alpha^{3} + 16\alpha^{4}.$

$y(5) = 1 + 2\alpha^{1} + 4\alpha^{2} + 18\alpha^{3} + 16^{4} + 5$

$y(6) = 4\alpha^{2} + 8\alpha^{3} + 16\alpha^{4} + \alpha^{5} + \alpha^{6}$

$y(7) = 8\alpha^{3} + \alpha^{4} + \alpha^{5} + \alpha^{6}$

$y(8) = 16\alpha^{4} + \alpha^{5} + \alpha 6$

$y(9) = \alpha^{5} + \alpha 6$

$y(10) = \alpha^{6}$

Q3 (i)

1) $x(n) = \begin{cases} (\frac{1}{4})^n, & n > 0 \\ (\frac{1}{3}) & n < 0 \end{cases}$

<u>Solution:</u> $x(n) = \begin{cases} (\frac{1}{4})^n, & n > 0 \\ (\frac{1}{3}) & n < 0 \end{cases}$

Z-transform

$$x(z) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n z^{-n} + \sum_{n=\infty}^{0} \left(\frac{1}{3}\right)^n z^{-n} - 1$$

using Geomatic series

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^n - 1$$

$$= \frac{1}{1-\frac{1}{4}z^{1}} + \frac{1}{1-2/3}z \quad -1$$

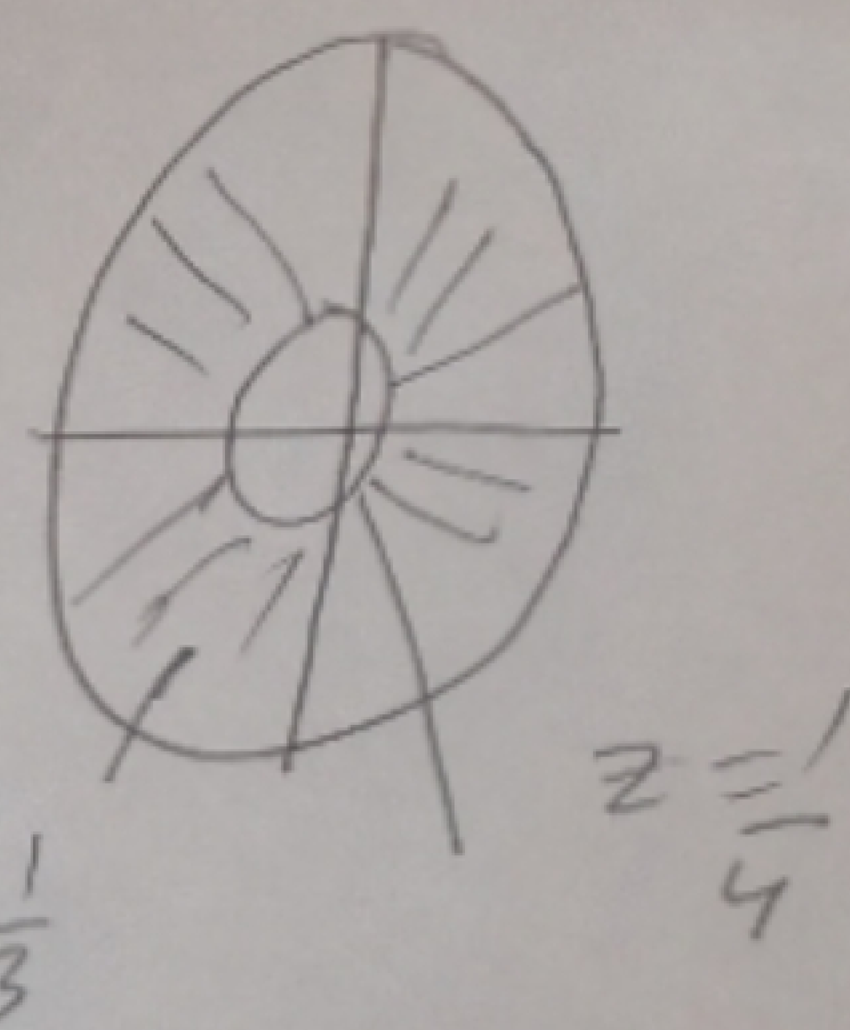
$$= \frac{1-\frac{1}{3}z + 1 - \frac{1}{4}z^{1} \quad -1}{(1-\frac{1}{4}z^{-1})(1.\frac{1}{3}z)}$$

$$= 1-\frac{1}{3}z + \left(-\frac{1}{4}\right)z^{-1} - \left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)$$

$$= \frac{\cancel{1}-\cancel{\frac{1}{3}z} + 1 - \frac{1}{4}\cancel{z^{-1}} + \cancel{\frac{1}{3}z} + \frac{1}{4}\cancel{z^{-1}} + \frac{1}{2}}{(1-\frac{1}{4}z^{-1}(1-\frac{1}{3}z}$$

$$= \frac{\frac{13}{12}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

Hence the Roc is $\frac{1}{4} < |z| < 3$.



Q3 (ii)

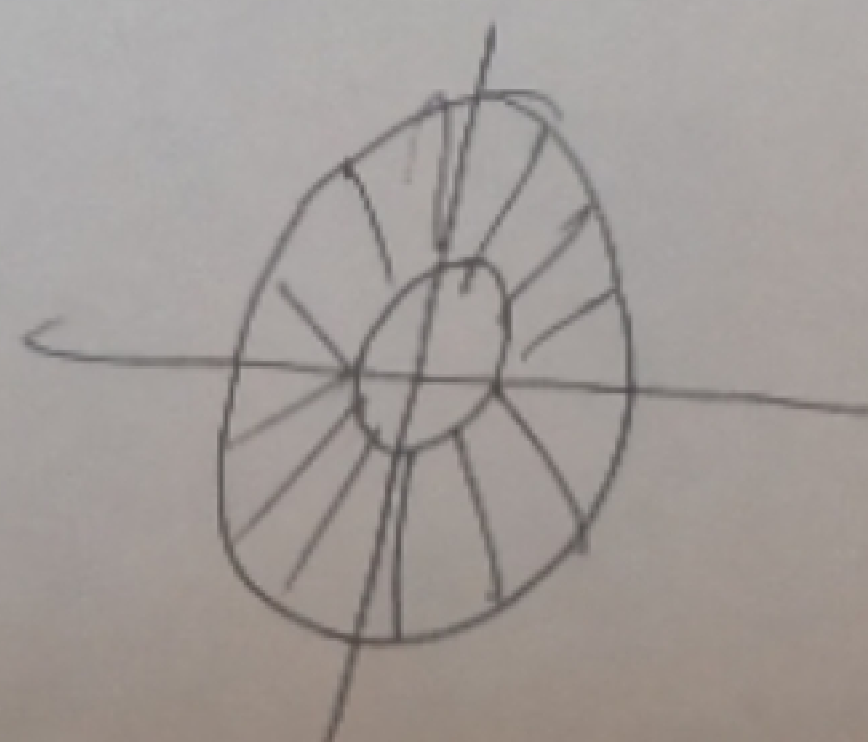
using Geomatic Series to Simplify it

$$= \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-3z^{-1}}$$

$$= \frac{1-3z^{-1}-1+\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})}$$

$$= \frac{-\frac{5}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})}$$

The Roc is $|z| > 3$



$z = 3$