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Subject	Calculus
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P. 1 calculus Noor-ul-wahab 12395

Q.1 (a) Identify $\dfrac{\sqrt{2+h}-\sqrt{2}}{h}$

Solution:-

$$\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h} = 0/0$$

Multiply and divide by $\sqrt{2+h}+\sqrt{2}$.

$$= \lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h} \times \frac{\sqrt{2+h}+\sqrt{2}}{\sqrt{2+h}+\sqrt{2}}$$

$$= \lim_{h\to 0} \frac{(\sqrt{2+h})^2-(\sqrt{2})^2}{h\left[\sqrt{2+h}+\sqrt{2}\right]}$$

$$= \lim_{h\to 0} \frac{\cancel{2}+h-\cancel{2}}{h\left[\sqrt{2+h}+\sqrt{2}\right]}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}\left[\sqrt{2+h}+\sqrt{2}\right]}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{2+h}+\sqrt{2}}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{2+0}+\sqrt{2}}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{2}+\sqrt{2}} \Rightarrow \frac{1}{2\sqrt{2}} \quad \text{Ans}$$

Q: I(b)— P. 2 Calculus Noor-ul-Wahab 12395

Find the 1st order derivatives of the function $Y = (x + 1/x)(x - 1/x + 1)$.

Solution:- $y = (x + 1/x)(x - 1/x + 1)$

$dy/dx = d/dx (x + 1/x)(x - 1/x + 1)$

$= (x + x^{-1}) \, d/dx (x - x^{-1} + 1) + (x - x^{-1} + 1) \, d/dx (x + x^{-1})$

$= (x + x^{-1})(1 + x^{-2}) + (x - x^{-1} + 1)(1 - x^{-2})$

$= (x + 1/x)(1 + 1/x^2) + (x - 1/x)(1 - 1/x^2)$

$= x + x \cancel{1/x^2} + 1/x + 1/x^3 + x - x \cancel{1/x^2} - 1/x + 1/x^3 + 1 - 1/x^2$

$= 2x + 1 - 1/x^2 + 1/x^3$ Ans

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P. 3 Calculus Noor-ul-wahab 12395

Q:2(a) A dynamatic blast blows up a heavy rock with launch velocity of 160m/sec reaches a hight of $S = 160t - 16t^2$ ft after t sec.

(i) How high does the rock go.

(ii) Find velocity and speed of The rock when it is 256 ft above The ground.

(iii) Find accelaration of rock at time 5 sec.

Solution:-

$$S = 160t - 16t^2 \text{ ft}$$

velocity is,

$$v = \frac{ds}{dt} = \frac{d}{dt}(160t - 16t^2)$$

$$= d/dt\ 160t - d/dt\ 16t^2$$

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P.4 Noor-ul-wahab 12395

$V = 160 - 32t$

Maximum hight: $V = 0$

So, $160 - 32t = 0$

$$\frac{160}{32} = \frac{\cancel{32}t}{\cancel{32}}$$

$t = 5$ sec

$S_{max} = S(5) = 160(5) - 16(5)^2$

$S_{max} = 400$ ft

(b) Given that :-

$S = 256$ ft

Then, $160t - 16t^2 = 256$

$16t^2 - 160t + 256 = 0$

$\cancel{16}/\cancel{16}(t^2 - 10t + 16) = 0/16$

$t^2 - 10t + 16 = 0$

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$t^2 - 8t - 2t + 16 = 0$

$t(t-8) - 2(t-8) = 0$

$(t-8)(t-2) = 0$

$t - 8 = 0 \quad , \quad t - 2 = 0$

$t = 8\text{ sec} \quad , \quad t = 2\text{ sec}$

Since,

$V = 160 - 32t$

$t_1 = 2s$

$V_{(2)} = 160 - 32(2)$

$V(2) = 160 - 64$

$V(2) = 96 m/s$ => velocity up.

$t_2 = 8s$

$V_{(8)} = 160 - 32(8)$

$= 160 - 256$

Q:3 (a) Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangent If so where ?

Solution:-

$$y = x^4 - 2x^2 + 2$$

Take derivative on both sides

$$d/dx\, y = d/dx\,(x^4 - 2x^2 + 2)$$

$$d/dx\, x^4 - 2\, d/dx\, x^2 + d/dx\, 2$$

$$= 4x^3 - 4x$$

If tangent is horizontal then

$\frac{dQ}{dx} = 0$ So, $4x^3 - 4x = 0$

$$4x(x^2 - 1) = 0$$

$4x = 0 \quad , \quad x^2 - 1 = 0 \quad \sqrt{x^2} = \sqrt{1}$

$x = 0 \quad , \quad x^2 = 1 \quad x = \pm 1$

$= x = \pm 0 \quad = x = \pm 1$

$x = 0, 1, -1$

there corresponding point is

$y = x^4 - 2x^2 + 2$

For, $x = 0$, $y = 0 - 0 + 2 \Rightarrow y = 2$

For, $x = 1$, $y = 1 - 2 + 2 \Rightarrow y = 1$

For, $x = -1$, $y = 1 - 2 + 2 \Rightarrow y = 1$

Hence, $(0, 2)$, $(1, 1)$ and $(-1, 1)$