QUIZ NO.1

NAME : SAIFULLAH JAVED

ID NO : 7954

DEPARTMENT : BS CIVIL ENGINEERING

SECTION : "B"

SUBJECT : DIFFERENTIAL EQUATION

SUBMITTED TO : MAAM SUMAILA

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QUESTION

A yarn merchant sells 3 brands: B_1, B_2 and B_3 of yarn, each of which is a blend of Pakistani, Egyption and American cotton in ratios: 1:2:1, 2:1:1 and 2:0:2. If Cost/kg of B_1, B_2 and B_3 is Rs 40, 50 and 60 respectievely. Find the cost/kg. of cotton of each country.

SOLUTION:

Let x, y, z be the cost/kg of Pakistani, Egyptian and American cotton respectively Then according to the given Conditions.

$$\frac{1}{4}x + \frac{2}{4}y + \frac{1}{4}z = 40$$

$$\frac{2}{4}x + \frac{1}{4}y + \frac{1}{4}z = 50$$

$$\frac{2}{4}x + \frac{2}{4}z = 60$$

$$1x + 2y + 1z = 160$$
$$2x + 1y + 1z = 200$$
$$1x + 1z = 120$$

In Matrix form we can write it as:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$B = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

$$AX = B$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A_1 = \begin{bmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{bmatrix}$$

Now

$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow |A| = (1)\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 0 + (1)\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}$$

$$\Rightarrow |A| = (2-1) + (1-4)$$

$$\Rightarrow |A| = 1 - 3 = -2$$

$$|A_1| = \begin{vmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow |A_1| = 160(1-0) - 2(200-120) + 1(200-120)$$

$$\Rightarrow |A_1| = -120$$

Now $$|A_2| = \begin{vmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{vmatrix}$$

$$\Rightarrow |A_2| = 1(200-120) - 160(2-1) + 1(2-200)$$

$$\Rightarrow |A_2| = -40$$

Now

$$|A_3| = \begin{vmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{vmatrix}$$

$$\Rightarrow |A_3| = 1(120-0) - 2(240-200) + 160(240-1)$$

$$\Rightarrow |A_3| = -120$$

Now According to Cramer's Rule.

$$x = \frac{|A_1|}{|A|} = \frac{-120}{-2} = 60$$

$$y = \frac{|A_2|}{|A|} = \frac{-40}{-2} = 20$$

$$z = \frac{|A_3|}{|A|} = \frac{-120}{-2} = 60$$

Hence

$$(x, y, z) = (60, 20, 60)$$