S M T W T F S Date:..........

Q1:(a) Estimate $\int \theta \sqrt[4]{1-\theta^2}\, d\theta$

Solution:-

$$\int \theta \sqrt[4]{1-\theta^2}\, d\theta$$

let $1-\theta^2 = u$

Then $1-\theta^2 = u$

$d/d\theta\, (1-\theta^2 = du/d\theta$

$0 - 2\theta = du/d\theta$

$\theta \, d\theta = -\frac{1}{2} \, du$

put $= \frac{1}{2} \, du = \theta \, du$

$\theta \, (1 - \theta^2) = u$

$\int -4 \sqrt{u/2} \quad du$

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$$-\frac{1}{2}\int 4\sqrt{u}\ du$$

$$-\frac{1}{2}\int u^{1/4}\ du$$

$$-\frac{1}{2}\ \frac{u^{1/4+1}}{1/4+1}+c$$

put $u = 1-\theta^2$

$$-\frac{1}{2}\ (1-\theta^2)^{5/4}\ +c$$

$$-\frac{1}{\cancel{4}}\ \frac{4^2\ (1\ \theta^2)^{5/4}}{5}+c$$

$$-\frac{2}{5}\ (1-\theta^2)^{5/4}+c$$

$$\frac{-2}{5}\ (1-\theta^2)^{5/4}+c$$

$$=\frac{-2}{5}\ \frac{(-\theta^2+1)^{5/4}}{5}+c\quad \text{Ans}$$

Estimate $\int_0^1 x^3(1+x^4)^3\,dx$

using Substitution method.

Solution:

$$\int x^3(x^4+1)^3\,dx$$

Substitute $u = x^4 + 1 \rightarrow \frac{du}{dx} = 4x^3 \Rightarrow dx = \frac{1}{4x^3}\,du$:

Now

$$\int u^3\,du$$

Apply power rule

$$\int u^n\,du = \frac{u^{n+1}}{n+1} \quad \text{with } n = 3$$

$$= \frac{u^4}{4}$$

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plug in solved Integrals

$$\frac{1}{4}\int u^3 \, du$$

$$= \frac{u^4}{16}$$

undo substitution $u = x^4 + 1$:

$$= \frac{(x^4 + 1)^4}{16}$$

the problem solved

$$\int x^3 (x^4 + 1)^3 \, dx$$

$$= \frac{(x^4 + 1)^4}{16} + C$$

Antiderivative computed by maximum

$$\int_0^1 f(x) \, dx = F(x) = \frac{(x^4 + 1)^4}{16} + C$$

$$\int_0^1 f(x) \, dx = \frac{15}{16}$$

Approximation ~~0.9375~~

0.9375 Ans

Q 2 (a) Find the Center and radius of the Spheria

$$x^2 + y^2 + Z^2 + 3x - 4Z + 1$$

Solution

$$x^2 + y^2 + Z^2 + 3x - 4Z + 1 = 0$$

$$(x^2 + 3x) + y^2 + Z - 4Z + 1 = 0$$

$$\left(x^2 + 3x + (3/2)^2\right) + (y - 0)^2 + \left(Z^2 - 4Z + \left(\frac{-4}{2}\right)^2\right)$$

$$= -1 + \left(\frac{3}{2}\right)^2 + \left(\frac{-4}{2}\right)^2$$

$$\left(x + \frac{3}{2}\right)^2 + (y^2 + 0) + (Z - 2)^2 = \frac{21}{4}$$

So (x_0, y_0, Z_0) = Center.

$$= (3/2, 0, 2)$$

Find radius $a = \sqrt{\frac{21}{4}}$

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Q 9 (B) the Region b/w the curve $y = \sqrt{x}$ $0 \le x \le 4$, revolved about the x axis to generate a solid. Apply the integration find the volume of solid

Solution Given that.

$$y = \sqrt{x}$$

$$0 \le x \le 4 \Rightarrow a \le x < b.$$

as $$V = \int_a^b \pi y^2 \, dx$$

$$V = \int_0^4 \pi (\sqrt{x})^2 \, dx$$

$$V = \pi \int_0^4 x \, dx = \pi \left. \frac{x^2}{2} \right|_0^4$$

$$V = \frac{\pi}{2} \left((4)^2 - 0 \right)$$

$$= 8\pi \text{ Ans}$$

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Q3 if $A = 2i - 4j + \sqrt{5}k$ and

$B = -2i + 4j - \sqrt{5}k$ then

illustrate the vector proje A^B

Solution:

$$proj_A B = \frac{B \cdot A}{\| A \|^2} A \rightarrow ①$$

$B \cdot A = (-2i + 4j - \sqrt{5}k)(2i - 4j + \sqrt{5}k)$

$B \cdot A \quad (-(4)(i \cdot i) - 16(j \cdot j) - (5)(k \cdot k)$

We know that.

$i \cdot i = j \cdot j = k \cdot k = 1$

$B \cdot A = (-4(1) - 16(1) - 5(1))$

$= -4 - 16 - 5$

$B \cdot A = -25 \longrightarrow$ (a)

$\|A\|^2 \left(\sqrt{(2i)^2 + (-4j)^2 + (\sqrt{5}k)^2}\right)^2$

$= \left(\sqrt{4 + 16 + 5}\right)^2$

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$\|A\|^2 = 25 \to (b)$

$A = 2i - 4j + \sqrt{5}\,k \to (c)$

put (a) (b), and (c) in equation (I)

$\text{proj}_A B = \frac{-25}{25}(2i - 4j + \sqrt{5}k)$

$'' = -1(2i - 4j + \sqrt{5}k$

$\Rightarrow \text{proj}_A B = -2i + 4j - \sqrt{5}k$ Ans

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Q4: Find the area region between the graph and the axis where $y = -x^2 + 5x - 4$ $[0, 2]$

Solution given that:-

$$y = fx = x^2 + 5x - 4$$

and $[a, b] = [0, 2]$

As $a = 0$
$b = 2$

So area under graph will

$$A = \int_a^b f(x)\,dx$$ By the value

$$A = \int_0^2 (-x^2 + 5x - 4)\,dx$$

$$= \frac{x^3}{3} + \frac{5x^2}{2} - 4 \Big]_0^2$$

$$= \left(\frac{-2}{3}\right)^3 + 5\left(\frac{2}{2}\right)^2 - 4(2) - \left(\frac{0}{3}\right)^3 + 5\left(\frac{0}{2}\right)^2 - 4(0)$$

$$= \left(\frac{-4}{3} + \frac{20}{2} - 8\right) - 0 + 0 - 0$$

$$= \frac{-4}{3} + 10 - 8$$

$$= \frac{-4}{3} + 10 - 8$$

$$= \frac{-4}{3} + 2 \Rightarrow \frac{-4+6}{3}$$

$$= A = \frac{2}{3}, \quad \boxed{\text{Ans} \rightarrow 0.6}$$

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Q(5)(A) Estimate the angle b/w

$A = i - 2j - 2k$ and $B = 6i + 3j + 2k$

Solution:

$$A = i - 2j - 2k$$

$$|A| \sqrt{1+4+4} = \sqrt{9} = 3$$

$$B = 6i + 3j + 2k$$

$$|B| \sqrt{36+9+4} = \sqrt{49} = 7$$

$$\theta = \cos^{-1}\left(\frac{A \cdot B}{|A| \cdot |B|}\right)$$

$$\theta = \cos^{-1}\left(\frac{(i - 2j - 2k)(6i + 3j + 2k)}{3 \times 7}\right)$$

$$\theta = \cos^{-1}\left(\frac{(1)(6) + (-2)(3) + (-2)(2)}{21}\right)$$

$$\theta = \cos^{-1}\left(-\frac{4}{21}\right)$$

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Q5: (B) Change into the Spherical cordinate equation for the Sphere:

$$x^2 + y^2 + (Z-1)^2 = 1$$

Solution:

$$x^2 + y^2 + (Z-1)^2 = 1$$

$$= (f \sin\phi \cos\phi)^2 + (f \sin\theta \sin\theta)^2$$

$$+ (f \cos\theta - 1)^2 = 1$$

$$= f^2 \sin\theta \cos\theta^2 + f^2 \sin^2\theta \sin^2\theta$$

$$+ f^2 \cos^2\theta + 1 \ 2f \cos\theta = 1$$

$$= f^2 \sin^2\theta (\cos^2\theta + \sin^2\theta)$$

$$+ f^2(\sin^2\theta) + f^2 \cos^2\theta + 2f\cos\theta - 1 - 1$$

$$- f^2 \sin^2\theta + \cos^2\theta) - 2f \cos\theta = 0$$

$$= f^2 = 2f \cos\theta$$

$$= f = 2\cos\theta$$ Ans

Thank you Sir