

Q1: (1) Estimate $\int \theta \sqrt[4]{1-\theta^2} d\theta$

Solution:- $\int \theta \sqrt[4]{1-\theta^2} d\theta$

$$\text{let } 1 - \theta^2 = u$$

$$\text{Then } 1 - \theta^2 = u$$

$$d/d\theta (1 - \theta^2) = du/d\theta$$

$$0 - 2\theta = du/d\theta$$

$$\theta d\theta = -\frac{1}{2} du$$

$$\text{put } \frac{1}{2} du = \theta d\theta$$

$$\theta (1 - \theta^2) = 4$$

$$\int -4 \sqrt{u} \frac{1}{2} du$$

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$$-\frac{1}{2} \int 4\sqrt{u} \, du$$

$$-\frac{1}{2} \int u^{1/4} \, du$$

$$-\frac{1}{2} \frac{u^{1/4+1}}{1/4+1} + C$$

$$\text{put } u = 1 - \theta^2$$

$$-\frac{1}{2} (1 - \theta^2)^{5/4} + C$$

$$-\frac{1}{2} \frac{4^2 (1 - \theta^2)^{5/4}}{5} + C$$

$$-\frac{2}{5} (1 - \theta^2)^{5/4} + C$$

$$-\frac{2}{5} (1 - \theta^2)^{5/4} + C$$

$$= -\frac{2}{5} \frac{(-\theta^2 + 1)^{5/4}}{5} + C \quad \text{Ans}$$

Estimate $\int_0^1 x^3 (1 + x^4)^3 dx$

using Substitution method.

Solution:

$$\int x^3 (x^4 + 1)^3 dx$$

Substitute $u = x^4 + 1 \rightarrow \frac{du}{dx} = 4x^3 \rightarrow dx = \frac{1}{4x^3} du$:

Now

$$\int u^3 du$$

Apply power rule

$$\int u^n du = \frac{u^{n+1}}{n+1} \quad \text{with } n=3$$

$$= \frac{u^4}{4}$$

plug in solved integrals

$$\frac{1}{4} \int u^3 du$$

$$= \frac{u^4}{16}$$

undo substitution $u = x^4 + 1$:

$$= \frac{(x^4 + 1)^4}{16}$$

The problem solved

$$\int x^3 (x^4 + 1)^3 dx$$

$$= \frac{(x^4 + 1)^4}{16} + C$$

Antiderivative computed by maximum

$$\int f(x) dx = F(x) = \frac{(x^4 + 1)^4}{16} + C$$

$$\int_0^1 f(x) dx = \frac{15}{16}$$

Approximation ~~0.9375~~

0.9375 Ans

Q9(a) Find the center and radius of the sphere

$$x^2 + y^2 + z^2 + 3x - 4z + 1$$

Solution

$$x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$$

$$(x^2 + 3x) + y^2 + z^2 - 4z + 1 = 0$$

$$\left(x^2 + 3x + \left(\frac{3}{2}\right)^2\right) + (y - 0)^2 + \left(z^2 - 4z + \left(\frac{-4}{2}\right)^2\right)$$

$$= -1 + \left(\frac{3}{2}\right)^2 + \left(\frac{-4}{2}\right)^2$$

$$\left(x + \frac{3}{2}\right)^2 + (y^2 + 0) + (z - 2)^2 = \frac{21}{4}$$

So $(x_0, y_0, z_0) = \text{Center.}$

$$= \left(\frac{3}{2}, 0, 2\right)$$

Find radius $a = \sqrt{\frac{21}{4}}$

Q9 (B) The Region b/w
the Curve

$$y = \sqrt{x} \quad 0 \leq x \leq 4, \text{ revolved}$$

about the x axis to generate
a solid. Apply the integration
find the volume of solid

Solution Given that.

$$y = \sqrt{x} \quad 0 \leq x \leq 4 \Rightarrow a \leq x \leq b.$$

$$\text{as } V = \int_a^b \pi y^2 dx$$

$$V = \int_0^4 \pi (\sqrt{x})^2 dx$$

$$V = \pi \int_0^4 x dx = \pi \frac{x^2}{2} \Big|_0^4$$

$$V = \frac{\pi}{2} ((4)^2 - 0)$$

$$= 8\pi \text{ Ans}$$

Q3 if $A = 2i - 4j + \sqrt{5}k$ and

$B = -2i + 4j - \sqrt{5}k$ then

illustrate the vector $\text{proj}_A B$

Solution:

$$\text{proj}_A B = \frac{B \cdot A}{\|A\|^2} A \rightarrow \textcircled{1}$$

$$B \cdot A = (-2i + 4j - \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$$

$$B \cdot A = -(4)(i \cdot i) - 16(j \cdot j) - (5)(k \cdot k)$$

We know that.

$$i \cdot i = j \cdot j = k \cdot k = 1$$

$$B \cdot A = (-4(1) - 16(1) - 5(1)) \\ = -4 - 16 - 5$$

$$B \cdot A = -25 \rightarrow \textcircled{a}$$

$$\|A\|^2 = \left(\sqrt{(20)^2 + (-4j)^2} + (\sqrt{5}k)^2 \right)^2 \\ = \left(\sqrt{4 + 16 + 5} \right)^2$$

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$$\|A\|^2 = 25 \rightarrow (b)$$

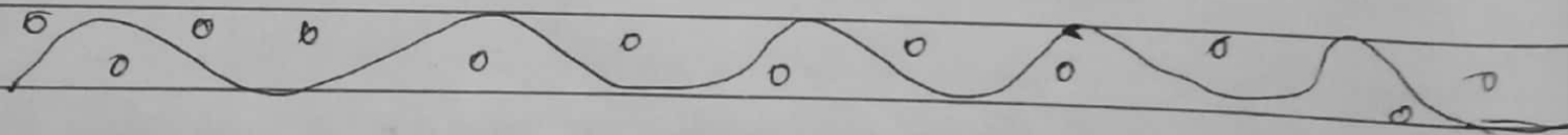
$$A = 2i - 4j + \sqrt{5}k \rightarrow (c)$$

put (a) (b), and (c) in equation (1)

$$\text{proj}_A B = \frac{-25}{25} (2i - 4j + \sqrt{5}k)$$

$$= -1(2i - 4j + \sqrt{5}k)$$

$$\Rightarrow \text{proj}_A B = -2i + 4j - \sqrt{5}k \text{ Ans}$$



Q4: Find the area region between the graph and the axis

where $y = -x^2 + 5x - 4$ $[0, 2]$

Solution given that:-

$$y = f(x) = -x^2 + 5x - 4$$

$$\text{and } [a, b] = [0, 2]$$

$$\text{As } a = 0$$

$$b = 2$$

So area under graph will

$$A = \int_a^b f(x) dx \quad \text{By the value}$$

$$A = \int_0^2 (-x^2 + 5x - 4) dx$$

$$= \frac{x^3}{3} + \frac{5x^2}{2} - 4x \Big|_0^2$$

$$= \left(\frac{-2}{3} \right)^2 + 5 \left(\frac{2}{2} \right)^2 - 4(2) - \left(\frac{0}{3} \right)^3 + 5 \left(\frac{0}{2} \right)^2 - 4(0)$$

$$= \left(\frac{-4}{3} + \frac{20}{2} - 8 \right) - 0 + 0 - 0$$

$$= \frac{-4}{3} + 10 - 8$$

$$= \frac{-4}{3} + 10 - 8$$

$$= \frac{-4}{3} + 2 \Rightarrow \frac{-4 + 6}{3}$$

$$= A = \frac{2}{3}, \text{ ~~Ans~~ } \boxed{\text{Ans} \rightarrow 0.6}$$

Q(5)(A) Estimate the angle b/w

$$A = i - 2j - 2k \text{ and } B = 6i + 3j + 2k$$

Solution:

$$A = i - 2j - 2k$$

$$|A| = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

$$B = 6i + 3j + 2k$$

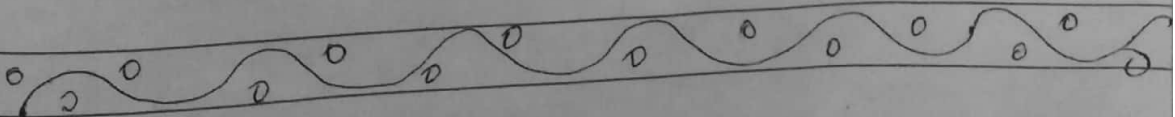
$$|B| = \sqrt{36 + 9 + 4} = \sqrt{49} = 7$$

$$\theta = \cos^{-1} \left(\frac{A \cdot B}{|A| \cdot |B|} \right)$$

$$\theta = \cos^{-1} \left(\frac{(i - 2j - 2k) \cdot (6i + 3j + 2k)}{3 \times 7} \right)$$

$$\theta = \cos^{-1} \left(\frac{(1)(6) + (-2)(3) + (-2)(2)}{21} \right)$$

$$\theta = \cos^{-1} \left(-\frac{4}{21} \right)$$



Q5: (B) Change into the spherical coordinate equation for the sphere

$$x^2 + y^2 + (z-1)^2 = 1$$

Solution:

$$x^2 + y^2 + (z-1)^2 = 1$$

$$= (r \sin \theta \cos \phi)^2 + (r \sin \theta \sin \phi)^2$$

$$+ (r \cos \theta - 1)^2 = 1$$

$$= r^2 \sin^2 \theta \cos^2 \phi + r^2 \sin^2 \theta \sin^2 \phi$$

$$+ r^2 (\cos^2 \theta - 2r \cos \theta + 1) = 1$$

$$= r^2 \sin^2 \theta (\cos^2 \phi + \sin^2 \phi)$$

$$+ r^2 (\cos^2 \theta) - 2r \cos \theta + r^2 = 1$$

$$= r^2 (\sin^2 \theta + \cos^2 \theta) - 2r \cos \theta = 0$$

$$= r^2 = 2r \cos \theta$$

$$= r = 2 \cos \theta \quad \text{Ans}$$

Thank you Sir