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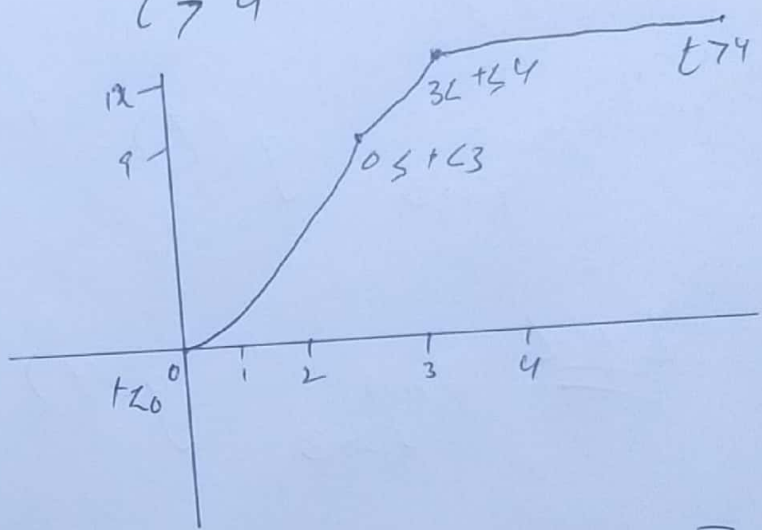
Q1 = The function $g(t)$ defind by

$$g(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \le t \le 3 \\ 2t+3 & 3 < t \le 4 \\ 12 & t > 4 \end{cases}$$

Sol:




From the graph. we conclude that the funtion $g(x)$ is continous at evey point. There is no point of discontinuity.

The all value ~~of~~ are exist of any point.

Q1 = Part = B

Find if they exist

(i) $\lim_{t \to 3} g$

Sol:

$\lim_{t \to 3} g(t) = ?$

L.H.L $\lim_{t \to 3} g(t) = \lim_{t \to 3} t^2$

$= (3)^2$

$= 9$

$\Rightarrow \lim_{t \to 3} g(t) = 9$

RHL $\lim_{t \to 3^+} g(t) = \lim_{t \to 3} (2t + 3)$

$= 2(3) + 3$

$= 6 + 3$

$= 9$

$$\Rightarrow \lim_{t \to 3} g(t) = 9$$

since L.H.L = R.H.L

so $\lim_{t \to 3} g(t) = 9$ exist

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Q2 = Find the maclaurin's series for

$$Y(x) = x^2 + \sin x$$

Sol: As by maclaurin series

$$f(x) = f(x) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{iv}(0) + \frac{x^5}{5!} \cdots$$

Here $f(x) = x^2 + \sin x$

$$f'(x) = 2x + \cos$$

$$f''(x) = 2 - \sin x$$

$$f'''(x) = 0 - \cos x$$

$$f^{iv}(x) = \sin x$$

Put $x = 0$ in all of the above equation

$f(0) = 0 + \sin 0 = 0 \Rightarrow \boxed{f(0) = 0}$

$f'(0) = 2(0) + \cos 0 = 0 + 1 = \boxed{f'(0) = 1}$

$f''(0) = 2 - \sin 0 = 2 - 0 \Rightarrow \boxed{f''(0) = 2}$

$f'''(0) = -\cos 0 = -1 \Rightarrow \boxed{f'''(0) = -1}$

Thus eq (1) $\Rightarrow$

$$f(x) = 0 + \frac{x}{1!} \times 1 + \frac{x^2}{2!} \times 2 + \frac{x^3}{3!} \times -1 + \cdots$$

$$f(x) = x^2 + \sin x = x + \frac{\cancel{2} x^2}{\cancel{2}} - \frac{x^3}{6} + \cdots$$

$$x^2 + \sin x = x + x^2 - \frac{x^3}{6} + \cdots$$

is the required maclaurin's series.

Q:3 Find $y''$ given

$1 + xy = x^2 + y^2$

Sol:: (A) $y'' = ?$

$$1 + xy = x^2 + y^2 \quad \text{--- (1)}$$

~~diff~~ (1) w.r.t $x$

$$\frac{d}{dx}(1 + xy) = \cancel{\frac{d}{dx}x^2} + \frac{d}{dx}(x^2 + y^2)$$

$$0 + \frac{d}{dx}(\underset{\mathrm{I}}{x}\,\underset{\mathrm{II}}{y}) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$$

$$x\frac{dy}{dx} + y\frac{d}{dx}x = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} + y \cdot 1 = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$$

$$(x - 2y)\frac{dy}{dx} = 2x - y$$

$$\Rightarrow \frac{dy}{\partial x} = \frac{2x-y}{x-2y}$$

$$y' = \frac{2x-y}{x-2y} \quad \text{—— (2)}$$

As an. diff (2) w.r.t x

$$y'' = \frac{d}{\partial x}\left(\frac{2x-y}{x-2y}\right) \quad \text{By Q. Rule}$$

$$y'' = \frac{(x-2y)\frac{d}{\partial x}(2x-y) - (2x-y)\frac{d}{\partial x}(x-2y)}{(x-2y)}$$

$$y'' = \frac{(x-2y\left(2\cdot 1 - \frac{dy}{\partial x}\right) - (2x-y)\left(1-2\frac{dy}{\partial x}\right)}{(x-2y)^2}$$

By eq (2)

$$y'' = \frac{(x-2y)\left(2-\frac{2x-y}{x-2y}\right) - (2x-y)\left(1-2\left(\frac{2x-y}{x-2y}\right)\right)}{(x-2y)^2}$$

$$= \frac{1}{(x-2y)^2}\left[(x-2y)\left(\frac{\cancel{2x}-4y-\cancel{2x}+y}{x-2y}\right)-(2x-y)\left(\frac{x-2y-2(2x-y)}{x-2y}\right)\right.$$

$$= \frac{1}{(x-2y)^2}\left[(x-2y)\left(\frac{-3y}{x-2y}\right) - (2x-y)\left(\frac{x-2y-4x+2y}{x-2y}\right)\right]$$

$$y'' = \frac{1}{(x-2y)^2}\left(\frac{-3y(x-2y)}{x-2y} - \frac{(-3-x)(2x-y)}{x-2y}\right)$$

$$y'' = \frac{1}{(x-2y)^2}\left(\frac{-3xy+6y^2}{x-2y} - \frac{-6x^2+3xy}{x-2y}\right)$$

$$y'' = \frac{1}{(x-2y)^2}\left(\frac{-3xy+6y^2+6x^2-3xy}{x-2y}\right)$$

$$y'' = \frac{6x^2+6y^2-6xy}{(x-2y)^3} \quad \text{Ans}$$

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Q=3 Part (ii) Find $y'$ by using logarthemic differentiation

$y = x^3(1+3)^9 e^{6x}$

Sol:

$y' = ?$

$y = x^3(1+x)^9 e^{6x}$

taking log on both side

$$\log y = \log \left(x^3 (1+x)^9 e^{6x}\right)$$

$$\log y = \log x^3 + \log (1+x)^9 + \log e^{6x}$$

$$\log y = 3 \log x + 9 \log (1+x) + \log e^{6x} \quad \text{——— (1)}$$

diff (1) w.r.t. $x$

$$\frac{d}{dx} \log y = \frac{d}{dx} 3 \log x + 9 \frac{d}{dx} \log (1+x) + \frac{d}{dx} \log e^{6x}$$

$$\frac{1}{y} \frac{dy}{dx} = 3 \cdot \frac{1}{x} + 9 \cdot \frac{1}{1+x} \frac{d}{dx} (1+x) + \frac{1}{e^{6x}} \frac{d}{dx} e^{6x}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{3}{x} + \frac{9}{1+x} (0+1) + \frac{1}{e^{6x}} e^{6x} \frac{d}{dx} 6x$$

$$\frac{1}{y} \frac{dy}{dx} = 3/x + \frac{9}{1+x} + 6$$

$$\frac{dy}{dx} = y \left[\frac{3}{x} + \frac{9}{1+x} + 6\right]. \quad \text{Ans}$$

OR

put $y = x^3 (1+x)^9 e^{6x}$

$$y' = x^3 (1+x)^9 e^{6x} \left[\frac{3}{x} + \frac{9}{1+x} + 6\right].$$