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Section : "B"

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OINU - Official :

Q No# 1

$$= \frac{dy}{dt} = e^{y-t} \sec(y)(1+t^2) \quad y(0)=0$$

Solution:

$$\Rightarrow \frac{dy}{dt} = e^{y-t} \sec(y)(1+t^2) \quad y(0)=0$$

$y(0)=0$ so; $x=0$, $y=0$

$$\Rightarrow \frac{1}{e^{y}\sec(y)} dy = (1+t^2) e^{-t} dt.$$

$\Rightarrow$ As $\cos(y) = 1/\sec(y)$

$$\Rightarrow \int e^{-y} \cos(y) dy = \int (1+t^2) e^{-t} dt.$$

using integration by parts:

$$\Rightarrow e^{-y} \int \cos y \, dx - \int \left(\int \cos y \cdot \frac{d}{dy} e^{-y} \right) =$$

$$(1+t^2) \int e^{-t} - \int \left(\int e^{-t} \cdot \frac{d}{dt} (1+t^2) \right) \Rightarrow \text{eq (i)}$$

NOW

$\Rightarrow$ L.H.S

$$\Rightarrow e^{-y} \int \cos y \, dx - \int \left(\int \cos y \cdot \frac{d}{dy} e^{-y} \right)$$

$$\Rightarrow e^{-y} \sin y - \int (\sin y \cdot e^{-y} (-1)).$$

P.T.O

$\Rightarrow e^{-y} \sin y + \int (\sin y \cdot e^{-y})$

$\Rightarrow e^{-y} \sin y + \int (e^{-y} \sin y).$

Again using integration by Parts:

$\Rightarrow e^{-y} \sin y + e^{-y}(-\cos y) - \int (\int \sin y \, \frac{d}{dt} e^{-y})$

$\Rightarrow e^{-y} \sin y + e^{-y}(-\cos y) - \int (-\cos y \, \frac{e^{-y}}{-1})$

$\Rightarrow e^{-y} \sin y - e^{-y} \cos y - \int (\cos y \, e^{-y}).$

Since.

$\Rightarrow \int (\cos y \, e^{-y}) = L.H.S$

Now:

since it is again same to the First one so L.H.S will becomes:

So;

$\Rightarrow L.H.S = e^{-y}(\sin y - \cos y) - L.H.S$

$2L.H.S = e^{-y}(\sin y - \cos y)$

$$L.H.S = \frac{e^{-y}(\sin y - \cos y)}{2}$$

~~Than~~ :

~~Answer~~.

P.T.O

Now taking R.H.S.:

$$\Rightarrow \int (1+t^2)\, e^{-t}\, dt.$$

$$\Rightarrow (1+t^2)\int e^{-t} - \int\left(\int e^{-t} \cdot \tfrac{d}{dt}(1+t^2)\right).$$

$$\Rightarrow (1+t^2)\, e^{-t} - \int(-e^{-t}\ (2t)$$

$$\Rightarrow -(1+t^2)\, e^{-t} + \int (2t)\, e^{-t}).$$

Againg using integration by Parts:

$$\Rightarrow -(1+t^2)e^{-t} + (2t\int e^{-t} - \int\left(\int e^{-t}\ \tfrac{d}{dt}\, 2t\right))$$

$$\Rightarrow -(1+t^2)\, e^{-t} + (-2t\, e^{-t} - \int(-e^{-t}\ 2$$

$$\Rightarrow -(1+t^2)\, e^{-t} + (-2t\, e^{-t} + \int(2e^{-t})).$$

$$\Rightarrow -(1+t^2)\, e^{-t}\ (-2t\, e^{-t} - 2e^{-t}) + c.$$

$$\Rightarrow -(1+t^2)e^{-t} - 2te^{-t} - 2e^{-t} + c.$$

$$\Rightarrow -e^{-t} - e^{-t} - 2te^{-t} - 2e^{-t} + c$$

$$\Rightarrow -(t^2+2t+3)\, e^{-t} + c = R.H.S.$$

Now take:

$$L.H.S = R.H.S$$

$$\Rightarrow \frac{e^{-y}(\sin y - \cos y)}{2} = -(t^2+2t+3)e^{-t} + c$$

We know that :

So;

$t = 0$, $y = 0$

Put it :

$\Rightarrow \frac{1}{2}(0-1) = -3 + c$

$\Rightarrow \boxed{c = \frac{5}{2}}$

Put the value of "c".

$$\Rightarrow \left[\frac{e^{y}}{2}(\sin y - \cos y) = -(x^2 + 2t + 3)e^{-t} + \frac{5}{2}\right]$$

Answer

★★★

Q NO # 2

$$: (\sqrt{x+y} + \sqrt{x-y})dx - (\sqrt{x+y} - \sqrt{x-y})dy = 0$$

Solution :

$$\Rightarrow (\sqrt{x+y} + \sqrt{x-y})dx - (\sqrt{x+y} - \sqrt{x-y})dy = 0$$

This is Homogenous Differtial eq in x & y :

$$\Rightarrow y = vx$$

$$\Rightarrow dy/dx = v + x\,dv/dx$$

$\Rightarrow$ Then eq 1 becomes:

$$\Rightarrow v + x\,dv/dx = \frac{\sqrt{x+vx} + \sqrt{x-vx}}{\sqrt{x+vx} - \sqrt{x-vx}}$$

NOW ;

$$\Rightarrow v + x\,dv/dx = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$\Rightarrow v + x\,dv/dx = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}} \times \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$\Rightarrow v + x\,dv/dx = \frac{1+\cancel{v} + 1-\cancel{v} + 2\sqrt{1-v^2}}{2v}$$

$$\Rightarrow v + x\,dv/dx = \frac{\cancel{2}(1+\sqrt{1-v^2})}{\cancel{2}v}$$

P.T.O

$$\Rightarrow v + x\,dv/dx = \frac{1+\sqrt{1-v^2}}{v}$$

$$\Rightarrow x\,dv/dx = \frac{1+\sqrt{1-v^2}}{v} - v)$$

$$\Rightarrow x\,dv/dx = \frac{1+\sqrt{1-v^2} - v^2}{v}$$

$$\Rightarrow x\,dv/dx = \frac{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})}{v}$$

$$\Rightarrow \frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = dx/x$$

So:

Taking integrals on b/s.

$$\Rightarrow \int \frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \int dx/x$$

Put $1+\sqrt{1-v^2} = t$

$$\Rightarrow \tfrac{1}{2}(1-v^2)^{-1/2}(-2v)\,dv = dt$$

$$\Rightarrow \frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\Rightarrow \int \frac{-dt}{t} = \int dx/x$$

Now:

$$\Rightarrow -\ln t = \ln x + \ln c$$

$$\Rightarrow -\ln(1+\sqrt{1+v^2}) = \ln cx$$

P.T.O.

$$\Rightarrow \ln\left(1+\sqrt{1-v^2}\right) = -\ln cx$$

$$\Rightarrow \cancel{\ln}\left(1+\sqrt{1-v^2}\right) = \cancel{\ln}(cx)^{-1}$$

$$\Rightarrow 1+\sqrt{1-v^2} = 1/cx$$

$$\Rightarrow 1+\sqrt{1-y^2/x^2} = 1/cx$$

$$\Rightarrow 1+\sqrt{\frac{x^2-y^2}{x^2}} = 1/cx$$

$$\Rightarrow x+\sqrt{x^2-y^2} = 1/c$$

$$\Rightarrow x+\sqrt{x^2-y^2} = c_1$$

$\because 1/c = c_1$

which is requried solution:

☆☆☆ XXX

P.T.O.

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Q No # 3

$\because (D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$

Solution :

$\Rightarrow (D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x.$

We know that ;

$\Rightarrow f(D)y = f(x)$

it is non-Homogenous linear eq. so ;

$\Rightarrow y = y_c + y_p$ —— (i)

$\Rightarrow$ Complementary Solution y_c.

$\Rightarrow D^4 + D^2 = 0 \Rightarrow D^2(D^2+1) = 0$

Either ;

$\Rightarrow D^2 = 0 \Rightarrow \boxed{D = 0}$

$\Rightarrow D^2 + 1 = 0 \Rightarrow D^2 = -1$

$\Rightarrow D = \sqrt{-1} \Rightarrow \boxed{D = i}$

or : $\boxed{D = 0 + i}$

$\Rightarrow D = \sqrt{-1} \Rightarrow \boxed{D = i}$:

$\Rightarrow$ Roots are real & complex.

$$\Rightarrow y_c = c_1 e^{0x} + e^{0x}(c_2 \cos x + c_3 \sin x)$$

$$\Rightarrow \boxed{y_c = c_1 + c_2 \cos x + c_3 \sin x}$$

$$\Rightarrow y_p = \frac{1}{f(D)} F(x.$$

NOW;

$$\Rightarrow y_p = \frac{1}{D^4 + D^2}(3x^2 + 4\sin x - 2\cos x).$$

$$\Rightarrow \frac{3x^2}{D^4 + D^2} + \frac{4\sin x}{D^4 + D^2} - \frac{2\cos x}{D^4 + D^2}$$

$$\Rightarrow f(D) = D^4 + D^2.$$

So;

$$\Delta = 0 \Rightarrow f(D) = 0$$

NOW;

$$\Rightarrow f(D) = 4D^3 + 2D.$$

so also for D;

$$\Rightarrow D = 0 \Rightarrow f'(D) = 0$$

$\Rightarrow$ again differentiating:

$$\Rightarrow f''(D) = 12D + 2$$

P.T.O

$\Rightarrow$ So for $D = 0$

$\Rightarrow f''(D) = 12(0) + 2 = \boxed{2}$

$\Rightarrow$ So replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$.

$\Rightarrow y_p = \frac{x^2 \, 3x^2}{12D+2} + \frac{x^2}{12D+2} \cdot 4\sin x - \frac{x^2}{12D+2} \, 2\cos x$

$\Rightarrow$ Put $D = 0$ in all :

$\Rightarrow y_p = \frac{x^2 \cdot 3x^2}{12(0)+2} + \frac{x^2}{12(0)+2} \cdot 4\sin x - \frac{2x^2 \cos x}{12(0)+2}$

$\Rightarrow y_p = \frac{3x^4}{2} + \frac{4x^2 \sin x}{2} - \frac{2x^2 \cos x}{2}$

$\Rightarrow \frac{3x^4}{2} + 2x \sin x - x^2 \cos x$

So:

Putting in eq (1)

$y = C_1 + C_2 \cos x + C_3 \cos x + 3/2 \, x^4 + 2x^2 \sin x - x^2 \cos x$.

So;

$$\boxed{y = C_1 + (C_2 - x^2)\cos x + (C_3 + 2x^2)\sin x + \frac{3}{2} x^4}$$

Ans :

The End !