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Subject = DSP.

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Module = 18

Question 1 a

The equation is

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2, 2$$

Hence.

$$y_h(n) = C_1 2^n + C_2 n 2^n.$$

The particular solution is

$$y_p(n) = K(-1)^n u(n)$$

Substituting this solution in to difference equation we obtain.

$$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2)$$
$$= (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

for

$n = 2$, $K(1+4+4) = 2$

$K = 2/9$ The total solution is

$$y(n) = \left[C_1 2^n + C_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n).$$

from the initial conditions.
we obtain $y(0) = 1$, $y(1) = 2$ then

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$$c_1 + \frac{2}{9} = 1$$

$$\Rightarrow c_1 = \frac{7}{9}$$

$$2c_1 + 2c_2 - \frac{2}{9} = 2.$$

$$\Rightarrow c_2 = \frac{1}{3}$$

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Question 1 b.

The characteristic equation is.

$$\lambda^2 - 0.7\lambda + 0.1 = 0.$$

$\lambda = \frac{1}{2}, \frac{1}{3}$ Hence.

$$y_h(n) = c_1 \frac{1}{2}^n + c_2 \frac{1}{3}^n.$$

~~Who~~ with $x(n) = \delta(n)$, we have.

$y(0) = 2.$

$y(1) - 0.7y(0) = 0 \Rightarrow \boxed{y(1) = 1.4}$

Hence.

$c_1 + c_2 = 2$ and.

$\frac{1}{2} c_1 + \frac{1}{5} c_2 = 1.4 = \frac{7}{5}.$

$c_1 + \frac{2}{5} c_2 = \frac{14}{5}.$

These equations yield.

unit step respons next page.

$c_1 = \frac{10}{3}, c_2 = -\frac{4}{3}.$

$$h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{3}\right)^n\right]u(n)$$

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The step response is

$$s(n) = \sum_{k=0}^{n} h(n-k).$$

$$= \frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3} \left(\frac{1}{2}\right)^{n} \sum_{k=0}^{n} 2^{k} - \frac{4}{3} \left(\frac{1}{5}\right)^{n} \sum_{k=0}^{n} 5^{k}$$

$$= \frac{10}{3} \left(\frac{1}{2}^{n} \left(2^{n+1} - 1\right) u(n) - \frac{1}{3} \right.$$

$$\left[\cdot 5^{n+1} - 1 \right] u(n)$$

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Question 2a.

Soln,

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$$= \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$A = 4$, $B = -3$, $C = -1$

Hence, $x(n) = \left[4(2)^n - 3 - n\right]u(n)$.

question 2b.

Sol:→

we have

$$x(n) = \frac{1}{2\pi j}\oint_c \frac{z^{n-1}}{1-az^{-1}}dz = \frac{1}{2\pi j}\oint_c \frac{z^n dz}{z-a}.$$

where 'c' is a circle at radius greater than |a| we shall evaluate this integral using $f(z) = z^n$ we distinguish two cases.

(1) → if $n \geq 0$, $f(z)$ has only zeros and hence no pole inside 'c' The only pole. 'c' is $z = a$ Hence.

$$x(n) = f(z_a) = a^n \quad n \geq 0$$

(2) → if $n < 0$ $f(z) = z^n$ has n^{th} order poles at $z = 0$ which is also inside 'c' Thus there are contributions from both ~~sides~~ poles. for $n = -1$ we have

$$x(-1) = \frac{1}{2\pi j}\oint_c \frac{1}{z(z-a)}dz = \left.\frac{1}{z-a}\right|_{z=0} + \left.\frac{1}{z^2}\right|_{z=a} = 0$$

By continuing in the same way we can show that $x(n) = 0$ for $n < 0$ thus.

$$x(n) = a^n u(n)$$

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Question 3a.

Sol:

At $\omega = 0$ we have.

$$H(0) = \frac{b_0}{(1-p)^2} = 1.$$

Hence

$$b_0 = (1-p)^2$$

At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-p)^2}{(1-pe^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{(1-p\cos(\pi/4) + jp\sin(\pi/4))^2}$$

$$= \frac{(1-p)^2}{(1-p/\sqrt{2} + jp/\sqrt{2})^2}$$

Hence

$$\frac{(1-p)^4}{[(1-p/\sqrt{2})^2 + p^2/2]^2} = 1/2.$$

or equivalently

$$\sqrt{2}(1-p)^2 = 1+p^2 - \sqrt{2}p$$

The value of $p = 0.32$ satisfies this equation. consequently. the system function for the desired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}.$$

The same principles can be applied for the design of bandpass fiters

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question 3b.

Sol.:→.

Clearly the filters must have poles at

$$P_{b_2} = re^{\pm j\pi/2}.$$

and zeros at $z=1$ and $z=-1$

Consequently the system function is

$$H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{z^2-1}{z^2+r^2}$$

The gain factor is determined by evaluating the frequency respons $H(\omega)$ of the filters at $\omega = \pi/2$. Thus.

$$H\left(\frac{\pi}{2}\right) = G \frac{2}{1-r^2} = 1.$$

$$G = \frac{1-r^2}{2}.$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$. Thus.

$$\left| H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \frac{2 - 2\cos(8\pi/9)}{1 + r^4 + 2r^2\cos(8\pi/9)} = 1/2.$$

Or equivalently.

$$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^4.$$

The value of $r^2 = 0.7$ satisfies this equation. There fore the system function desired filter is

$$H(z) = 0.15 \frac{1 - z^{-2}}{1 + 0.7z^{-2}}.$$

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Question 4a.

A finite-duration sequence of length L is given as.

$$x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & \text{otherwise.} \end{cases}$$

Sol:-

The fourier transform of this sequence.

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega}}$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega L/2)} e^{-j\omega(L-1)/2}$$

The magnitude and phase of $X(\omega)$ are illustrated $L=10$ The N-point DFT. of $x(n)$ is simply $X(\omega)$ evaluated. at the set of N equally spaced frequencies $\omega_k = 2\pi k/N$ $k = 0, 1, \ldots, N-1$ Hence.

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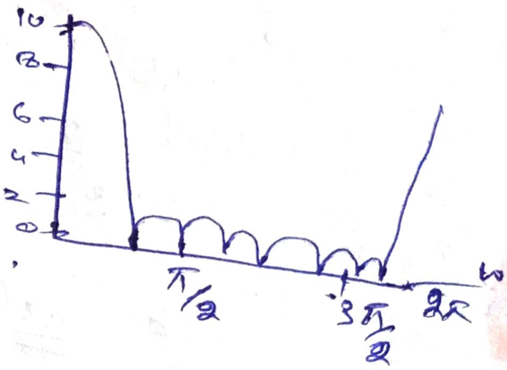
$$X(k) = \frac{1-e^{-j2\pi kL/N}}{1-e^{-j2\pi k/N}} \qquad k=0,1,\ldots N-1$$

$$= \frac{\sin(\pi kL/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)/N}$$

if N is selected such that N = L then DFT.

$$X(k) = \begin{cases} L, & k=0 \\ 0, & k=1,2\ldots L-1. \end{cases}$$



Thus there is only one non zero value in DFT this is apparent from observation of $X(\omega)$ since $X(\omega)=0$ at the frequencies $\omega_k = 2\pi k/L$ $k \neq 0$

Question 4 b.

$x1(n) = \{ \underset{\uparrow}{2}, 1, 2, 1 \}$.

$x2(n) = \{ \underset{\uparrow}{1}, 2, 3, 4 \}$.

Sol:-

Each seequomce consists of four nongero paints. for the purposes of illustrating the operations invalved in circular convalution it is descriable to graph each seqeuence as paint on a circle. Thus the sequomces $x1(n)$ and $x2(n)$ are graphed.

Now $x_3(m)$ is abtained by circularly convalving $x_1(n)$ and $x_2(n)$

$$x_3(0) = \sum_{x=0}^{3} x_1(n) x_2((L-n))_N .$$

$x2((-n))$ is simply the seequence $x_2(n)$ falded and graphed on a circle.

$$\boxed{x_3(0) = 14.}$$

for $m=1$ we have.

$$x_3(1) = \sum_{x=0}^{3} x_1(n) x_2((1-n))_4.$$

it is easily verifred that $x_2((1-n))_4$ is simply the seequomce $x_2((1-n))_4$ rotated by one unit in time as illustrated.

For $m=2$.
we have.

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4.$$

Now $x_2((2-n))_4$ is the folded sequence

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