

Course title

Electro magnetic  
Theory

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Module

4<sup>th</sup>

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Q1: Transform the vector  $B = y_i(x+z)$   
located at point  $(-2, 6, 3)$   
into cylindrical coordinate

Ans: Part (a)

Solution:-

$$B = y_i(x+z)j$$

$$\text{Point } (-2, 6, 3)$$

Then

$$B = y_i(x_j + z_j)j$$

$$B = y_x i j + y_z i j$$

$$P = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}(y/x)$$

$$z = z$$

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$$r = \sqrt{(-2) + (6)^2}$$

$$\boxed{5.83}$$

$$\phi = -71.565$$

$$\boxed{z = 3}$$

$$B = (5.83, -71.56, 3)$$

Q1

Part (B)

Convert the point (3,4,5) from

Cartesian coordinates.

Sol:

For spherical we have to find

$$r, \phi$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r = \sqrt{3^2 + 4^2 + 5^2} = 7.07$$

$$r = 7.07$$

$$\phi = \tan^{-1} y/x$$

$$= \tan^{-1} 4/3$$

$$\phi = 53.13$$

$$\theta = \cos^{-1} \frac{z}{\rho}$$
$$= \cos^{-1} \frac{5}{7.07}$$

$$\theta = 45^\circ$$

$$(\rho, \theta, \phi) = (7.07, 45^\circ, 53.13^\circ)$$

Q1  
Part (c):-

Find the spherical coordinates

of ~~B~~<sup>A</sup> (2, 3, -1)

Q1:  
part c:  
solution:-

$$A (2, 3, -1)$$

$$\rho = (\sqrt{x^2 + y^2 + z^2})$$
$$= \sqrt{2^2 + 3^2 + (-1)^2}$$

$$\theta = \cos^{-1} \frac{z}{\rho}$$

$$\cos \left( \frac{-1}{3.74} \right)$$

$$\boxed{\theta = 105.5^\circ}$$

$$\phi = \tan^{-1} (y/x)$$

$$\boxed{\phi = 56.3^\circ}$$

Q1:

Part (D):-

Find the "spherical" cartesian

Coordinates of B(4.25, 120)

Answer:-

point B is actually given  
is spherical

$$(\rho, \theta, \phi)$$

we have to find  $(x, y, z)$

$$\begin{aligned} x &= \rho \sin \theta \cos \phi \\ &= 4.25 \sin 25^\circ \cos 120^\circ \\ &= 0.84 \end{aligned}$$

$$\begin{aligned} y &= \rho \sin \theta \sin \phi \\ &= 4.25 \sin 25^\circ \sin 120^\circ \\ &= 1.40 \end{aligned}$$

$$z = 8 \cos \theta = 4 \cos 25^\circ$$

$$z = 3.62$$

$$(x, y, z) = (-0.84, 1.46, 3.6)$$

Q1:

part(e) :-

Find the force between two charges when they brought in contact and separated by 4cm apart, charges are  $2\text{nc}$  and  $-1\text{nc}$  in  $\mu\text{N}$ .

Solution:

$$F = \frac{k q_1 q_2}{r^2}$$

$$\text{where } k = \frac{1}{4\pi\epsilon_0}$$

$$F = \frac{2 \times 10^{-9} \times 1 \times 10^{-9}}{9\pi \times 8.85 \times 10^{-12} \times (4 \times 10^{-2})^2}$$

$$F = -1123 \times 10^{-5}$$

Then configuration

$$F = -11.23 \text{ } \mu\text{W}$$

Q1  
part (F):

Find the electric field intensity of two charges  $-2\text{ } \mu\text{C}$  and  $1\text{ } \mu\text{C}$  separated by distance  $2\text{m}$  in air

Solution

$$q_1 = -2\text{ } \mu\text{C} \text{ and } q_2 = 1\text{ } \mu\text{C}$$

$$\text{So } 2\text{m } \epsilon_1$$

$$\epsilon_1 = \frac{q_1}{2\pi\epsilon_0 r^2} = \frac{-2}{4\pi \times 8.85 \times 10^{-12} \times 1^2}$$

$$\epsilon_1 = -1.0798 \times 10^{10} \text{ N/C}$$

$$\epsilon_2 = \frac{-1}{4\pi\epsilon_0 \times 10^2} = -8.9 \times 10^9 \text{ N/C}$$

$$\epsilon_2 = -8.9 \times 10^9 \text{ N/C}$$

Q1  
Part (G)

Determine the charges that  
produce an electric field  
strength of 40 v/cm at a  
distance of 30cm in vacuum  
(in  $10^{-8}C$ )

Solution :-

$$E = 40 \text{ v/m} = 40 / 10^{-2} \text{ m}$$

$$E = 4000 \text{ v/m}$$

$$r = 30 \times 10^{-2} \text{ m}$$

Find  $q = ?$

Now

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

$$q = E \times 4\pi\epsilon_0 r^2$$

$$= 4000 \times 4 \times 3.14 \times 8.85 \times 10^{-12} (30 \times 10^{-2})^2$$

$$q = 4 \times 10^{-8} C$$



Q1

part (h)

A charges of  $2 \times 10^{-7} \text{ C}$  is acted upon by a force of  $0.1 \text{ N}$ . determine the distance to the other charges of  $4.5 \times 10^{-7} \text{ C}$ , both the charges are in vacuum.

Solution:

$$r = ?$$

To find  $r$  we

$$q_1 = 2 \times 10^{-7} \text{ and}$$

$$q_2 = 4.5 \times 10^{-7}$$

where,

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

$$r^2 = \frac{q_1 q_2}{4\pi\epsilon_0 F}$$

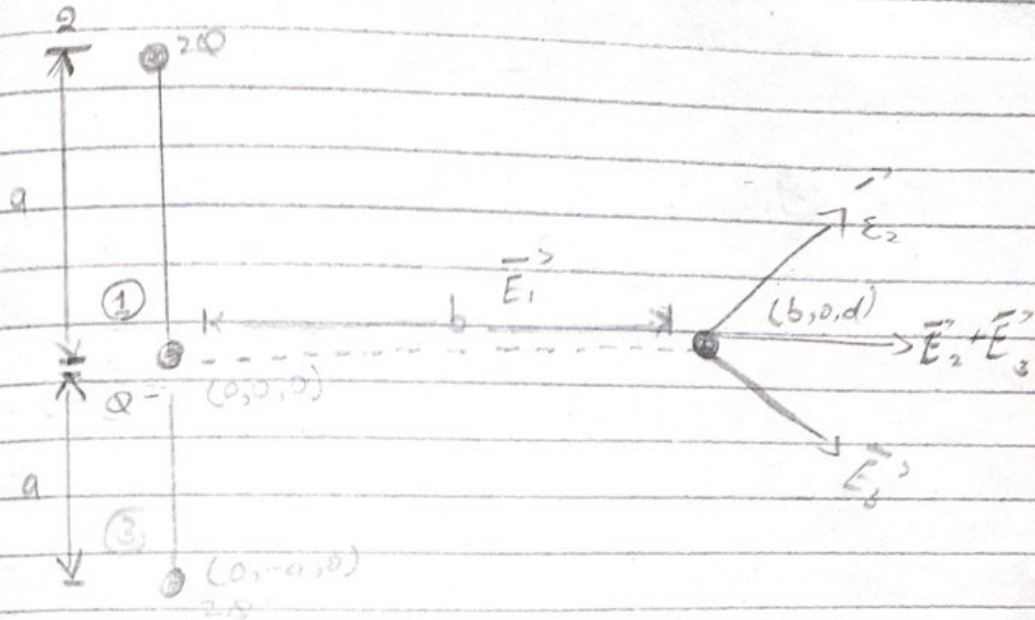
Now putting values.

$$r^2 = \frac{2 \times 10^{-7} \times 4.5 \times 10^{-7}}{4\pi\epsilon_0 \times 0.1}$$

$$\boxed{r^2 = 0.09 \text{ m}} \quad R = 0.0899 \text{ m}$$

Find  $E$  at  $P$ .  $(0, a, 0)$

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$$r = b\hat{x} + 0\hat{y} + 0\hat{z}$$

$$r_1 = 0\hat{x} + 0\hat{y} + 0\hat{z}$$

$$r_2 = 0\hat{x} + 0\hat{y} + 0\hat{z}$$

$$r_3 = 0\hat{x} + -a\hat{y} + 0\hat{z}$$

$$r = r_1 = b\hat{x} + 0\hat{y} + 0\hat{z}$$

$$r = r_2 = b\hat{x} - a\hat{y} + 0\hat{z}$$

$$r = r_3 = b\hat{x} + 0\hat{y} + 0\hat{z}$$

$$|r - r_1| = b$$

$$|r - r_2| = \sqrt{b^2 + a^2}$$

$$|r - r_3| = \sqrt{b^2 + a^2}$$

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Now

$$E = \frac{Q}{4\pi\epsilon_0 r}$$

For charge 1

$$E_1 = \frac{-Q}{4\pi\epsilon_0} \frac{\vec{r} - r_1}{r^2 - r_1} = \frac{-Q}{4\pi\epsilon_0} \left[ \frac{bax}{b} \right] \left[ \frac{1}{(b)^2} \right]$$

For charge 2

$$E_2 = \frac{2Q}{4\pi\epsilon_0} \left[ \frac{bax - a \cdot ay}{\sqrt{b^2 + a^2}} \right] \left[ \frac{1}{(\sqrt{b^2 + a^2})^2} \right]$$

For charge 3

$$E_3 = \frac{2Q}{4\pi\epsilon_0} \left[ \frac{bax + a \cdot ay}{\sqrt{b^2 + a^2}} \right] \left[ \frac{1}{(\sqrt{b^2 + a^2})^2} \right]$$

$$E = E_1 + E_2 + E_3$$

$$E = \frac{-Q}{4\pi\epsilon_0} \left[ \frac{bax}{b} \right] \left[ \frac{1}{b^2} \right] + \frac{2Q}{4\pi\epsilon_0} \left[ \frac{bax - a \cdot ay}{\sqrt{b^2 + a^2}} \right] \left[ \frac{1}{(b^2 + a^2)} \right]$$
$$+ \frac{2Q}{4\pi\epsilon_0} \left[ \frac{bax + a \cdot ay}{\sqrt{b^2 + a^2}} \right] \left[ \frac{1}{(b^2 + a^2)} \right]$$

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$$E = \frac{Q}{4\pi\epsilon_0} \left[ \frac{1}{\left[\frac{bax}{b}\right] \left[\frac{1}{b^2}\right]} + \frac{2}{\left[\frac{bax - a \cdot ay}{\sqrt{b^2 + a^2}}\right] \left[\frac{1}{b^2 + a^2}\right]} + \frac{2}{\left[\frac{bax + a \cdot ay}{\sqrt{b^2 + a^2}}\right] \left[\frac{1}{b^2 + a^2}\right]} \right]$$

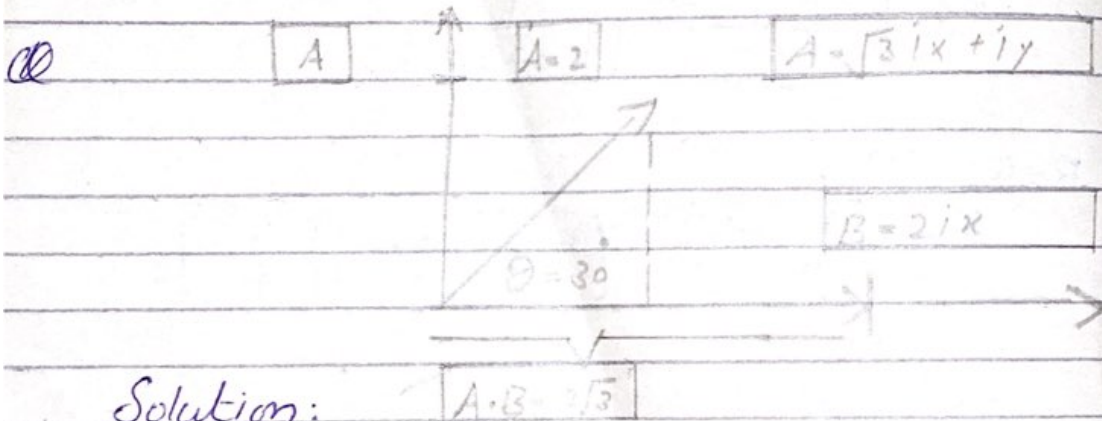
$$E = \frac{Q}{4\pi\epsilon_0} \left[ \frac{1}{\left[\frac{bax}{b}\right] \left[\frac{1}{b^2}\right]} + \frac{2}{\left[\frac{bax - a \cdot ay}{\sqrt{b^2 + a^2}}\right] \left[\frac{1}{b^2 + a^2}\right]} + \frac{2}{\left[\frac{bax + a \cdot ay}{\sqrt{b^2 + a^2}}\right] \left[\frac{1}{b^2 + a^2}\right]} \right]$$

$$\left[ \frac{1}{b^2 + a^2} \right]$$

Q2

(a)

Find the angle between the vectors  
shown in figure.



Solution:

Part (a)

$$A = \sqrt{3}ix + iy$$

$$|A| = 2$$

$$B = 2ix$$

$$|B| = 2$$

So

$$A \cdot B = \sqrt{3}$$

Now

$$A \cdot B = |A||B| \cos \theta$$

$$\cos \theta = \frac{A \cdot B}{|A||B|}$$

$$\theta = \cos^{-1} \left( \frac{\sqrt{3}}{1 \times 2} \right)$$

$$\theta = 30$$

Q2

Part (B)

Find the gradient of each of the following function where  $a$  and  $b$  are constant.

(i)  $f = ax^2 + by^3z$

Solution:

$$f = ax^2 + by^3z$$

$$\nabla f = \left( \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k \right) (ax^2 + by^3z)$$

$$\nabla f = \frac{\partial}{\partial x} ax^2 i + \frac{\partial}{\partial y} by^3z j + \frac{\partial}{\partial z} by^3z k$$

Then now

$$\nabla f = 2ax i + 3by^2z j + by^3 k$$

Then we find

$$\nabla f = 2ax i + 3by^2z j + by^3 k$$

$$\text{ii) } f = ar^2 \sin \phi + brz \cos 2\phi$$

Solution :

gradient in cos. of  
spherical ...

$$\nabla f = \frac{\partial f}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \phi} \hat{\phi} + \frac{1}{r \sin \phi} \frac{\partial f}{\partial \theta} \hat{\theta}$$

$$\frac{\partial f}{\partial r} = \frac{\partial}{\partial r} (ar^2 \sin \phi + brz \cos 2\phi) r^{n+1} \hat{r}$$

$$+ \frac{1}{r} \frac{\partial}{\partial \phi} (ar^2 \sin \phi + brz \cos 2\phi) \hat{\phi}$$

$$+ \frac{1}{r \sin \phi} \frac{\partial}{\partial \theta} (ar^2 \sin \phi + brz \cos 2\phi) \hat{\theta}$$

Then

$$\nabla f = (\partial ar^2 \sin \phi + bz \cos 2\phi) r^{n+1} \hat{r}$$

$$+ \frac{1}{r \sin \phi} (ar^2 \cos \phi - 2brz \sin \phi) \hat{\phi}$$

$$\nabla f = (\partial ar^2 \sin \phi + bz \cos 2\phi)$$

$$(ar^2 \cos \phi - 2brz \sin \phi) r^{n+1} \hat{\phi}$$

$$+ (ar^2 \cos \phi - 2brz \sin \phi)$$

in the case of cylindrical.

$$\nabla f = \frac{\partial f}{\partial \rho} \hat{\rho} + \frac{1}{\rho} \frac{\partial f}{\partial \phi} \hat{\phi} + \frac{\partial f}{\partial z} \hat{z}$$

$$\nabla f = 0 \hat{\rho} + \frac{1}{\rho} (a \rho^2 \cos \phi - 2 b \rho z \sin 2\phi) \hat{\phi} + (b \cos 2\phi) \hat{z}$$

When the first term is zero (0)

$$A \hat{\rho} = \frac{1}{\rho} (a \rho^2 \cos \phi - 2 b \rho z \sin 2\phi) \hat{\phi} + (b \cos 2\phi) \hat{z}$$