SUBJECT: STRUCTURAL ANALYSIS I

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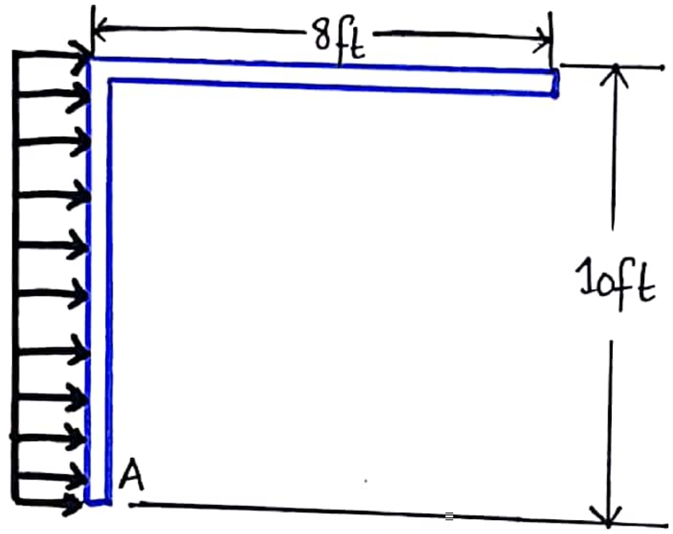
SECTION:B

QUESTION No. 1

Determine the vertical displacement of free end point C on the frame shown in figure. Take $E = 29(10^3)$ ksi and $I = 600\ in^4$ for both members. Use method of Virtual Work.

$\Delta_C = ?$
$E = 29 \times 10^3$ ksi
$I = 600\ in^4$



Solution:

Finding reactions

$$\circlearrowleft + \quad \Sigma M_A = 0$$

$$-4(10)(5) + A_y(8) = 0$$

$$A_y = 25\ k$$

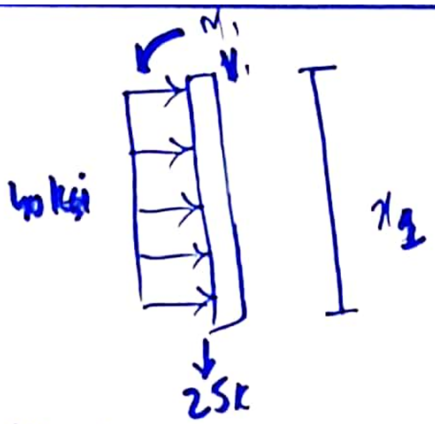
$$\Sigma F_y = 0 \uparrow^+$$

$$25 + A_y = 0$$

$$A_y = -25\ k$$

$$\Sigma F_x = 0 \xrightarrow{+}$$

$$40 - A_x = 0 \Rightarrow A_x = 40k$$

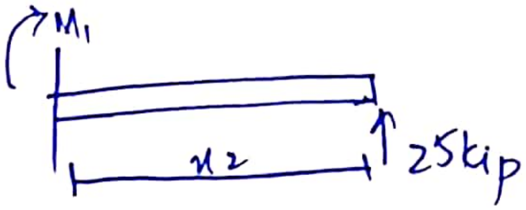


⇒ Rect. moments

$$\circlearrowleft + \sum M_1 = 0$$

$$-40(x_1) + 4(x_1)\left(\frac{x_1}{2}\right) + M_1$$

$$M_1 = 40x_1 - 2x_1^2$$

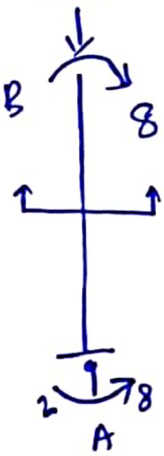


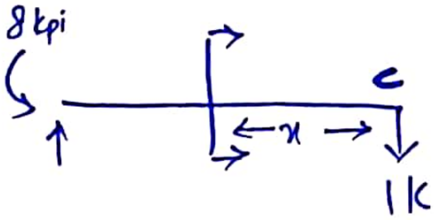
$$-25(x_2) + M_2 = 0$$

$$M_2 = 25x_2 \text{ kip}$$

Now

Vertical Moments





Members	BA	CB
Origin	B	C
limit	0-10	0-8
M	$2x^2$	0
m	8	x

By virtual working

$$1 . \Delta_c = \int_0^{10} \frac{(2x^2)(8)}{EI} dx + \int_0^{8} \frac{(0)(x)}{EI}$$

$$= \frac{16x^3}{3} \Big|_0^{10} + 0$$

$$= \frac{16 \times 1000}{3} / EI$$

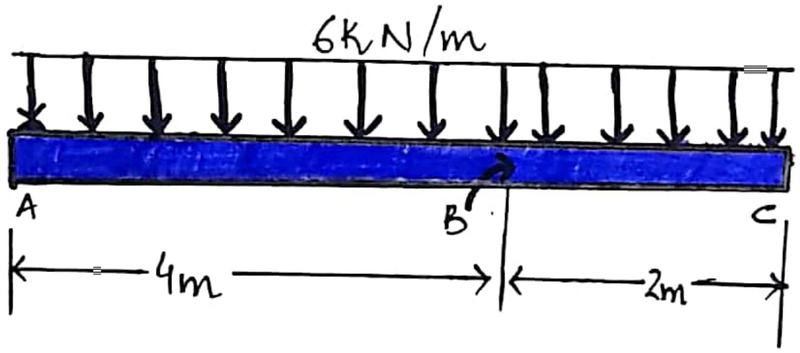
$$= \frac{5333.33}{EI} = \frac{533.33}{29 \times 10^3 \times 600}$$

$$= 3.06 \times 10^{-4} \text{ in}$$

QUESTION No. 2

Determine the slope and displacement at point B. Assume the support at A is a pin and C is a roller. Take $E = 200\ GPa$, $I = 60(10^6)\ mm^4$. Use Castigliano's Theorem.

Figure:



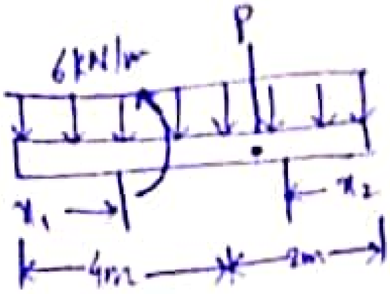
SOLUTION:

Given Data

$E = 200\ GPa$
$I = 60(10)^6\ mm^4$

Required:

Slope and displacement = ?



$R_1 + R_2 = 0$ —(1)

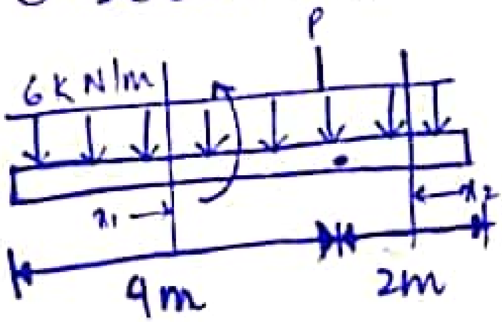
$\Sigma M_A = 0 \circlearrowright +$

$1 + R_2(6) = 0$

$\Rightarrow -0.16667$ put in (1)

$R_1 + (-0.16667) = 0$

$R_1 = 0.16667$ KN



$R_1 + R_2 = 1$

$\circlearrowright + \Sigma M_A = 0$

$-(1)(4) + R_2(6) = 0$

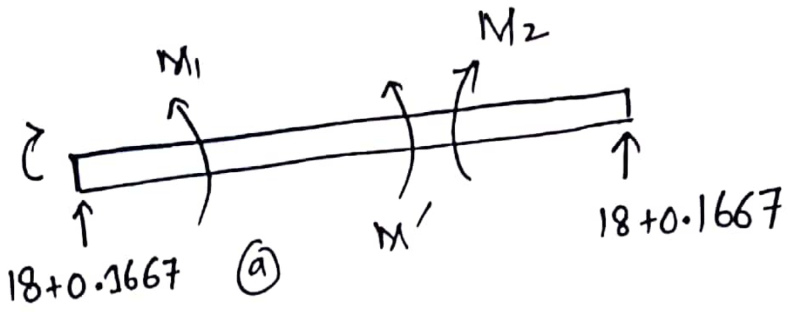
$R_1 = 0.6667$ KN

$R_2 = 1 - 0.6667$ KN

$R_2 = 0.333$ KN

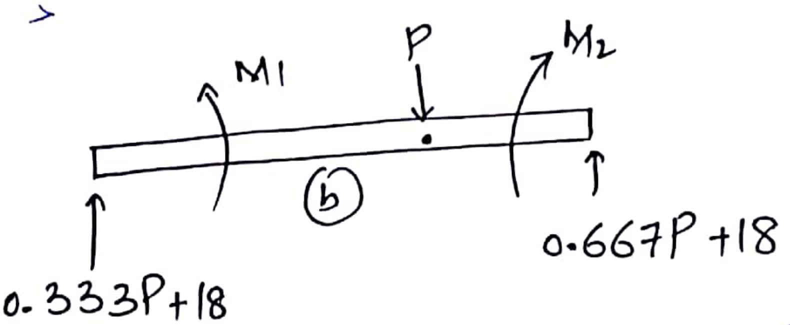
$M_1 = (18 + 0.1667\,M')x_1 - 2x_1^2$

$M_2 = (18 - 0.1667\,M')\,x_2 - 2x_2^2$



$M_1 = (0.333\,P + 18)\;x_1 - 2x_1^2$

$M_2 = (0.667P + 18)\;x_2 - 2x_1^2$



The displacement functions shown in the fig. a above

$\frac{\partial M_1}{\partial M'} = 0.1667\,x_1$ and $\frac{\partial M_2}{\partial M'} = 0.1667x_2$, set $M'=0$ then

$M_1 = \left[18 + 0.1667(0)\right] x_1 - 2x_1^2$

$\Rightarrow M_1 = (18x_1 - 2x_1^2)$

$\Rightarrow M_2 = (18x_2 - 2x_2^2)$

$\phi_B = \int_0^2 M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{Ei} = \int_0^4 \frac{(18x_1 - 2x_1^2)(0.1667x_1)\,dx_1}{Ei} + \int_0^2 \frac{(18x_2 - 2x_2^2)(0.1667x_2)\,dx_2}{Ei}$

$\phi_B = \frac{42.65}{Ei} + \frac{6.66}{Ei}$

$$\phi_B = \frac{49.31}{Ei}$$

$$\phi_B = \frac{49.31}{(200 \times 10^6 \, kPa)(0.0006)} = 0$$

$$\boxed{\phi_B = 0.4411 \text{ rad}}$$

→ For the displacement functions are shown in figure "6"

$$\frac{\partial M_1}{\partial M_p} = 0.333 x_1 \text{ and } \frac{\partial M_2}{\partial P} = 0.6667 x_2 \text{ also}$$

Set $P = 0$

then $M_1 = (18 x_1 - 2 x_1^2) \, kN\text{-}m$

$M_2 = (18 x_2 - 2 x_2^2) \, KN.m$

thus

$$\Delta B = \int_0^L M \left(\frac{\partial M}{\partial P} \right) \frac{dx}{Ei}$$

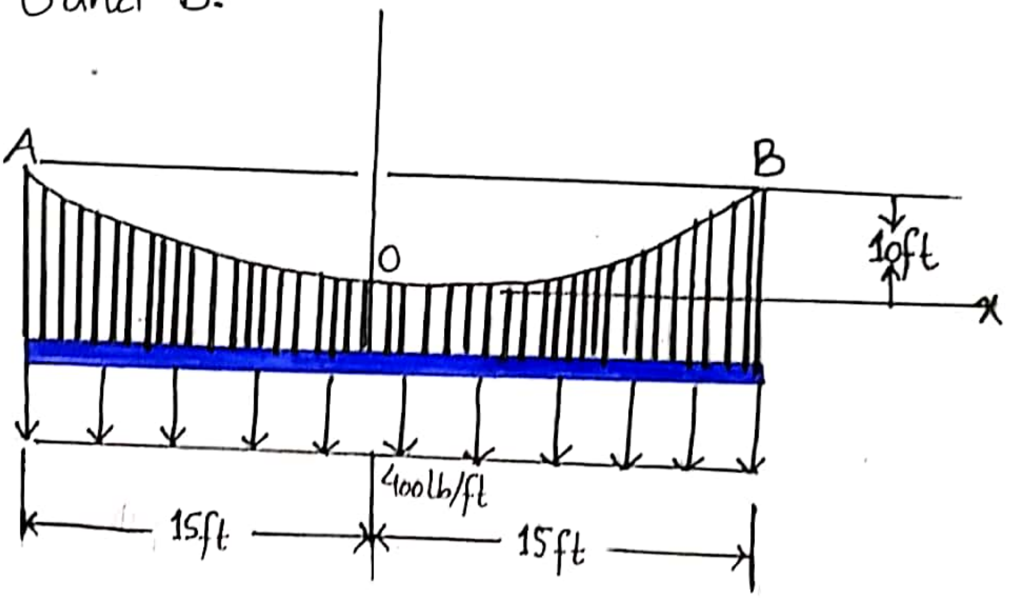
$$\Delta B = \int_0^4 \frac{(30 x_1 - 2x_1^2)(0.333 x_1)\, dx}{Ei} + \int_0^2 \frac{(30 x_2 - 2x_2^2)(0.6667 x_2)\, d}{Ei}$$

$$\Delta B = \frac{218.5}{Ei} \Rightarrow \frac{218.5}{(200 \times 10^6)(0.00006)} = \boxed{0.018 m \Rightarrow 18mm} \downarrow$$

QUESTION No.3

The cable is subjected to the uniform loading. If slope of the cable at point O is zero, determine the equation of the curve and the force in the cable at O and B.

Figure



Solution:

From Eq: 5-9

$$y = \frac{h}{L^2}x^2 = \frac{10}{15^2}x^2$$

$$y = 0.0444x^2$$

From Eq 5-8

$$T_o = F_H = \frac{w_o L^2}{2h} = \frac{400(15^2)}{2(10)}$$

$T_o = 4500$ lb $\div$ing by 1000

$T_o = 4.5K$

From Eq 5-10

$$T_B = T_{max} = \sqrt{F_H^2 + (w_o L)^2}$$

$$= \sqrt{(4500)^2 + (400)^2(15)^2}$$

$$T_B = T_{max} = \sqrt{20250000 + (400 \times 15)^2}$$

$$T_B = T_{max} = 7500\,lb \quad \div ing \text{ by } 1000$$

$$T_B = T_{max} = 7.5K$$

Also From Eq. 5-11

$$T_B = T_{max} = w_0 L \sqrt{1 + \left(\frac{L}{2h}\right)^2}$$

$$T_{max} = (400)(15)\sqrt{1 + \left(\frac{15}{20}\right)^2}$$

$$T_{max} = 6000\sqrt{1 + \frac{225}{400}}$$

$$T_{max} = 6000\,(1.25)$$

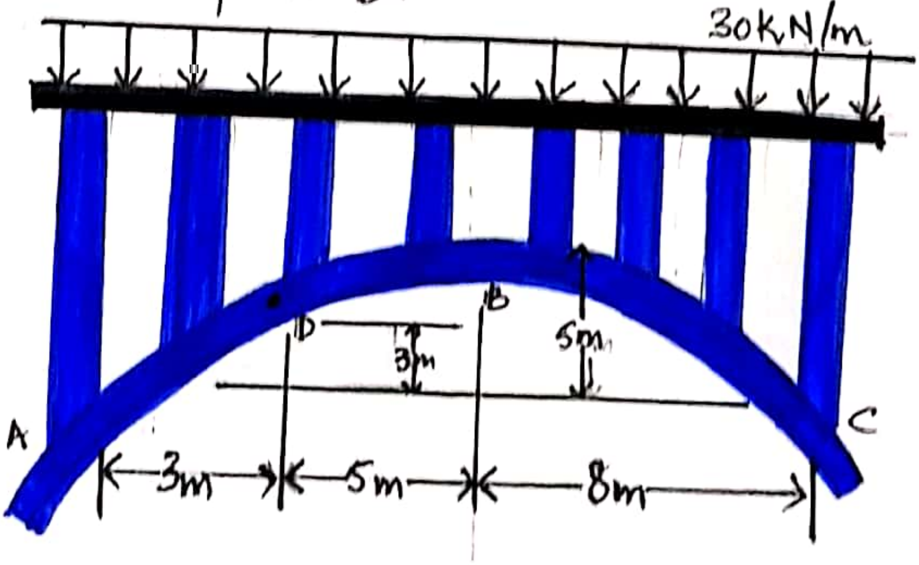
$$T_{max} = 7500\,lb \quad \div ing \text{ by } 1000$$

$$T_B = T_{max} = 7.5K$$

QUESTION No.4

The three hinged spandrel arch is subjected to the uniform load of 30kN/m. Determine internal moment arch at point D.

Figure:



SOLUTION

Member AB;

$$\circlearrowright + \Sigma M_A = 0$$

$$\Rightarrow Bx(5) + By(8) - 240(4) = 0$$

Member BC;

$$\circlearrowright + \Sigma M_C = 0$$

$$\Rightarrow -Bx(5) + By(8) + 240(4) = 0$$

Solving

$$Bx = 192\ KN, \quad By = 0$$

Segment BD:

$$\circlearrowright + \Sigma MD = 0$$

$$\Rightarrow 192(2) - 150(2.5) - M_D = 0$$

$$M_D = 9\ KN.m$$