PAPER:

DIFFERENTIAL EQUATIONS

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SECTION:

B

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Q No 1: PART A:

i) $w = \sin(x+ct) + \cos(2x+2ct)$

Ans of Q no 1 Part A:

SOLUTION:

$$\frac{\partial w}{\partial t} = \cdot\cos(x+ct) + (-\sin(2x+2ct)+2c$$

$$\frac{\partial^2 w}{\partial t^2} = -\sin(x+ct) + c^2 - \cos(2x+2ct) + 4c^2 \text{ --- } 1$$

$$\frac{\partial w}{\partial x} = \cdot\cos(x+ct) - \sin(2x+2ct) + 2$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

$$= \left[-\sin(x+ct) - 4\cdot\cos(x+ct)\right]$$

$$\frac{\partial^2 w}{\partial t^2} = +c^2\left[-\sin(x+ct) - 4(\cos(2x+2ct)\right]$$

$$c^2 * \frac{d^2 w}{dx^2}$$

" Hence it is the solution of wave equation,,

QNO 1 PART B:

ii :- $w = \tan(2x + ct)$

Ans of Qno1 Part B:

SOLUTION:

$$\frac{\partial w}{\partial w} = \frac{\partial w}{\partial t}(\tan(2x + ct))$$

$$= \sec^2(2x + ct)\cdot(c)$$

$$= c\sec^2(2x + ct)$$

$$\frac{\partial^2 w}{\partial t^2} = c\frac{\partial}{\partial t}(\sec^2(2x + ct))$$

$$= c\frac{\partial}{\partial t}((\cos(2x + ct))^{-2}$$

$$= c\frac{\partial}{\partial t}-2(\cos(2x + ct))^{-3}(-\sin(2x+ct))$$

$$\frac{\partial^2 w}{\partial t^2} = 2c^2 \sec^2(2x+ct)\sin(2x+ct)$$

Now w.r.t x

$$\frac{\partial w}{\partial x} = \frac{\partial}{\partial x}(\tan(2x+ct))$$

$$= \sec^2(2x+ct)(2)$$

$$\frac{\partial w}{\partial x} = 2\sec^2(2x+ct)$$

$$\frac{\partial^2 w}{\partial x^2} = 2\frac{\partial}{\partial x}(\sec^2(2x+ct))$$

$$= 2\frac{\partial}{\partial x}(\cos(2x+ct)^{-2})$$

$$\frac{\partial^2 w}{\partial x^2} = 2(-2)(\cos(2x+ct))^{-3}$$

$$(-\sin(2x+ct)(2)$$

RESULT:

Hence it is not the solution of wave equation.

Q No 2:

Expand the following function in a Fourier series.

$$F(x) = x, \ -\pi < x \le 0$$
$$= 2x, \ 0 \le x \le \pi$$

Ans of Q No 2:

SOLUTION:

$$F(x) = \begin{cases} x \ ; & -\pi < x \le 0 \\ 2x \ ; & 0 \le x \le \pi \end{cases}$$

we have to find the Fourier co-efficients a_0, a_n, b_n

so:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx$$

$$a_0 = \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi}$$

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$$a_0 = \frac{1}{\pi}\left[0 - \frac{\pi^2}{2}\right] + \frac{2}{\pi}\left[\frac{\pi^2}{2} - 0\right]$$

$$a_0 = -\frac{\pi}{2} + \pi = \frac{\pi}{2}$$

$$a_0 = -\frac{\pi}{2} + \pi = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\cos nx\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0} (x\cos x\,dx) + \frac{1}{\pi}\int_{0}^{\pi} (2x\cos x)\,dx$$

$$a_n = \frac{1}{\pi}\left\{x\left(\frac{\sin x}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left[-\frac{\cos nx}{n^2}\right]\right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos nx}{n^2}\right] + \frac{2}{\pi}\left[\frac{\cos nx}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right] = \frac{(-1)^n - 1}{\pi n^2}$$

So

$$a_n = \begin{cases} \dfrac{-2}{\pi n^2} & \text{if } n \text{ is odd} \\ 0 & ; \text{if } n \text{ is even.} \end{cases}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin x\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx + \frac{2}{\pi}\int_{0}^{\pi} x\sin x\,dx$$

$$b_n = \frac{1}{\pi}\left[x\left(-\frac{\cos x}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+\frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$b_n = \frac{1}{\pi}\left[-\frac{\pi\cos\pi}{n}\right] + \frac{2}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right]$$

$$b_n = -\frac{3\cos nx}{n} = \frac{3(-1)^{n+1}}{n}$$

The follow is the

fourier series:

P.T.O

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$= \frac{\pi}{4} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2x-1)}{(2n-1)^2}$$

$$+ 3 \sum^{\infty} (-1)^{n+1} \frac{\sin nx}{n}$$

Q No 3:

Solve the initial value Problem.

$$y'' - 4y + 13y = 8\sin 3x$$

$$y(0) = 1 \text{ and } y'(0) = 2$$

Ans of Q no 3:

SOLUTION:

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 8\sin 3x$$

let $D = \frac{d}{dx}$ $D^2 = \frac{d}{dx^2}$

So:

$$D^2y - 4Dy + 13y = 8\sin x$$

$$\Rightarrow (D^2 - 4D + 13)y = 8\sin 3x \longrightarrow (1)$$

General solution for eq (1) is as follow.

$$y = y_c + y_p \longrightarrow (1)$$

$$D^2 - 4D + 13 = 0$$

$= D^2 - 4D + 4 + 9 = 0$

$= (D-2)^2 + 9i^2 = 0$

$= (D-2)^2 - (3i)^2 = 0$

$= (D-2-3i)(D-2+3i) = 0$

$= D-2-3i = 0$

and

$D-2+3i = 0$

So

$D-2-3i = 0$ and $D-2+3i=0$

$D = 2+3i$ $D = 2-3i$

So $D = 2 \pm 3i$

Imaginary roots.

$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x)$

For P. I we have

$$y_p \left(\frac{8 \sin 3x}{D^2 - 4D + 13} \right.$$

$$y_p = 8 \cdot \frac{\text{Im Part of } e^{i \cdot 3x}}{(i3)^2 - 4(i3) + 13}$$

$$= 8 \frac{\text{Im Part of } e^{i3x}}{-9 - 12i + 13}$$

$$= 8 \frac{\text{Im Part of } e^{i3x}}{4 - 12i}$$

$$= \frac{8 \text{ Im Part of } e^{i3x}}{4(1-3i)}$$

$$\frac{2}{(1-3i)} \cdot \frac{(1+3i)}{(1+3i)} \text{ Im Part of } e^{i3x}.$$

$$y_p = \frac{2+6i}{10} \text{ im Part of } e^{i3x}$$

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$$y_p = \frac{2+6i}{10} \text{ im Part of. } e^{i3x}$$

$$y_p = \frac{2(1+3i)}{10} \text{ im Part}(\cos 3x + i\sin 3x)$$

$$y_p = \frac{(1+3i)}{5} \quad (\cos 3x + i\sin 3x)$$

$$y_p = \frac{(3\cos 3x + \sin 3x)}{5}$$

$$A = y = e^{2x}(C_1 \cos 3x + C_2 \sin 3x) + \frac{1}{5}(3\cos 3x + \sin 3x)$$

$$y' = 2e^{2x}(C_1 \cos 3x + C_2 \sin 3x) + e^{2x}(-3C \sin 3x + 3C_2 \cos 3x)$$
$$+ \frac{1}{5}(-9\sin 3x + 3\cos 3x)$$

For $y(0) = 1$

$$y = e^0(C_1 \cos(0) + C_2 \sin 0) + \frac{1}{5}(3\cos 0 + \sin 0)$$

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$$C_1 = 1 - \frac{3}{5} = \frac{2}{5}$$

$$C_1 = \frac{2}{5}$$

For $y'(0) = 2$

$$2 = 2e^0(c_1 \cos 0 + c_2 \sin) + e^0(-3c_1 \sin$$
$$+ 3c_2 \cos 0) + \frac{1}{5}(-9 \sin 0 + \cos 0)$$

$$2 = 2(c_1 + 0) + (0 + 3c_2) + \frac{1}{5}(0 + 3)$$

$$2 = 2c_1 + 3c_2 + \frac{3}{5}$$

$$3c_2 = 2 - \frac{3}{5} + 2c_1$$

$$3C_2 = 2 - \frac{3}{5} \cancel{+} 2c_1$$

As $C_1 = \frac{2}{5}$

$$3C_2 = 2 - \frac{3}{5} - \left(2 \cdot \frac{2}{5}\right)$$

$$= 3C_2 = \frac{1 - 3 - 4}{5} = \frac{3}{5}$$

$$C_2 = \frac{1}{3} \cdot \frac{3}{5}$$

$$C_2 = \frac{1}{5}$$

$$y = e^{2x}\left(\frac{2}{5}\cos 3x + \frac{1}{5}\sin 3x\right) + \frac{1}{5}(3\cos 3y + \sin 3x).$$

RESULT:

$$y = e^{2x}\left(\frac{2}{5}\cos 3x + \frac{1}{5}\sin 3x\right) + \frac{1}{5}(3\cos 3x + \sin 3x)$$

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QNo 4 :

Solve :

$$(D^2 - DD')z = \cos x \cos 2y$$

Solution:

This can also be written

as :

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = E \cos x . \cos 2y$$

Now as we have:

Corresponding A.E is $m^2 - m = 0$

when $D/D' = m$ so $m = 1$ $m = 1$

$$C.F = \phi_1(y) + \phi_2(y + x)$$

Now

$$P.I = \frac{1}{(D^2 - DD')} \cos x \cos 2y$$

$$= \frac{1}{2} \frac{1}{D^2 - DD'} [\underbrace{\cos(x + 2y)}_{\mathrm{I}} + \underbrace{\cos(x - 2y)}_{\mathrm{II}}]$$

$$P.I = \frac{1}{2}\left[\frac{1}{-1+2}\cos(x+2y) + \frac{1}{-1-2}\cos(x-2y)\right]$$

(when $\cos(ax+by)$ replaing D^2 by $-a^2$, D'^2 by $-b^2$ and $DD' = -ab$)

$$P.I \ \frac{1}{2}\left[\cos(x+2y) - \frac{1}{3}\cos(x-2y)\right]$$

The complete solution.

$$z = \phi_1(y) + \phi_2(y+x) + \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$$

RESULT:

$$z = \phi_1(y) + \phi_2(y+x) + \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$$