"Assignment"

"Final Term Paper"

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paper:- Advanced Engineering Mathematics

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Q1 (a): Find the general solution of

$$y'' - 6y' - 7y = 0$$ ①

Sol:

$$y'' - 6y' - 7y = 0$$

$$y'' + ay' + by = 0$$

$$a = -6, \quad b = -7$$

The Characteristic equation

$$\lambda^2 + a\lambda + b = 0$$

$$\lambda^2 - 6\lambda - 7 = 0$$

$$\lambda^2 - 7\lambda + \lambda - 7 = 0$$

$$\lambda(\lambda - 7) + 1(\lambda - 7) = 0$$

$$(\lambda - 7)(\lambda + 1) = 0$$

$$\lambda - 7 = 0, \quad \lambda + 1 = 0$$

$$\lambda_1 = 7, \quad \lambda_2 = -1$$

Roots are real and Distinct

So

Case 1 :-

Here

$$\boxed{y = C_1 e^{7x} + C_2 e^{-x}}$$

P.T.O

Q1 (b):

Find an ODE

$y'' + ay' + by$ for the given basis

e^{4x}, e^{-4x}

Solution:

$y_1 = e^{4x}$

$y_2 = e^{-4x}$

$y = C_1 y_1 + C_2 y_2 \longrightarrow (1)$

Now putting the value of y_1 & y_2 in eq (1)

$y = C_1 e^{4x} + C_2 e^{-4x}$

$y_1 = C_1 e^{\lambda_1 x} + C_2 e^{-\lambda_2 x}$

$\lambda_1 = 4$

$\lambda_2 = -4$

$\lambda_1 - 4 = 0 \quad , \quad \lambda_2 + 4 = 0$

$(\lambda - 4)(\lambda + 4) = 0$

$\lambda^2 + 4\lambda - 4\lambda - 16 = 0$

$\lambda^2 - 16 = 0$

$\sqrt{\lambda^2} = \sqrt{16}$

$\boxed{\lambda = \pm 4}$

So the equation will be.

$\boxed{y'' + 4y' - 4y = 0}$

So $a = 4$, $b = -4$

P.T.O

Q2:

Apply $(D+5I)(D-I)$ to the function e^{5x}

Solution: $(D+5I)(D-I);\ e^{5x}$

$= D^2 - DI + 5DI - 5I^2$

As we know that

$\sqrt{I} = 1$

$= (D^2 + 4D - 5(1))(e^{5x})$

$= D^2(e^{5x}) + 4D(e^{5x}) - 5e^{5x}$

$= D[5e^{5x}] + 4[5e^{5x}] - 5e^{5x}$

$= 5\cdot 5e^{5x} + 4\cdot 5e^{5x} - 5e^{5x}$

$= 25e^{5x} + 20e^{5x} - 5e^{5x}$

$= \boxed{40e^{5x}}$

P.T.O

Q3:- Find the general solution of

$$x^2y'' + 3xy' + y = 0$$

Sol:: As,

$$x^2y'' + axy' + y = 0$$

$a = 3$, $b = 1$

The auxiliary equation

$$m^2 + (a-1)m + b = 0$$

$$m^2 + (3-1)m + 1 = 0$$

$$m^2 + 2m + 1 = 0$$

$$m^2 + 2m + 1 = 0$$

$$(m+1)^2 = 0$$

$$(m+1)(m+1) = 0$$

$$m+1 = 0 \quad , \quad m+1 = 0$$

$$m_1 = -1 \quad , \quad m_2 = -1$$

So

$$m = m_1 = m_2 = -1$$

The roots are equal & Real

$$y = (c_1 + c_2 \ln x)x^m$$

$$y = (c_1 + c_2 \ln x)x^{-1}$$

P.T.O

Q4:- Find the general solution of

$$y'' + 3y' + 2y = 30e^{2x} \longrightarrow ①$$

Sol:- For Homogenous Solution

$$y'' + 3y' + 2y = 0$$

The Auxilary eq is

$$\lambda^2 + 3\lambda + 2 = 0$$

$$\lambda^2 + 2\lambda + \lambda + 2 = 0$$

$$\lambda(\lambda+2) + 1(\lambda+2) = 0$$

$$(\lambda+2)(\lambda+1) = 0$$

$$\lambda+2 = 0 \qquad \lambda+1 = 0$$

$$\lambda_1 = -2 \quad , \quad \lambda_2 = -1$$

Roots are Real and Distinct

$$y_h = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$$

$$y_h = C_1 e^{-2x} + C_2 e^{-x}$$

Now For y_p From the table we take choice for y_p in our Question the Choice is Ist Choice.

$$y_p = c e^{2x} \longrightarrow ②$$

Taking Ist and 2nd Derivative.

$$y_p' = 2ce^{2x} , \quad y_p'' = 4ce^{2x}$$

put in eq ①

P.T.O

(6)

$4ce^{2x} + 3(2ce^{2x}) + 2(ce^{2x}) = 30e^{2x}$

$12ce^{2x} = 30e^{2x}$

Now Comparing

$12c = 30 \Rightarrow \boxed{c = \frac{30}{12}}$

put in eq (2)

$$y = \frac{30}{12} e^{2x}$$

So the general solution

$$y = y_h + y_p$$

$$\boxed{y = C_1 e^{2x} + C_2 e^{-x} + \frac{30}{12} e^{2x}}$$

Q5 (a):- Let $f(t) = 1$, $t \geq 0$ find $\mathcal{L}(f(t))$ then also find $\mathcal{L}(t^2 - 2t)$

Solution:- Given that $f(t) = 1$

As we know that

$$f(s) = \mathcal{L}f(t) = \int_0^\infty e^{-st}(f(t)\,dt$$

$$F(s) = \int_0^\infty e^{-st}(1)\,dt$$

P.T.O

$$F(s) = \int_0^\infty e^{-st}\,dt - \frac{e^{-st}}{-s}\Big|_0^\infty$$

$$= -\frac{1}{s}\left[e^{-st}\Big|_0^\infty\right] = -\frac{1}{s}\left[\frac{1}{e^{st}}\Big|_0^\infty\right]$$

$$F(s) = -\frac{1}{s}\left[\frac{1}{e^\infty} - \frac{1}{e^0}\right]$$

$$F(s) = \frac{-1}{s}\left[0-1\right] = \frac{1}{s}$$

So $\boxed{F(s) = \frac{1}{s}}$

Q: 5(a) Find $\mathcal{L}(t^2-2t)$

Solution: $\mathcal{L}(t^2-2t) = \mathcal{L}(t)^2 - 2\,\mathcal{L}(t)$

As,

$$\mathcal{L}(t) = \frac{1}{s^2}, \quad \mathcal{L}(t)^2 = \frac{2!}{s^3}$$

$$\mathcal{L}(t^2-2t) = \frac{2!}{s^3} - 2\frac{1}{s^2}$$

$$= \frac{2!}{s^3} - \frac{2}{s^2}$$

$$\frac{2! - 2s}{s^3}$$

$\boxed{\text{P.T.O}}$

Q5:(b): Find the Laplace transform of

$$e^{-t}\sinh 5t$$

Solution:-

$$F(s) = \mathcal{L}f(t) = \mathcal{L}\left[e^{-t}\sinh 5t\right]$$

$$\Rightarrow \mathcal{L}\left[e^{-t}\left[\frac{e^{5t}-e^{-5t}}{2}\right]\right]$$

$$= \mathcal{L}\left[\frac{e^{4t}-e^{-6t}}{2}\right]$$

$$= \frac{1}{2}\left[\mathcal{L}e^{4t} - \mathcal{L}e^{-6t}\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{1}{s-4} - \frac{1}{s+6}\right]$$

$$= \frac{1}{2}\left[\frac{s+6-s+4}{(s-4)(s+6)}\right]$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{10}\,5}{(s-4)(s+6)}\right]$$

$$\boxed{F(s) = \frac{5}{(s-4)(s+6)}}$$

Ans

END