Q # 1

(i) The order of Matrix AB is $m \times n$.

(ii) The number of non-zero row in one Echelon form is Rank

(iii) if $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is Singular matrix then $a =$ 8

(iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$= -2i^2 - i^2 \qquad \because i^2 = -1$$

$$= -2(-1) - (-1)$$

$$= 2 + 1 = 3$$

$$= \underline{3}$$

v) Scalor matrix.

vi) $\frac{dy}{dx} + 2xy = y$

Separting veriabels

$\Rightarrow \frac{dy}{dx} = y - 2xy$

$\Rightarrow \frac{dy}{dx} = y(1-2x)$

$\Rightarrow \frac{dy}{y} = (1-2x)\,dx$

Apply integration on both side

$\int \frac{1}{y}\,dy = \int 1\,dx - \int 2x\,dx$

$\ln y = x - \cancel{2}\frac{x^2}{\cancel{2}} + c$

$\boxed{\ln y = x - x^2 + c}$

ix)

$$u = \frac{x^2}{2}$$

$$e^{\int \frac{x^2}{2} dx} = e^{x^3/6}$$

$$e^{x^3/6} y' + e^{x^3/6}\left(\frac{x^2}{2}\right) y = \frac{1}{2} e^{x^3/6}(x^2+3)$$

$$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6} + C}{2e^{x^3/6}}$$

$$y(0) = \frac{0+3}{2} = \frac{3}{2}$$

$$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + \frac{3}{2}$$ Ans

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x)
$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Expand by C_1:

$$1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - 1\begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1\begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix}$$

$$\Rightarrow 1(bc^2 - cb^2) - 1(ac^2 + a^2c) + 1(ab^2 - a^2b)$$

P.T.O

$= bc^2 - cb^2 - ac^2 - a^2c + ab^2 - a^2b$

$= ab^2 - cb^2 + a^2c - a^2b - ac^2 + bc^2$

$a^2c - a^2b + ab^2 - cb^2 + bc^2 - ac^2$

$a^2(c-b) + b^2(a-c) + c^2(b-a)$ Ans

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Q # 02 A part:-

Express the Determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product of factors which are linear in a, b, c.

Sol:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

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Q# 06

$ab^2c^3 - ab^3c^2 - a^2bc^3 - a^3bc^2 + a^2cb^3 - a^2b^2c$

Common $a \cdot b \cdot c$

$\Rightarrow abc(bc^2 - b^2c - ac^2 - a^2c + ab^2 - a^2b)$

$\Rightarrow abc[bc(c-b) - ac(c-a) + ab(b-a)]$

Ans

Q #2 B part

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Soln :-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Characteristic equ $\longrightarrow |A - \lambda I| = 0 \longrightarrow$ (A)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

Now take determinant

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by R_1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \longrightarrow B$$

Again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ Expand by R_1

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\Big[\big((3-\lambda)(2-\lambda) - (-1)(-1)\big) + 1\big((-1)(2-\lambda) - (-1)(-1)\big)$$
$$- 1\big((-1)(-1) - (-1)(3-\lambda)\big)\Big]$$

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$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1)+(-2+\lambda-1)-(1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5)+(-3+\lambda)-(4-\lambda)$$

$$= 3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$= \boxed{-\lambda^3+8\lambda^2-18\lambda+8} \longrightarrow (a)$$

$$\Rightarrow +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$$

$$\Rightarrow \boxed{-\lambda^2+6\lambda-8} \longrightarrow (b)$$

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$$\Rightarrow -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$-\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$\Rightarrow -\left[-1(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right.$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= \boxed{-\lambda^2+6\lambda-8} \rightarrow C$$

Put (a), (b) & (c) in (B)

$$(2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$$

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$-2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda$

$-\lambda^2+6\lambda-8-\lambda^2+16\lambda-8$

$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2$

$-36\lambda-8\lambda+6\lambda+6\lambda+16-16$

$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$

By Synthetic division we get

$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$

$(\lambda=0)$

$\lambda-2=0 \Rightarrow \boxed{\lambda=2}$

$\lambda^2-8\lambda+16=0$

By factorization method

$\lambda^2-4\lambda-4\lambda+16=0$

$\lambda(\lambda-4)-4(\lambda-4)=0$

$(\lambda-4)(\lambda-4)=0$

$\lambda=4$, $\lambda=4$

$\boxed{\lambda_1=0,\ \lambda_2=2,\ \lambda_3=4,\ \lambda_4=4}$

Ans

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Q#3:-

$$(x^2+3y^2)\,dx - 2xy\,dy = 0$$

$$x=2 \quad , \quad y=6$$

Solu: $(x^2+3y^2)\,dx - 2xy\,dy = 0$

$$\Rightarrow (x^2+3y^2)\,dx = 2xy\,dy$$

Dividing both Sides by $2xy\,dx$ we get

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^2}{2xy} + \frac{3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \longrightarrow (1)$$

let $y = vx$

Diff: $dy = v\,dx + x\,dv$

Dividing by dx

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \longrightarrow (a)$$

put (a) in (*)

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{\cancel{x}v} + 3\frac{v\cancel{x}}{\cancel{x}}\right]$$

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiplying both sides by "2"

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

P. T. O

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

Multiplying both sides by $\frac{dx}{dv}$

we get

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

multiplying both sides by $\frac{v}{x(1+v^2)}$

we get

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Take "$\int$" on both sides

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx + C$$

$\ln|1+v^2| = \ln x + \ln c$

Take "e" on both Sides

$e^{\ln|1+v^2|} = e^{\ln|x \cdot c|}$

$1+v^2 = xc$

$1+v^2 = x.c$

put $v = y/x$

$1+\left(\frac{y}{x}\right)^2 = x \cdot c$

$\frac{x^2+y^2}{x^2} = xc$

$x^2+y^2 = x^3c \longrightarrow$ (**)

put $x=2$, $y=6$ in eq (**)

$(4)+(36) = 8c$

$\boxed{c = 5} \longrightarrow$ put in (**)

So $x^2+y^2=5x^3$

$y^2=5x^3-x^2$

$y^2=x^2(5x-1)$

Taking "$\sqrt{\ }$" on both Sides

$$\boxed{y=+x\sqrt{5x-1}\quad , \quad y=-x\sqrt{5x-1}}$$

or

$$\boxed{y=\pm x\sqrt{5x-1}}$$

Ans