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Section B

Ans/:

$$x^3y''' + 3x^2y' + 3y = 10x + 10/x$$

Solution:

Let: $y = x^m$

$$y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

$$y''' = m(m-1)(m-2)x^{m-3}$$

putting values now.

$$x^3 m(m-1)(m-3)x^{m-3} + 3x^2 mx^{m-1} + 3x^m = 0$$

$$m(m-1)(m-3)x^m + 3mx^{m+1} + 3x^m = 0$$

$$x^m\left[(m^2-m)(m-3) + 3mx + 3\right] = 0$$

Q no 8:

$$x^3y''' + 4x^2y'' - 5xy' - 15y = x^4$$

Solution:

$$x^3y''' + 4x^2y'' - 5xy' - 15y = 0$$

Let suppose.

$$y = x^m$$

$$y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

$$y''' = m(m-1)(m-2)x^{m-3}$$

Now putting values.

$$x^3 m(m-1)(m-2)x^{m-3} + 4x^2 m(m-1)x^{m-2} - 5xmx^{m-1} - 15x^m = 0$$

$$m(m-1)(m-3)x^m + 4m(m-1)x^m - 5mx^m - 15x^m = 0$$

Taking x^m common.

$$x^m[(m^2-m)(m-2) + 4(m^2-m) - 5m - 15] = 0$$

$$m^2 - 2m^2 - m^2 + 2m + 4m^2 - 4m - 5m - 15 = 0$$

$$m^3 + m^2 - 7m - 15 = 0$$

$m = 1$	$m = 9$	$m = 3$
$-1 + 1 + 7 - 15$	$8 + 4 - 14 - 15$	$27 + 9 - 21 - 15 \neq 0$
	$-12 - 14 - 15$	

Ans3:

$$x^2y'' + 8xy' - 6y = 10x^2$$

Solution:

$$y = x^m$$

$$y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2}$$

$$x^2 m(m-1)x^{m-2} + 8xmx^{m-1} - 6x^m = 0$$

$$x^m m(m-1) + 8mx^m - 6x^m = 0$$

$$x^m\left[m^2 - m + 2m - 6\right] = 0$$

$$m^2 + m - 6 = 0$$

$$m(m+3) - 2(m+3) = 0$$

$$(m+3)(m-2) = 0$$

$$m = -3 \quad , m = 2$$

$$y_n = C_1x^{-3} + C_2x^2 .$$

Ano 3:

$$y'' + 3/x y' - 6/x^2 y = 10$$

$$f(x) = 10$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} C_1 x^{-3} & C_2 x^2 \\ -3C_1 x^{-4} & x_2 x \end{vmatrix}$$

$$= 2C_1 C_2 x^{-2} + 3C_1 C_2 x^{-2}$$

$$= 5C_1 C_2 x^{-2}$$

$$W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix} = \begin{vmatrix} 0 & C_2 x^2 \\ 10 & 2C_2 x \end{vmatrix} = -10 C_2 x$$

$$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix} = \begin{vmatrix} C_1 x^{-3} & 0 \\ -3C_1 x^{-4} & 10 \end{vmatrix} = 10 C_1 x^{-3}$$

$$u_1' = \frac{W_1}{W} = \frac{-10 C_2 x^2}{5 C_1 C_2 x^{-2}} = -2/C_1 x^4$$

$$u_2' = \frac{W_2}{W} = \frac{10 C_1 x^{-3}}{5 C_1 C_2 x^{-3}} = 2/C_2 x^{-1}$$

So:

$$u_1' = -2/C_1 x^4$$

$$u_1 = -2/C_1 = \int x^4 dx = -2/5 C_1 - x^5$$

$$u_2 = 2/C_2 \int 1/x \, dx = 2/C_2 \ln x.$$

$$y_p = \frac{-2}{5C_1} x^5 C_1 x^{-3} + 2/C_2 \ln x C_2 x_2$$

$$y_p = -2x^2 + 2\ln(x)x^2$$

$$y = C_1x^3 + C_2x^2 - 2x^2 + 2\ln x(x^2)$$

Qno5:

$$(x+1)^2 y'' - 3(x+1)y' + 4y = x^2$$

Solution: Let

$$y = x^m$$

$$y' = m x^{m-1}$$

$$y'' = m(m-1) x^{m-2}$$

$$(x^2+1+2x)m(m-1)x^{m-2} - 3(x+1)mx^{m-1} + 4x^m = 0$$

$$= x^2 m(m-1)x^{m-2} + m(m-1)x^{m-2} + 2xm(m-1)x^{m-2} - 3(xmx^{m-1} + mx^{m-1}) + 4x^m$$

$$= m(m-1)x^m + m(m-1)x^m + 2m(m-1)x^{m-1} - 3mx^m - 3mx^{m-1} + 4x^m = 0$$

$$= m(m-1)x^m - 3mx^m + 4x^m + 2m(m-1)x^{m-1} + m(m-1)x^{m-2} = 0$$

$$m(m-1) - 3m + 4 = 0$$

$$m^2 - m - 3m + 4 = 0$$

$$m^2 - 4m + 4 = 0$$

$$a = 1, \quad b = -4, \quad c = 4$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{+4 \pm \sqrt{-6 - 4(1)(4)}}{2}$$

$$= 2$$

Ans 5:

$$y_1 = C_1x^2 \quad , y_2 = C_2x^2\ln x.$$

Solution:

$$y'' - \frac{3}{(x+1)}y' + \frac{4}{(x+1)^2} = \frac{x^2}{(x+1)^2} = \left(\frac{x}{x+1}\right)^2$$

$$f(x) = \frac{x^2}{(x+1)^2}$$

$$W = \begin{vmatrix} C_1x^2 & C_2x^2\ln x \\ 2C_1x & C_2x + C_2 2x\ln x \end{vmatrix}$$

$$W = \begin{vmatrix} 0 & C_2x^2\ln x \\ x^2/(x+1)^2 & C_2x + C_2 2x\ln x \end{vmatrix}$$

$$= \frac{-x^2}{(x+1)^2} C_2x^2\ln x$$

$$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix} = \begin{vmatrix} C_1x^2 & 0 \\ 2C_1x & \frac{x^2}{(x+1)^2} \end{vmatrix}$$

$$W_2 = \frac{C_1x^4}{(x+1)^2}$$

$$u_1 = \frac{W_1}{W} \quad , \quad u_2 = \frac{W_2}{W}$$

$$Y_p = v_1y_1 + v_2y_2$$

$$Y = Y_n + Y_p$$