

# "ASSIGNMENT ; 01"

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SECTION	✳	B
SUBJECT	✳	DIFFERENTIAL EQUATIONS
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# \* "APPLICATION OF ODE'S and PDE'S<sup>"01"</sup> IN ENGINEERING";

## \* "Differential equations"

The differential equations have wide applications in various engineering and Sciences disciplines. It is practically important for engineers to be able to model physical problems using mathematical equations, so that solve these equations and the behavior of the systems concerned can be studied.

## "Classification by Type";

"i"; Ordinary Differential equations.  
ODE'S.

"ii"; Partial Differential equations.  
PDE'S.

## \* Ordinary Differential Equation;

If a Differential Equations contains only ordinary derivatives of one or more dependent variables with respect to a single independent variables, it is said to be an ordinary differential equation or (ODE) for short.

"For example";

$$\left( \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0 \right), \left( x \cdot \frac{dy}{dx} = y - 1 \right)$$

## \* APPLICATION OF ODE'S

"1"; Newton's law of cooling.

"2"; Beams.

"3"; Physical Applications.

"4"; Radio Active Elements.

"5"; Electrical circuits.

"6"; Modelling Free Mechanical oscillation.

"7"; Modelling Forced Mechanical oscillation.

"8"; Computer exercise or Activity.

"9"; Modelling with First order Equations.

## \* General Applications of ODE's

→ ODE's are used in various fields such as;

- Radioactivity and carbon dating.
- Equation of Series RL circuits.
- Economics.
- Bernoulli Equation.
- Population Dynamics.
- Euler-Lagrange Equation.

## \* Physical Applications of ODE's

$$\text{Its velocity (v)} = \frac{dx}{dt}$$

$$\text{Its acceleration (a)} = \frac{dv}{dt} \quad \underline{\text{OR}}$$

$$\frac{d^2x}{dt^2} \quad \underline{\text{OR}} \quad v \cdot \frac{dv}{dx}$$

If the body be moving along a curve,  
Then;

$$\text{Velocity (v)} = \frac{ds}{dt} \quad \underline{\text{OR}} \quad v \cdot \frac{dv}{ds} \quad \underline{\text{OR}} \quad \frac{d^2s}{dt^2}$$

# \* Partial Differential Equation

If a differential equation contains partial derivatives of one or more dependent variables of two or more independent variables, it is said to be a partial differential equation or (PDE) for short.

"For example";

$$\left( \frac{\partial x}{\partial y} + \frac{\partial y}{\partial x} + 2 = 0 \right), \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial t^2} = 2 \frac{\partial u}{\partial t} \right)$$

## \* Application of PDE's

PDE's are used to model many systems in many different fields of science and engineering.

- "1"; Laplace Equation.
- "2"; Heat Equation.
- "3"; Wave Equation.

## \* LAPLACE EQUATION;

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- Laplace Equation is used to describe the steady state distribution of Heat in body.
- Also used to describe the steady state distribution of electrical charge in a body.

$$\frac{\partial^2 u(x, y, z)}{\partial x^2} + \frac{\partial^2 u(x, y, z)}{\partial y^2} + \frac{\partial^2 u(x, y, z)}{\partial z^2} = 0$$

## \* HEAT EQUATION;

- The function  $u(x, y, z, t)$  is used to represent the temperature at time "t" in a physical body at a point with coordinates  $(x, y, z)$ .
- $\alpha$  is the thermal diffusivity. It is sufficient to consider the case " $\alpha = 1$ ".

$$\frac{\partial u(x, y, z, t)}{\partial t} = \alpha \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right]$$

## \* Wave EQUATION;

- The function  $u(x, y, z, t)$  is used to represent the displacement at time "t" of a particle whose position at rest is  $(x, y, z)$ .

$$\frac{\partial^2 u(x, y, z, t)}{\partial t^2} = c^2 \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right]$$

- The constant "c" represent propagation speed of wave