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Semester :- 12th

Subject :- Earthquake Engg

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:- 2 CR. HRS

Q no 1 :-

A beam shown in figure 1 is pulled for 1/2 inch in the downward direction and then suddenly released to vibrate freely. Take $E = 29000$ ksi and $I = 150\ in^4$. δst = Deflection due to (7487) lb static load. Also draw graph to show the variation of displacement with time and the variation of equivalent static force with time

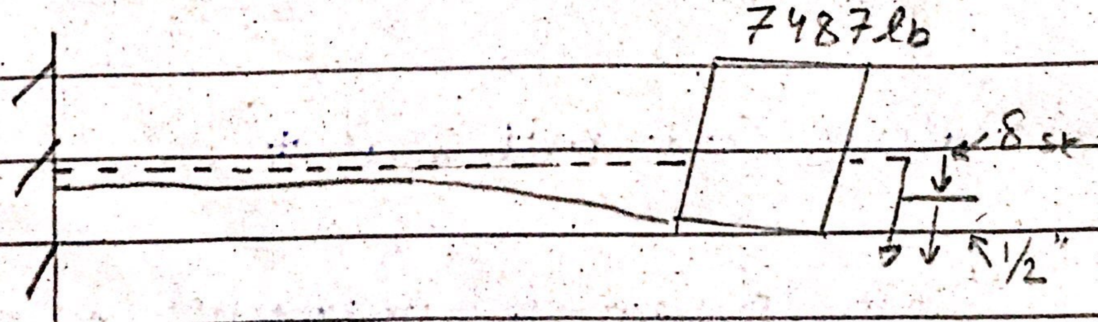


Figure 1

Given Data:-

$E = 29{,}000$ ksi , $I = 150$ in^4

δ = deflection due to 7487 lb static load. Beam is pulled 1/2" downward

Required :-

Natural time period = of system

Develope & Solve Eq of motion.

Draw graph to show the variation of displacement with time & the variation of equivalent static force with time.

Solution:-

General EOM for SDOF System is

$$ku + c\dot{u} + m\ddot{u} = P(t)$$

Since system is undamped = $C = 0$

undergoing free vibration = $P(t) = 0$

Hence general EOM becomes

$$ku + m\ddot{u} = 0 \quad -①$$

$$K = \frac{3EI}{L^3} = \frac{3 \times 29000\ k/in^2 \times 150\ in^4}{(10 \times 12\ in)^3}$$

$$K = \cancel{\frac{3EI}{L^3}} = \frac{7.55k}{in} = K = 7.55208$$

→ In Order to eliminate chances of mistake during calculation it is more appropiate to use fundamental units like lb, ft, sec or kg, m, sec

$K = 7.55208\ k/in \Rightarrow 9.0625\ lb/ft$

$$m = \frac{w}{g} = \frac{7487}{32.12} = 233.09\ \text{slug}$$

$$W_n = \sqrt{\frac{K}{m}} \Rightarrow \sqrt{\frac{90625}{233.09}}$$

$W_n = 19.71$ rad/sec

$$T_n = \frac{2\pi}{W_n} \Rightarrow \frac{2\pi}{19.71} = 0.138 \text{ Secs.}$$

Put m & k in Eq ①

$$90625\, u + 233.09\, \ddot{u} = 0$$

where k is in $\frac{lb}{ft}$ & m is in in/lb/sec/ft^2.

⇒ General solution for EOM for undamped free vibration is

$$u(t) = u(0)\cos(W_n t) + \frac{\dot{u}(0)}{24_n} \sin(W_n t)$$

$$u(0) = \frac{1}{2''} = \frac{1}{24} \text{ ft} \;\&\; \dot{u}(0) = 0$$

$$u(t) = \frac{1}{24} \times \cos(19.71) + 0 \left(\frac{1}{24}\right) \times \cos(19.71)$$

Equivalent static force at any time 't' is

$$f_s(t) = K \cdot u(t) = \frac{90625 \times \cos(19.71t)}{24}$$

$$= 3776 \cos(19.71t)$$

Amplitude of dynamic displacement u_0 for undamped free vibration is

$$u_0 = \sqrt{\left[(u(0))^2 + \left(\frac{u(0)}{w_n}\right)^2\right]}$$

$$= \sqrt{\left(\left(\frac{1}{24}\right)\right)^2 + 0}$$

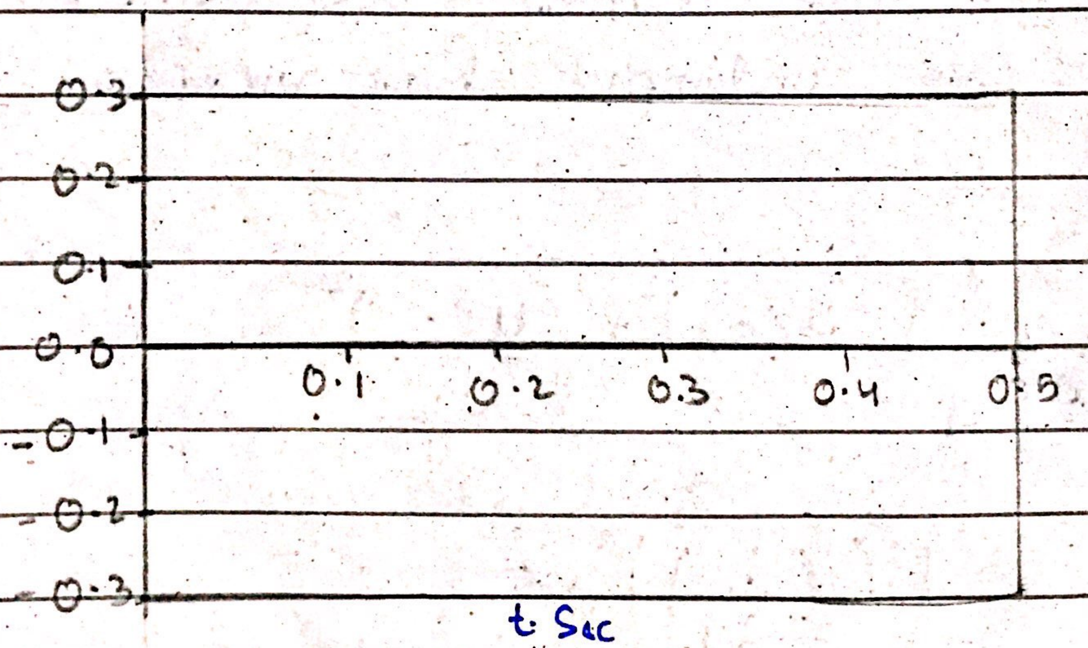
$$= \frac{1}{22} \text{ ft}$$

Amplitude of equivalent static force f_{so}

$$K u_o = 90625 \times \frac{1}{24}$$

$$K u_o = 3776.$$

Graph variation of displacement with time.



$$u(t) = \left(\frac{1}{24}\right) \cos\left((19.7\cancel{1})(t)\right)$$

Qno2 :-

ξ (damping Ratio) of reinforced concrete with considrable working = 3- 5% = 3%.

Using Data of beam given in Question 1

Required :-

- Develop & Solve the equation showing variation in equivalent static force with time
- Draw Graph to show variation of displacement with time and the variation of equivalent static force with time.

Solution :-

EOM damped free viberation is

$$Ku + C\dot{u} + m\ddot{u} = 0 \quad \text{—} (1)$$

From Question 1

$K = 90625 \frac{lb}{ft}$ & $m = 233.09 \frac{lb \, sec^2}{ft}$

$W_n = 19.71$

$C = \xi \times 2m W_n$

$= (0.03 \times 2 (233.09)(19.71)$

$C = 275.65$ lb.sec/ft.

Put values in Eq ①

$90625 + 275.65 + 233.09 = 0$

Solution to the EOM for damped free vibration is

$$u(t) = e^{-\xi W_n t} \left[(u)(\cos)(W_2 t) + 1/W_n \left[\dot{u}(0) + u(0) \, \xi W_n \right] \sin W_D t \right.$$

(9)

$W_D = 19.71$ rad/sec

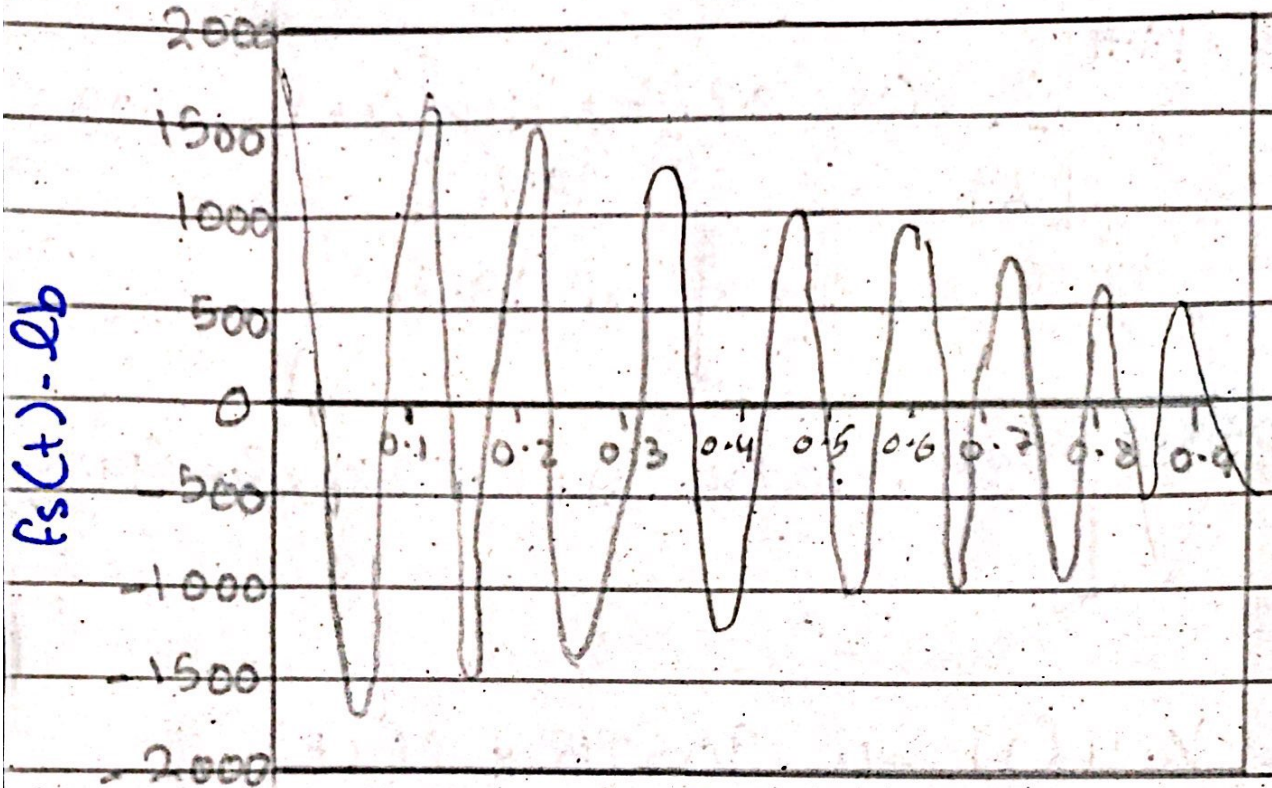
$$u(t)\, e^{-0.03 \times 19.71}\left[\frac{1}{24} \times \cos(19.71)t + \frac{1}{19.71} \times \left\{0 + \frac{1}{24} \times 0.03 \times 19.71 \times \right\} \sin(19.71)\right]$$

$$u(t) = e^{-0.5913}\left[0.041 \times \cos(19.71t) + 0.0281 \times \sin(19.71t)\right]$$

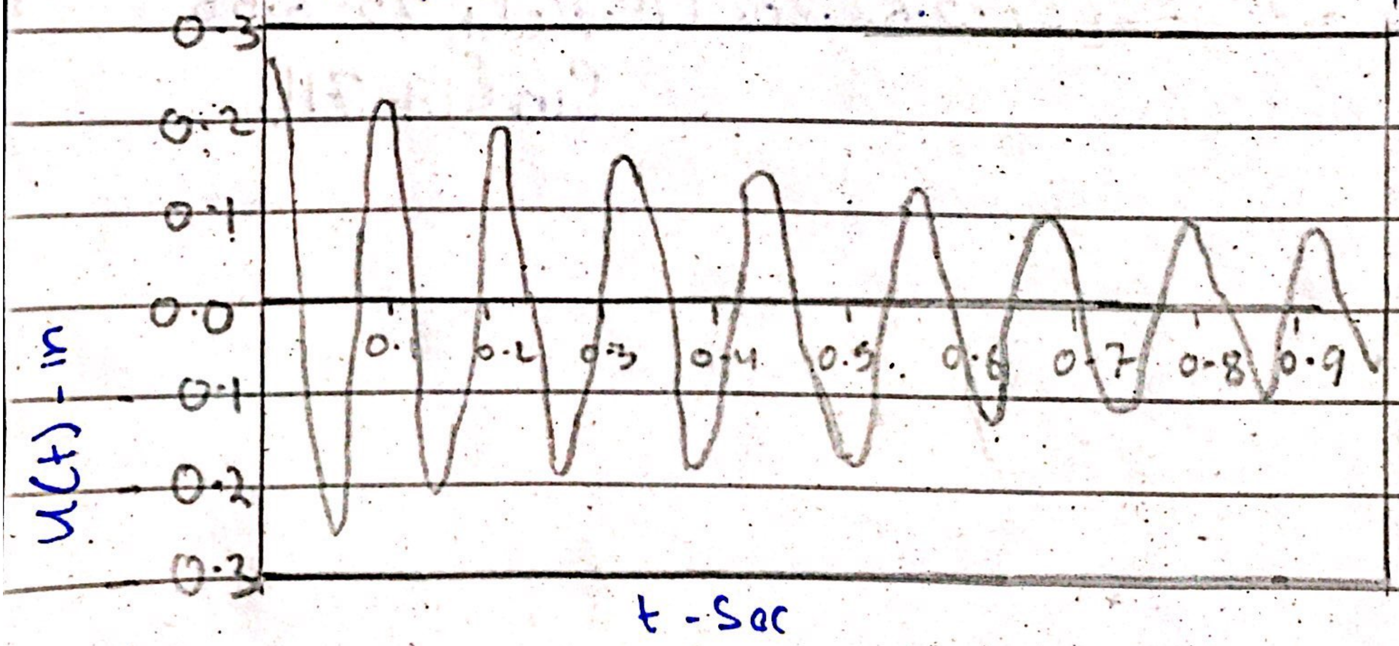
$$fs(t) = K \cdot u(t) \Rightarrow 90625 \times u(t)$$

$$fs(t) = e^{-0.5913}\left[(90625 \times 0.041)\cos(19.71t) + 90625 \times 0.0281 \times \sin(19.71)\right]$$

$$fs(t) = e^{-0.5913}\left[3715.62 \cdot \cos(19.71t) + 2546.56 \sin(19.71t)\right]$$



Variation of Equivalent static force with time.



Variation of displacement with time

Qno3:-

A free vibration test was conducted on an empty water tank shown in figure ......
....

Given Data:-

Force = 60 Kips

Displacement of tank = $\frac{7487}{1000}$ = 7.487"

Time taken to complete 7 cycles = 3.57 sec, amplitude of displacement = 2.286 cm = 0.9"

Required Data:-

(a) Damping ratio
(b) Natural period of un-damped vibration
(c) Stifness of structures
(d) weight of tank
(e) Damping coefficient.
(f) Number of cycles to reduce the displacement amplitude to 0.5"

⇒ Solution:

→ Displacement of tank $u_1 = 7.487$

→ After 7 cycles ie, after $J = 7$, $u_{j+1} = u_8 = 0.9$

ⓐ Damping ratio $= \xi = ?$

$$j = \frac{1}{2\pi\xi} \ln\left[\frac{u_1}{u_1+1}\right]$$

$$7 = \frac{1}{2\pi\xi} \ln\left[\frac{7.487}{0.9}\right]$$

$$\frac{\cancel{2\pi}\,\xi\,\cancel{7}}{\cancel{2\pi}\,\cancel{7}} = \frac{8.31}{2\pi \times 7}$$

$$\xi = 0.18 \quad = 18\%$$

ⓑ Natual period of undamped vibration $= T_n = ?$

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As, the 7 cycles of vibrations are completed in 3.57 sec

$\Rightarrow$ Time required to complete one cycle, $T_D = \frac{3.57}{7} = 0.51 sec$

$T_D = 0.51$ sec

Now,

$$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$$

$$\frac{2\pi}{\omega_D} = \frac{2\pi}{(\omega_n \sqrt{1-\zeta})}$$

$$T_D = \frac{T_N}{(1-\zeta)^2}$$

$$\Rightarrow T_n = T_D \times \sqrt{1-\zeta^2}$$

$$T_N = 0.51 \times \sqrt{1-(0.18)^2}$$

$$\boxed{T_n = 0.50 \text{ Sec}}$$

ⓒ Stifness of structure, $K = ?$

$$K = \frac{60 \times \cos 60^\circ}{7.487} = 4.0 \ k/in$$

$K = 4.0 \ k/in = 48\,000 \ lb/ft$

ⓓ weight of tank, $W = ?$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{\frac{W}{g}}} = \sqrt{\frac{K \cdot g}{W}}$$

$$\Rightarrow \omega_n^2 = \frac{Kg}{W}$$

$$W = \frac{K \times g}{\omega_n^2}$$

Also $\omega_n = \dfrac{2\pi}{T_n}$

$$W = \frac{Kg}{\left(\frac{4\pi^2}{T_n^2}\right)}$$

$$= Kg \times \frac{T_n^2}{4\pi^2}$$

Scanned with CamScanner

$$W = \left[\frac{48000\ lb}{ft} \times \frac{32.2\,ft}{sec^2} \right] \times \frac{(0.50\,sec)^2}{4\pi^2}$$

$$= 1957.52\ lb = 1.95k$$

ⓔ Damping coefficient, $c = ?$
It is known that

$$\xi = \frac{c}{2m\omega_n}$$

$$\Rightarrow c = \xi \times 2m\omega_n$$
$$= \xi \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$c = \frac{0.18 \times 4 \times \pi \left(\frac{1957.52}{32.2}\right)}{0.51}$$

$$\boxed{c = 269.71\ lb.sec/ft}$$

(f) Number of cycles to reduce the displacement amplitude to 0.5"

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$j = \frac{1}{2 \times \pi \times 0.18} \ln\left[\frac{7.487}{0.5}\right]$$

$j = 2.38$ so 3 circles