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Section:- A

Subject:- Defferential Equation

Question Solved by 2 Methods

Method # 01

Given data

→ Pakistani, Egyptian & Americian cotton Ratio ⇒

A		B		C
1	:	2	:	1
2	:	1	:	1
2	:	0	:	2

A, B & C cost ⇒ 40, 50 & 60 Rs Respectively

We write the given data in ~~the~~ a ~~linear~~ System of Linear Equation.

$$\left\{\begin{array}{l} a + 2b + c = 40 \\ 2a + b + c = 50 \\ 2a + 2c = 60 \end{array}\right. \quad \text{or} \quad \left\{\begin{array}{l} x + 2y + z = 40 \\ 2x + y + z = 50 \\ 2x + 2z = 60 \end{array}\right.$$

Now we use Gauss - Jordan - Elimention method to find the values of x, y & z

where x, y & z are the cost of a kg of each country.

kilogram

↳

Q# $\quad x + 2y + z = 40$ (2)

$2x + 1y + z = 50$

$2x + 2z = 60$

Augmented Matrix of the given System.

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 40 \\ 2 & 1 & 1 & 50 \\ 2 & 0 & 2 & 60 \end{array}\right].$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 40 \\ 0 & -3 & -1 & -30 \\ 0 & -4 & 0 & -20 \end{array}\right]. \sim \begin{array}{l} R_2 - 2R_1 \\ R_3 - 2R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 40 \\ 0 & 1 & 1/3 & 10 \\ 0 & -4 & 0 & -20 \end{array}\right] \sim -\frac{1}{3} R_2$$

$0+0 \quad -4+4 \quad 0+4/3 \quad {-20 \atop +40}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 40 \\ 0 & 1 & 1/3 & 10 \\ 0 & 0 & 4/3 & 20 \end{array}\right] \sim R_3 + 4R_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 40 \\ 0 & 1 & 1/3 & 10 \\ 0 & 0 & 1 & 15 \end{array}\right] \sim \frac{3}{4} R_3$$

Here we find out the values of

$x = 15$

$y = 5$

$$\boxed{\begin{array}{l} X = 15 \\ Y = 5 \\ Z = 15 \end{array}}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 25 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 15 \end{array}\right] \sim \begin{array}{l} R_1 - R_3 \\ R_2 - 1/3 R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 15 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 15 \end{array}\right] \sim R_1 - 2R_2$$

Method # 02

Solution:- 1 : 2 : 1 , 2 : 1 : 1 , 2 : 0 : 2

40 50

P	E
A	E

P	P
A	E

P	P
A	A

Let x, y and z be the cost/kg Pak, Egyptain, American cotton respectively then according to the given conditions:

$$\left.\begin{aligned} \frac{1}{4}x + \frac{2}{4}y + \frac{1}{4}z &= 40 \\ \frac{2}{4}x + \frac{1}{4}y + \frac{1}{4}z &= 50 \\ \frac{2}{4}x + \frac{2}{4}z &= 60 \end{aligned}\right] \rightarrow (A)$$

$$\left.\begin{aligned} 1x + 2y + 1z &= 160 \\ 2x + 1y + 1z &= 200 \\ 1x + 1z &= 120 \end{aligned}\right] \rightarrow (B)$$

In matrix form, we can write as

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

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Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$

$$AX = B$$

$\Rightarrow$ $A_1 = \begin{bmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{bmatrix}$

$A_3 = \begin{bmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{bmatrix}$

First $|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$ Expand by R_1

$$= 1\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= 1(1\times1 - 1\times0) - 2(2\times1 - 1\times1) + 1(2\times0 - 1\times1)$$

$$= -2$$

Now

$|A_1| = \begin{vmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{vmatrix}$ Expand by R_1

$$= 160\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} + 1\begin{vmatrix} 200 & 1 \\ 120 & 0 \end{vmatrix}$$

$A_1 = -120$

Similarly

$$|A_2| = \begin{vmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{vmatrix} \quad \text{Expand by } R_1$$

$$= 1\begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} - 160\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 200 \\ 1 & 120 \end{vmatrix}$$

$$|A_2| = -40$$

$$|A_3| = \begin{vmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{vmatrix} \quad \text{Expand by } R_1$$

$$= 1\begin{vmatrix} 1 & 200 \\ 0 & 120 \end{vmatrix} - 2\begin{vmatrix} 2 & 200 \\ 1 & 120 \end{vmatrix} + 160\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= 1(120-0) - 2(240-200) + 160(0-1)$$

$$|A_3| = -120$$

Now according to Crammer's Rule

$$x = \frac{|A_1|}{|A|} = \frac{-120}{-2} = 60$$

$$y = \frac{|A_2|}{|A|} = \frac{-40}{-2} = 20$$

$$z = \frac{|A_3|}{|A|} = \frac{-120}{-2} = 60$$

$(x, y, z) = (60, 20, 60)$

Pakistani = 60

Egyptian = 20

American = 60