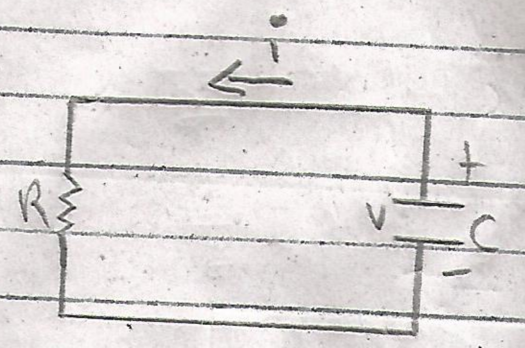
$-40e^{-4t}$

Sol:-



a) Find R & C

The time constant

$$\tau = \frac{1}{e} = RC = \frac{1}{4}$$

Now we know that

$$i = C\frac{dv}{dt}$$

$$0.2e^{-4t} = C(10)\frac{e^{-4t}}{4} = 10(-4)e^{-4t}$$

$$\frac{0.2\cancel{e^{-4t}}}{-40\cancel{e^{-4t}}} = \frac{C \cdot -40\cancel{e^{-4t}}}{-40\cancel{e^{-4t}}}$$

$$C = -0.005$$

we write it as

$$\boxed{C = 5\ mF}$$

Now

$$R = \frac{V(t)}{i_R(t)} = \frac{10e^{-4t}}{0.2e^{-4t}}$$

$$R = 50\,\Omega$$

b) for the time Constant

we know that
time Constant

$$\tau = R \times C$$

$$= 50\Omega \times 0.005C$$

$$= 0.25s$$

c) Now to find initial energy in the capacitor

we know that

$$w(0) = \frac{1}{2} C V_0^2$$

c) $\Rightarrow \frac{1}{2}(5\times10^{-3})(10)^2$

$$= \frac{1}{2}(5\times10^{-3})(100)$$

$$= \frac{1}{2}(500\times10^{-3})$$

$$= 250\times10^{-3}\ J$$

$$\therefore 10^{-3} = m$$

$$w_c(0) = 250\ mJ$$

d) Now to find the obtain time to dissipate 50% of initial energy we use.

$$w_c(t) = \frac{1}{2}Cv^2(t)$$

To dissipate 50%

$$w_c(t) = \frac{w_c(0)}{2}$$

from (c) the initial energy of capacitor

$$\frac{w_c(0)}{2} = \frac{1}{2} C v^2(t)$$

$$\frac{2\cancel{5}0 \times 10^{-3} J}{\cancel{2}} = \frac{1}{2} 5 \times 10^{-3} (10 e^{-4t})^2$$

$$125 \times 10^{-3} J = \frac{1}{2} \times 5 \times 10^{-3} \times 100 \times e^{-8t}$$

$$\frac{125 \times \cancel{10^{-3}} J}{500 \times \cancel{10^{-3}}} = \frac{500 \times \cancel{10^{-3}} \times e^{-8t}}{500 \times \cancel{10^{-3}}}$$

$$0.5 = e^{-8t}$$

$$\tau = \frac{1}{e}$$

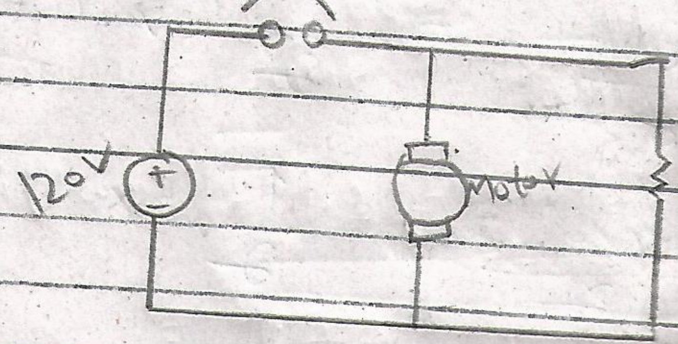
$$\ln 0.5 = -8t \ln e$$

$$-0.6931 = -8t$$

$$t = 86.643 \text{ms}$$

Q2:-

Sol:-



Given:-

$V_S = 120V$
$L = 50H$
$R = 100\Omega$
$t = 100ms$

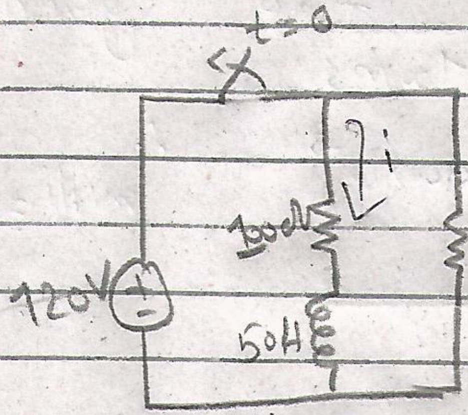
when the circuit is closed:-

Steady state condition apply & the inductor act as short circuit

The current that flows through the 100Ω resistor at $t=0$ (Before the Circuit opens) given by

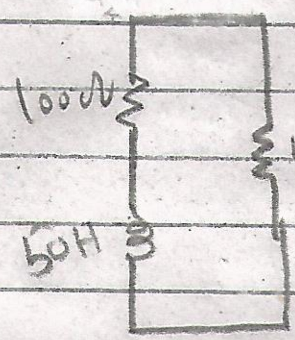
$V = iR$

$$i = \frac{120}{100} = 1.2A$$



$$i = \frac{V}{R} = \frac{120}{100} = 1.2A$$

After the switch is open



we know that transient response is always decaying

$$\tau = \frac{L}{R} = \frac{50}{100+400} = 0.1s$$

Now the inductor will discharge & the circuit has no source

$$i(t) = i(0)e^{-t/\tau} = 1.2e^{-t/0.1}$$

$$i(t) = 1.2e^{-10t}$$

So now the given time 100ms

$$i(100ms) = 1.2\, e^{-10 \times 100 \times 10^{-3}} \qquad \because milli = 10^{-3}$$

$$= 1.2\, e^{-1}$$

$$= 0.441\, A$$

$$= 441\, mA$$

Q3:-

Sol:- Given that :-

$$V_C(t) = 30 - 10e^{-20t} + 30e^{-10t}\ V$$
$$i_L(t) = 40e^{-20t} - 60e^{-10t}\ mA$$

we aim that response of this circuit is (overdamp) So now Given that

$S_1 = -10$, $S_2 = -20$

we know that

$$S_1 + S_2 = -2\alpha$$

$$-10 - 20 = -2\alpha$$

$$\frac{-30}{-2} = \frac{-2\alpha}{-2}$$

$$\alpha = \frac{R}{2L} = 15$$

$$\boxed{\alpha = 15}$$

now

$$S_1 - S_2 = 2\sqrt{(15)^2 - \omega_0^2}$$

$$-10 + 20 = 2\sqrt{(15)^2 - \omega_0^2}$$

$$10 = 2\sqrt{15^2 - \omega_0^2} \rightarrow$$

So

$$S_1 = -15 + \sqrt{15^2 - w_0^2}$$

$$-10 = -15 + \sqrt{15^2 - w_0^2}$$

$$15^2 - w_0^2 = 25$$

$$w_0 = \sqrt{225 - 25} = \sqrt{200}$$

$$w_0 = \frac{1}{\sqrt{LC}}$$

$$LC = 1/200$$

Since we have a series RLC $i_L = i_C = C\frac{dv_c}{dt}$

$$i_L(t) = 40e^{-20t} - 60e^{-10t} \text{ mA}$$

$$m = 10^{-3}$$

$$i_L(t) = 40 \times 10^{-3} e^{-20t} - 60 \times 10^{-3} e^{-10t} \text{ A}$$

$$= 0.04e^{-20t} - 0.06e^{-10t}$$

$$i_L = C\frac{dv_C}{dt}$$

$$= 0.04e^{-20t} - 0.06e^{-10t} = C\frac{d}{dt}\left(30 - 10e^{-20t} + 30e^{-10t}\right)$$

$=$

The derivative of this give us.

$$0.04e^{-20t} - 0.06e^{-10t} = C\left(200e^{-20t} - 300e^{-10t}\right)$$

$$C = \frac{0.04e^{-20t} - 0.06e^{-10t}}{200e^{-20t} - 300e^{-10t}}$$

$$C = 2\times10^{-4}\,F$$ (i)

Now for "L"

Previous

$$LC = \frac{1}{200} \qquad \frac{1}{LC} = 200$$

$$\frac{1}{L(2\times10^{-4})} = 200$$

$$L = 25\,H$$ (ii)

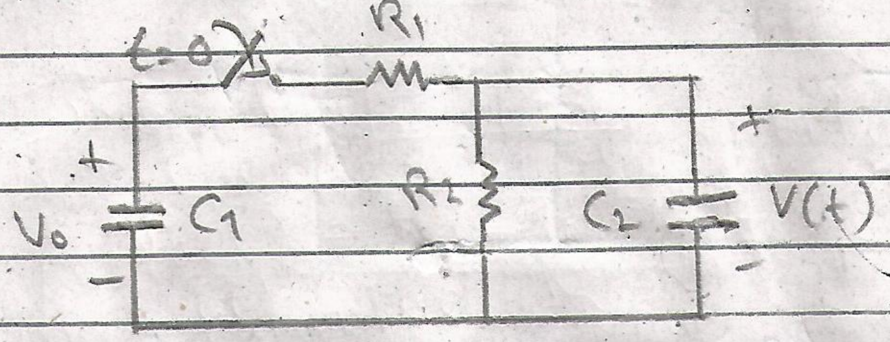
Previous

$$\alpha = \frac{R_{eq}}{2L} = 15$$

$$\frac{R}{2(25)} = 15 \qquad \frac{R}{50} = 15$$

$$R = 750\,\Omega$$ (iii)

Q4:-



Given data:-

$C_1 = 0.5 \mu f$

$C_2 = 5 \mu f$

$R_1 = 5 M\Omega$

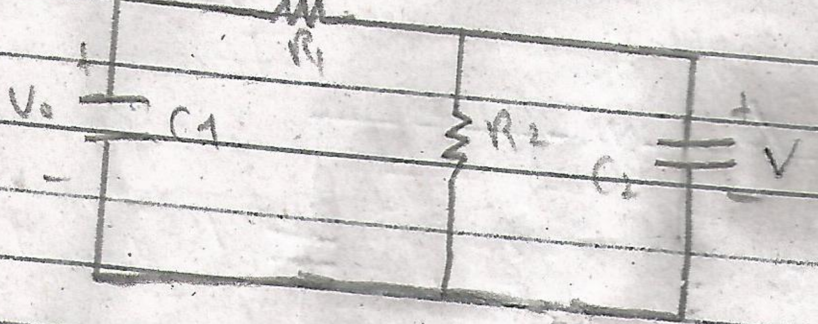
$R_2 = 2.5 M\Omega$

$V_0 = 60 u(t) V$

for $t<0$ the switch is open therefore C_1 is detached from network.

$V(0) = 0 V$

for $t>0$ the switch is closed at $t=0$ C_1 is connected to the circuit as shown



Applying KCL at node b/w R_1 & R_2

$$-i_{R1} + i_{R2} + i_{C2} = 0$$

thus $i_{R1} = i_{R2} + i_{C2}$

$$\frac{V_0 - V}{R_1} = \frac{V}{R_2} + C_2 \frac{dv}{dt}$$

multiply by R_1, thus

$$V_0 - V = \frac{R_1 V}{R_2} + R_1 C_2 \frac{dv}{dt}$$

Rearrange

$$V_0 = V\left(1 + \frac{R_1}{R_2}\right) + R_1 C_2 \frac{dv}{dt}$$

Substitute

$$V_0 = V\left(1 + \frac{5}{2.5}\right) + \left(5\times10^6 \times 5\times10^{-6} \frac{dv}{dt}\right)$$

thus $V_0 = 3v + 25\frac{dv}{dt}$ 2

when the steady state condition apply, the capacitor act as an open circuit and no current will flow through it

$$i_{c_2} = C_2\frac{dv}{dt} = 0$$

thus $\frac{dv}{dt} = 0$

Substitate in (2)

$$V_0 = 3V \quad ---3$$

where V_s is the steady state voltage across the capacitor but the unit step function for $t>0$ is given by

$$u(t) = 1 \qquad t>0$$

thus

$$V_0 = 60u(t) = 60V \qquad t>0$$

Substitute

$$60 = 3V_s$$

thus

$$V_s = 20V$$

The voltage across the capacitor $v(t)$ is given

$$v(t) = V_s + Ae^{-t/\tau}$$

the equivalent resistance of the circuit $R_1 // R_2$

$$R_{eq} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{5 \times 2.5}{5 + 2.5} = \frac{5}{3}\, M\Omega$$

the time constant for capacitor C_2

$$\tau = R_{eq} C_2 = \frac{5 \times 10^6}{3} \times 5 \times 10^{-8} = \frac{25}{3}\, s$$

Now

$$v(t) = 20 + Ae^{-3t/25}$$

$$V(0) = 20 + A$$

$$A = -20$$

$$v(t) = 20 - 20e^{-3t/25}$$

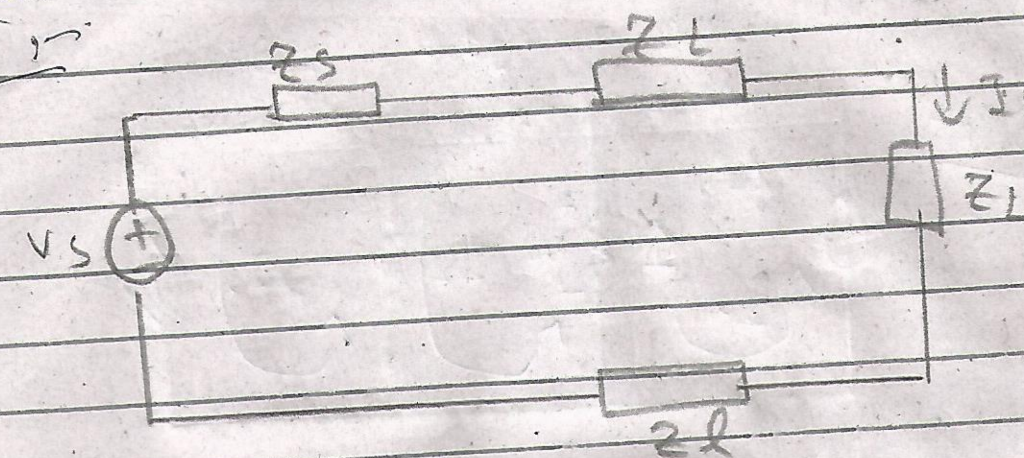
$$v(t) = 20 - 20e^{-3t/25}$$

$$v(t) = 20(1 - e^{-3t/25})\,V$$

$$t > 0$$

$$v(t) = 20(1 - e^{-3t/25})\,V \quad t > 0$$

Q5 :-



using

$$I_L = \frac{V_s}{Z}$$

So this in series so

$$I_L = \frac{V_s}{Z_s + 2Z_l + Z_L} = \frac{115 \angle 0}{(1 + j0.5) + 2(0.4 + j0.3) + (23.2 + j18.9)}$$

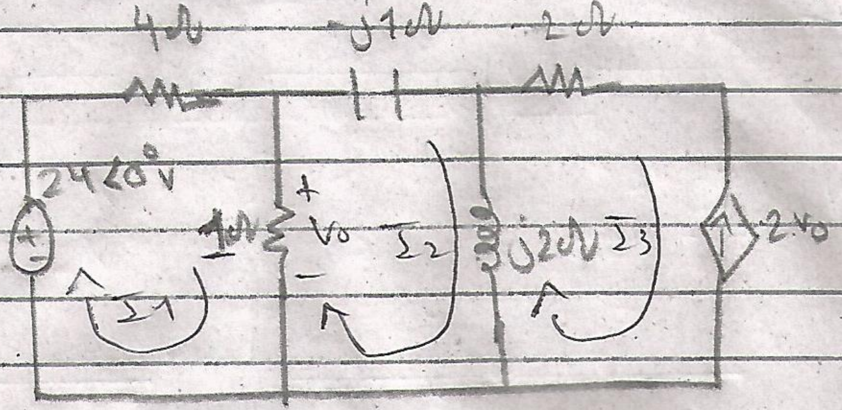
$$= \frac{115 \angle 0}{32.02 \angle 38.66} = 3.592 \angle -38.66$$

thus load current is

$3.592 \angle -38.66$

Q6:-

Sol:-



KVL at mesh 1

$$24\angle 0^\circ + 4I_1 + I_1 - I_2 = 0$$

$$\| 5I_1 - I_2 = 24\angle 0^\circ \|$$

KVL at mesh 2

$$I_2 - I_1 - jI_2 + j2(I_2 - I_3) = 0$$

$$-I_1 + (1 - j + j2)I_2 - j2I_3 = 0$$

$$\| -I_1 + (1+j)I_2 - j2I_3 = 0 \|$$

KVL at mesh 3

$$I_3 = -2V_0$$

$$V_0 = I_1 - I_2$$

$$I_3 = -2(I_1 - I_2) = -2I_1 + 2I_2$$

$$\| 2I_1 - 2I_2 + I_3 = 0 \|$$

BY solving them

$I_1 = 6.48\angle -1.55^\circ A$

$I_2 = 8.45\angle -5.95^\circ A$

$I_3 = 4.1\angle -19.98^\circ A$

Now to find voltage across dependent source

$j2(4.1\angle -19.98 - 8.45\angle -5.95^\circ + 2(4.1\angle -19.98^\circ) + V_s) = 0$

$V_s = 14.78\angle 126.34^\circ V$

Now

$I = 2V_o$

$= 2(6.48\angle -1.55 - 8.45\angle -5.95^\circ)$

$= 4.1\angle 160.020 \; A$

$S = VI = (14.78\angle 126.340^\circ) \times (4.1\angle -16.02)$

$= 60.6\angle -33.68 \; VA$

$S = 50.43 - j33.61 \; VA$

So $P = 50.43 W$

$Q = 33.61 \; VAR$

the average reactive & complex power Delivered