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Subject Differential Equation

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Q.No. 1

Find the fourier Series representation of $f(t) = 1 + t, \; -\pi < t < \pi$.

Solution ;

$$f(t) = 1+t, \; -\pi < t < \pi$$

Using formula ;

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t$$

(Eq 1)

$$a_0 = 1/2\pi \int_{-\pi}^{\pi} f(t)\,dt$$

$$a_0 = 1/2\pi \int_{-\pi}^{\pi} (1+t)\,dt$$

$$a_0 = 1/2\pi \left| t + t^2/2 \right|_{-\pi}^{\pi}$$

$$a_0 = 1/2\pi \left(\pi - (-\pi) + \pi^2/2 - \left(\frac{-\pi^2}{2}\right) \right)$$

$$a_0 = 1/2\pi \left(2\pi + \frac{2\pi^2}{2} \right)$$

(2)

$$a_0 = 1/2\pi \left(2\pi + \pi^2 \right)$$

$$a_n = 1/\pi \int_{-\pi}^{\pi} (1+t)(\cos nt)\,dt$$

$$a_n = 1/\pi \left((1+t)\frac{\sin nt}{n} - \frac{\cos nt}{n^2} \Big|_{-\pi}^{\pi} \right)$$

$$a_n = -1/n^2\pi \left(\cos n\pi - \cos n(-\pi) \right)$$

$$a_n = \frac{-1}{n^2\pi} \left(-1 - (-1) \right)$$

$$a_n = 0$$

$$b_n = 1/\pi \int_{-\pi}^{\pi} (1+t) \sin nt \,dt$$

$$b_n = 1/\pi \left((1+t) \int_{-\pi}^{\pi} \sin nt - \int \left(\int \sin nt \, d/dt \right.\right.$$

$$\left. (1+t)\,dt \right)$$

$$b_n = 1/\pi \left(\frac{(1+t)(-\cos nt)}{n} \Big|_{-\pi}^{\pi} + \frac{\sin nx}{n^2} \Big|_{-\pi}^{\pi} \right)$$

$$bn = \frac{-1}{n\pi}\left((1+\pi)(\cos n\pi) - ((1+(-\pi))(\cos(\pi))\right)$$

$$bn = \frac{-1}{n\pi}\left(\cos n\pi + \pi \cos n\pi - \cancel{\cos} n\pi + \cos n\pi\right)$$

$$bn = \frac{-1}{n\pi}\left(2\pi \cos n\pi\right)$$

Here $\cos n\pi = \frac{(-1)^{n+1}}{n}$

$$bn = 2/n \, (-1)^{n+1}$$

So, equation becomes,

$$"f(x) = \frac{1}{2\pi}\left(2\pi + \pi^2\right) + 0 + \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin t"$$

Q No 2

Calculate the characteristics equation the eigen value of the system. where A is given by.

Solution;

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Eigen Values;

Step # 1,

We have;

$(A-\lambda I)x = 0$ A Given Matrix

Step # 02,

We have;

The characteristics equation is given by;

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

Step #3

$$\lambda^3 - \left| \begin{matrix}\text{sum of diagonal} \\ \text{element}\end{matrix} \right| \lambda^2 + \left| \begin{matrix}\text{sum of} \\ \text{diagonal} \\ \text{minor}\end{matrix} \right| \lambda - |A| = 0, \text{(B)}$$

Sum of diagonal element $= 1 + 1 + 2 = 4$

Sum of diagonal minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= (-6) + (2) + (1)$

$= -6 + 2 + 1$

$= -3$

By Putting values in eq (B).

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \quad \text{— (C)}$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1 \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0 \begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1 \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$$= 1(2-8) - 0 + 1(6-0)$$
$$= -6 + 6$$
$$= 0$$

By Putting Values in (c);

$$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$$

$$\lambda^3 - 4\lambda^2 - 3\lambda = 0$$

$$\lambda(\lambda^2 - 4\lambda - 3) = 0$$

$$\lambda = 0$$

$$\lambda^2 - 4\lambda - 3 = 0$$

Using Quadratic formula;

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a = 1, \quad b = -4, \quad c = -3$$

$$\lambda = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16+12}}{2}$$

$$= \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4 + \sqrt{28}}{2}, \quad \lambda = \frac{4 - \sqrt{28}}{2}$$

We have eigen values;

$$\lambda = \left(0, \frac{4+\sqrt{28}}{2}, \frac{4-\sqrt{28}}{2}\right)$$

Required solution

Q No. 3

Solve the following system of linear equation.

$$5x + 0 + 4z + 2m = 3$$
$$x - y + 2z + m = 1$$
$$4x + y + 2z + 0 = 1$$
$$x + y + z + m = 0$$

Solution;

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \; R_4 R_2$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & +6/5 & +4/5 & 3/5 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \; -1/5 \times R_3$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 0 & 7/5 & 8/5 & 1/5 \end{array}\right] \quad 5\times R_3 \ \& \ 5\times R_4$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 5R_3 \ \& \ 5R_4$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 1/5 \times R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -1 & 6/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_2 \times 5$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_3 - R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad \begin{array}{l} R_3 \leftrightarrow R_4 \\ 1/7 \times R_3 \\ 1/3 \times R_4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad R_2 \times -5$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & -5 & 26/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad 5/4 \times R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 126/84 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$(x, y, z, m) = (3/4,\ 31/21,\ -11/21,\ 1/3)$$

$x = 3/4$, $y = 31/21$,

$z = -11/21$, $m = 1/3$

Q. No 4

$$u(x,t) = \sin(x+2t)$$

is a solution of the one-dimensional wave equation.

Solution;

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$u(x,t) = \sin(x+2t)$ is solution of $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$

If we satisfy the above equation

$$\frac{\partial u}{\partial t} = 2\cos(x+2t$$

Again,

$$\frac{\partial^2 u}{\partial t^2} = -2\sin(x+2t)\frac{d}{dt}(x+2t)$$

$$\Rightarrow \frac{\partial^2 u}{\partial t^2} = -4\sin(x+2t) \quad \text{—(A)}$$

Now,

$$\frac{\partial u}{\partial x} = \cos(x+2t$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)$$

$$\Rightarrow \frac{\partial^2 u}{\partial x^2} = -\sin(x+2t) \quad \text{—(B)}$$

Comparing A & B

$$c = 2$$

$$\Rightarrow -4\sin(x+2t) = -c^2 \sin(x+2t)$$

$$\Rightarrow -4\sin(x+2t) + c^2 \sin(x+2t) = 0$$

This is possible is $c = \pm 2$

$$-4\sin(x+2t) + (\pm 2)^2 \sin(x+2t) = 0$$

$$\Rightarrow 0 = 0$$

So, This $u(x,t) = \sin(x+2t)$ is a solution of one dimensional wave equation.