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Section ⇒ "C"

Subbject ⇒ Structural AnalysisII

Submitted to ⇒ Engr. Adeed Khan

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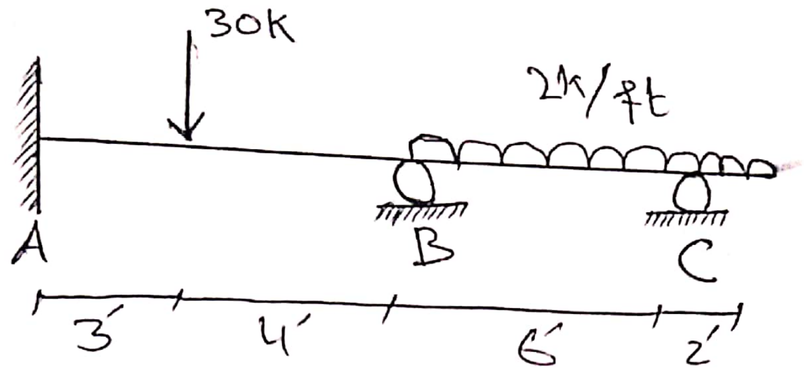
Ans; 01 ①

Beam:




• $EI$ = Constant

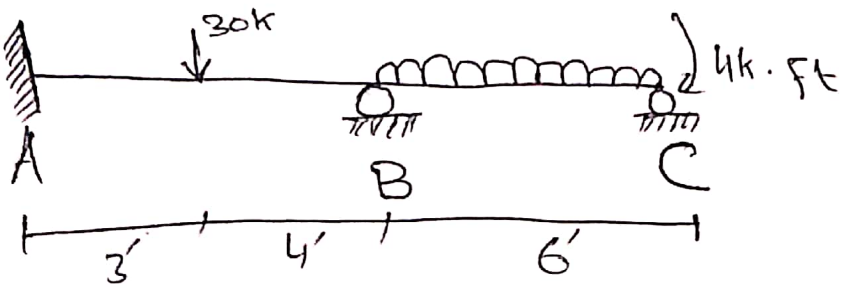
• Stiffness Method:

Step; 01

• Kinematic Indeterminacy;

$K.I = 5^\circ$

We have to reduce the extended portion




$$\frac{2(2)}{1} = 4k.ft$$

Now

$K.I = 2^\circ$

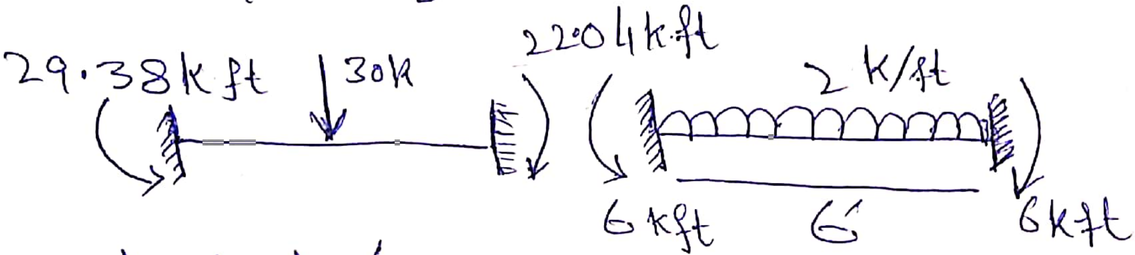
## Step; 02

Determining unknown joint Displacement.


$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \qquad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

## Step; 03

Compute [ADL] Matrix




* For point load (not at mid point)

  - For left End

$$Pab^2/L^2 = 30(3)(4)^2/(7)^2 = 29.38 \text{ k.ft}$$

  - For right End

$$Pa^2b/L^2 = 30(3)^2(4)/(7)^2 = 22.04 \text{ k.ft}$$

* For uniformly Distributed load

  - $wL^2/12 = 2(6)^2/12 = 6$ k.ft
  - $ADL_1 = +22.04 - 6 = 16.04$ k.ft
  - $ADL_2 = 6$ k.ft

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Step; 04

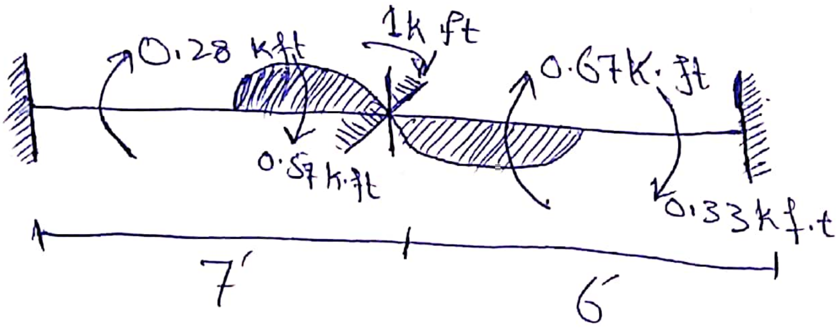


Now computing [S] Matrix;

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

(a);

$D_1 = 1k$ , $D_2 = 0$



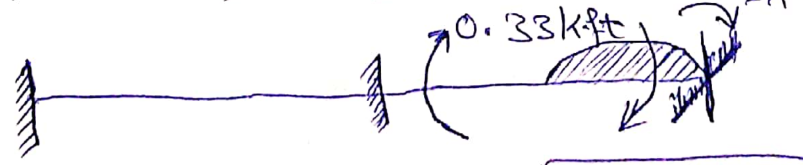

$\frac{4EI}{7} = 0.57$ $\frac{2EI}{6} = 0.33$

$\frac{4EI}{6} = 0.67$ $\frac{2EI}{7} = 0.28$

$S_{11} = 0.57 + 0.67$

$\boxed{S_{11} = 1.24 EA}$ $\boxed{S_{21} = 0.33EA}$

(b); $D_1 = 0$ , $D_2 = 1k$




$4EI/6 = 0.67$ $\boxed{S_{12} = 0.33}$

$2EI/6 = 0.33$ $\boxed{S_{22} = 0.67}$

$$S = \begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$

## Step; 05

Now Computation $[D]$ Matrix;

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}$$

$$= \frac{1}{\begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}} \times \text{Adj } A \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}$$

$$|S| = (1.24 \times 0.67) - (0.33 \times 0.33)$$

$$= 0.8308 - 0.1089$$

$$|S| = 0.7219$$

$$\text{Adj } A = \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}$$

Now;

$$\begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix} = \begin{bmatrix} 0 - 16.04 \\ 4 - 6 \end{bmatrix} = \begin{bmatrix} -16.04 \\ -2 \end{bmatrix} E$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{|S|} \times \text{Adj}A \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$= \frac{\begin{bmatrix} 0.67 & -0.33 \\ 0.33 & 1.24 \end{bmatrix}}{0.729} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.919 & -0.452 \\ -0.452 & 1.70 \end{bmatrix} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} -13.83 \\ 3.85 \end{bmatrix}$$

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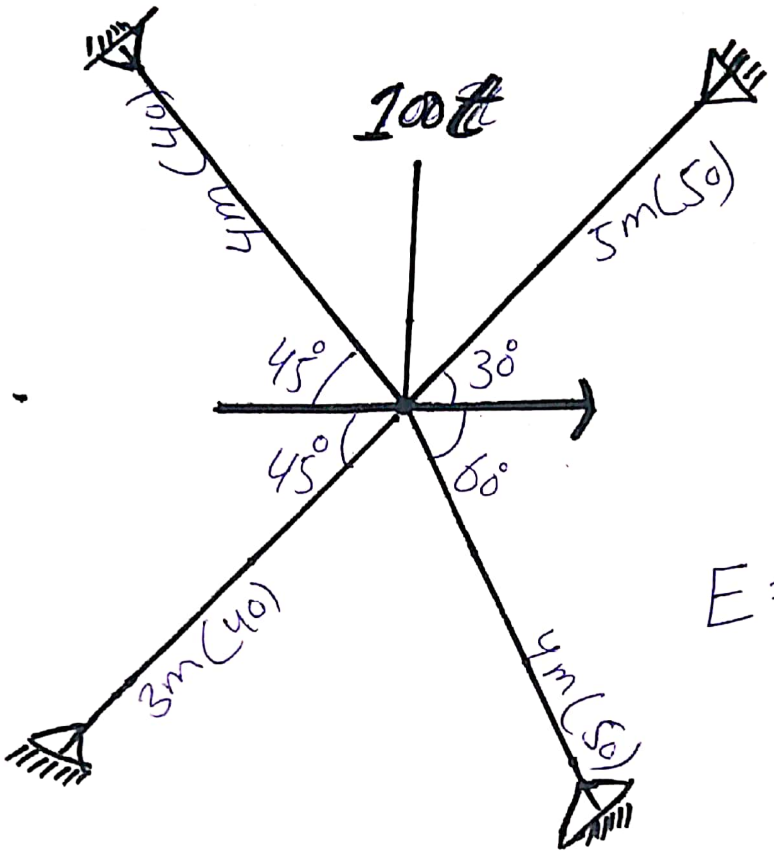
# Ans# 02;

## "Pin - jointed frame"




$E = 2000 t/cm^2$

## : Stiffness Method :→

Sol:

For A;

• $\sin 45 = \frac{P}{H} = \frac{P}{4}$

$P = 2.828 m$

• $\cos 45° = \frac{b}{H} = \frac{b}{4}$

$b = 2.828 m$

**For B;**

- $\sin 45 = \frac{P}{H} = \frac{P}{3}$

$P = 2.12m$

- $\cos 45 = \frac{b}{H} = \frac{b}{3}$

$b = 2.12m$

**For C;**

- $\sin 60 = \frac{P}{H} = \frac{P}{4}$

$(\sin 60)(4) = P$

$P = 3.46$

- $\cos 60 = \frac{b}{H} = \frac{b}{4}$

$(\cos 60)(4) = b$

$b = 2$

**For D;**

- $\sin 30 = \frac{P}{5}$

$P = 2.5m$

- $\cos 30 = \frac{b}{5}$

$b = 4.33m$

Now;

- $EA(A) = 2000 \times 40 = 80,000t$
- $EA(B) = 2000 \times 40 = 80,000t$
- $EA(C) = 2000 \times 50 = 100,000t$
- $EA(D) = 2000 \times 50 = 100,000t$

## Step#01

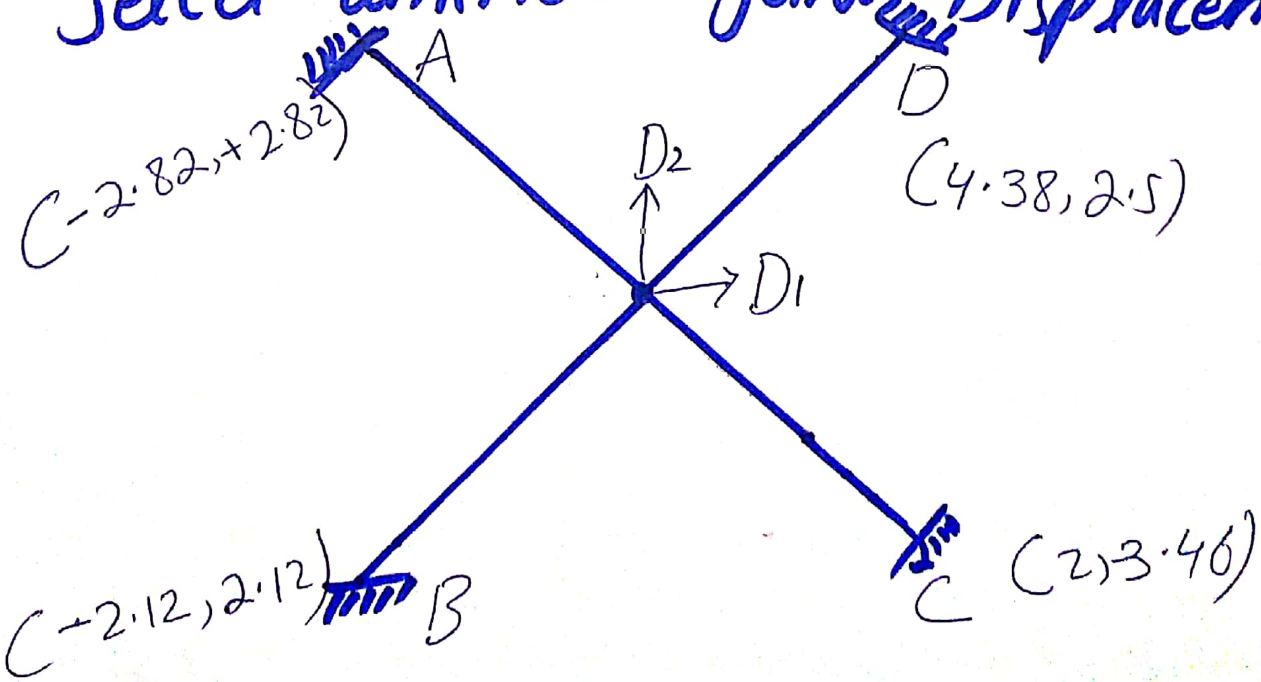
Kinematic indeterminacy

$K.I = 2j - r = 2(5) - 8$

$K.I = 2°$

## Step#02

**Select unknown joint Displacement:**



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$\boxed{\text{Step \# 03}}$

$$[AMD]_{4\times 2} \; [S]_{2\times 2}$$

(a); $D_1 = 1K$ , $D_2 = 0$

$$AMD = \frac{EA}{L^2}(X_K - X_j)$$

$$AMD_{11} = \frac{80000}{(400)^2} \times (0+282) = 141$$

$$AMD_{21} = \frac{80,000}{(300)^2} \times (0+212) = 188.44$$

$$AMD_{31} = \frac{10,0000}{(400)^2} \times (0-433) = -173.2$$

$$AMD_{41} = \frac{100,000}{(400)^2} \times (0-200) = -125$$

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## Now:-

$$S_{11} = \sum_{i=1}^{m} \frac{EA}{L^3}(X_K - X_j)^2$$

$$= \frac{80,000}{(400)^3}(282)^2 + \frac{80,000}{(300)^2}(212)^2$$

$$+ \frac{100,000}{(500)^3}(-433)^2 + \frac{100,000}{(400)^3}(-200)^2$$

$$S_{11} = 99.405 + 133.107 + 149.991 + 62.5$$

$$\boxed{S_{11} = 445.063}$$

$$S_{12} = S_{21} = \sum_{i=1}^{m} \frac{EA}{L^3}(X_K - X_j)(Y_K - Y_j)$$

$$= \frac{80,000}{(400)^3}(282)(-282) + \frac{80,000}{(300)^3}(212 \times 212)$$

$$+ \frac{100,000}{(500)^3}(433)(0-250) + \frac{100,000}{(400)^3}(-200)(0-346)$$

$$\boxed{S_{12} = S_{21} = 12.237}$$

(b);

$$D_1 = 0 \quad , \quad D_2 = 1K$$

$$AMD = \frac{EA}{L^2}(Y_2 - Y_1)$$

$$AMD_{12} = \frac{800,000}{(400)^2}(-282) = -1141$$

$$AMD_{22} = \frac{80,000}{(300)^2}(212) = 188.44$$

$$AMD_{32} = \frac{100,000}{(500)^2}(-250) = -100$$

$$AMD_{42} = \frac{100,000}{(400)^3}(346) = 216.25$$

Now:

$$S_{22} = \sum_{i=1}^{m} \frac{EA}{L^3}(Y_K - Y_j)^2$$

$$= \frac{80,000}{(400)^3}(-282)^2 + \frac{80,000}{(300)^3}(212)^2$$

$$+ \frac{100,000}{(500)^3}(-250)^2 + \frac{100,000}{(400)^3}(346)^2$$

$$\boxed{S_{22} = 469.628}$$

## Step # 04

$$[D] = [S]^{-1} \times [AD]$$

$$\begin{bmatrix} D1 \\ D2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

## Step # 05

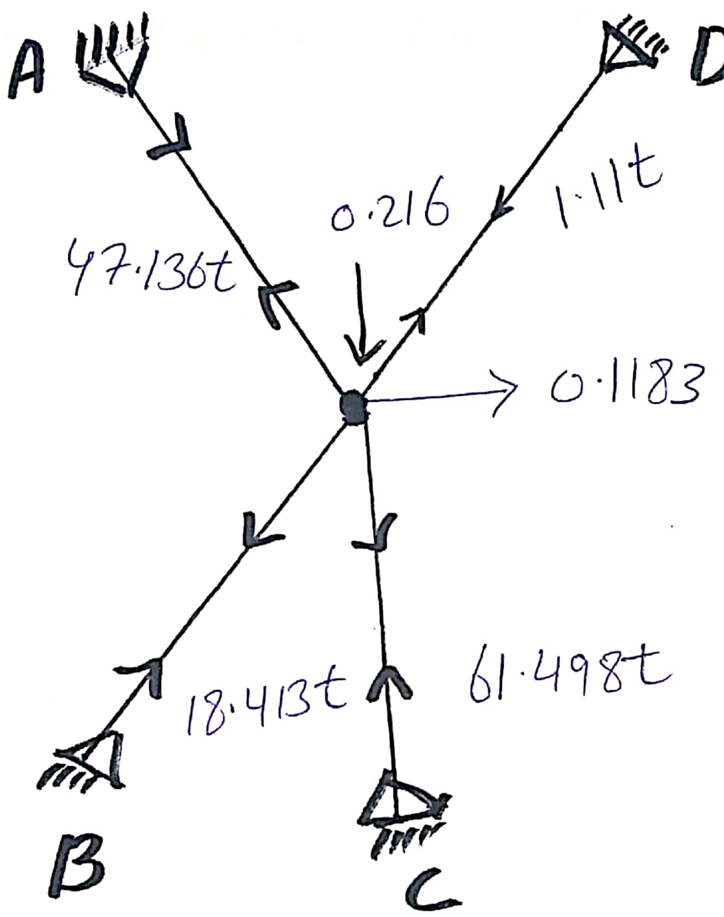
## [AM]

$$\begin{bmatrix} AM1 \\ AM2 \\ AM3 \\ AM4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} X \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

$$\begin{bmatrix} 141 \times 0.1183 + (-141)(-0.216) \\ (188.44)(0.1183) + (188.44)(-0.216) \\ (173.2)\ (0.1183) + (-100)(-0.216) \\ (-125)\ (0.1183) + (216.25)(-0.216 \end{bmatrix}$$

$$\begin{bmatrix} AM1 \\ AM2 \\ AM3 \\ AM4 \end{bmatrix} = \begin{bmatrix} 16.68 + 30.46 \\ 22.29 - 40.70 \\ -20.49 + 21.60 \\ -14.79 + 46.71 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 47.136t \\ -18.413t \\ 1.11t \\ -61.498t \end{bmatrix}$$




X

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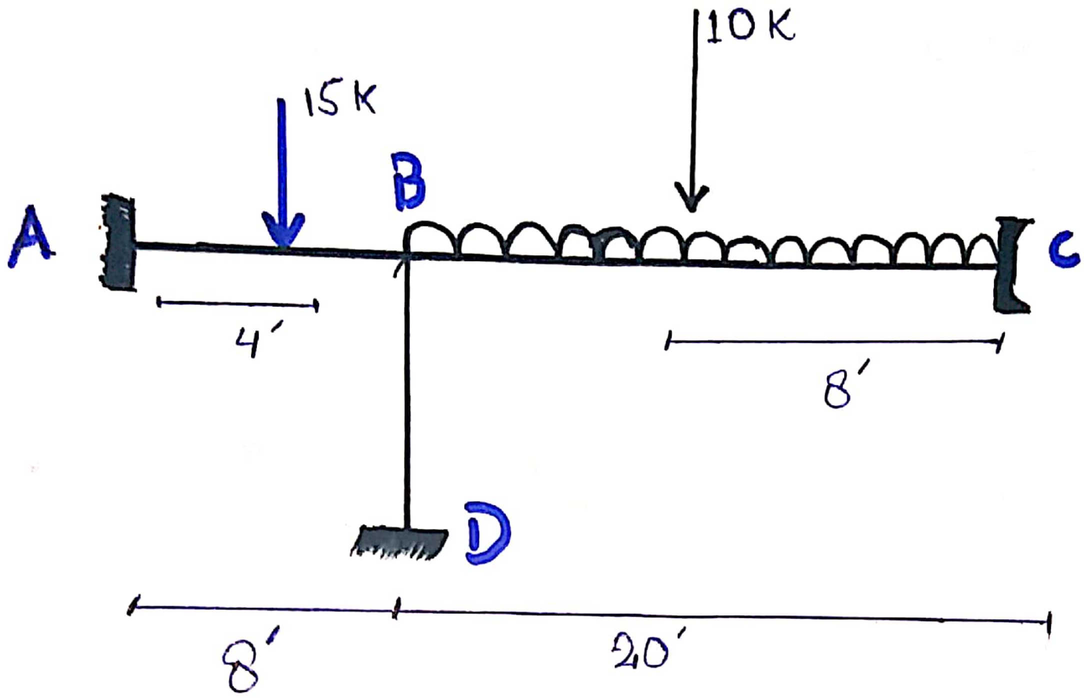
Ans # 03

# Rigid-Jointed Frame




- E = Constant
- Using Stiffiness Method.

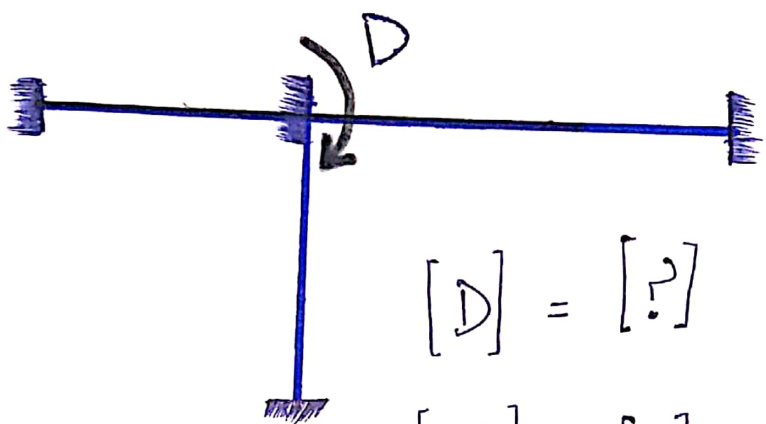
## Step # 01

* Kinetic Indeterminancy

K · I = 1°

Step #(02)

Determination of Unknown Joint Displacment

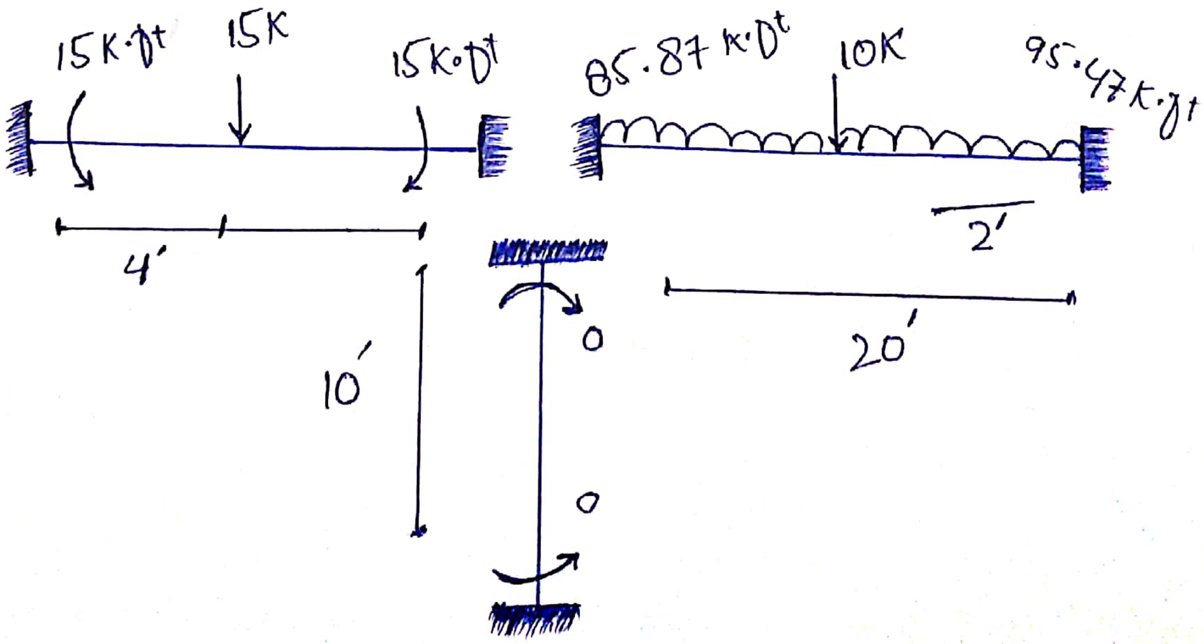



$[D] = [?]$

$[AD] = [0]$

Step #(03)

Compute [ADL] Matrix;



⇒ Point Load at Centre

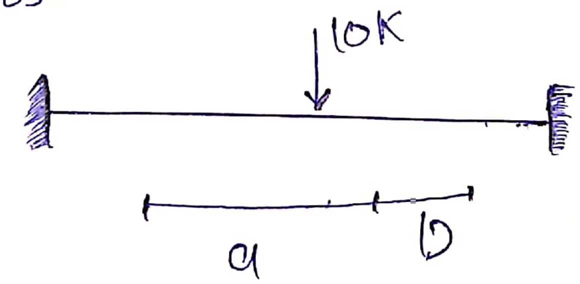
$$PL/8 = 15(8)/8 = 15 K \cdot ft$$

⇒ Uniformly Distributed Load.

$$wL^2/12 = 2(20)^2/12 = 66.67 K \cdot ft$$

⇒ Point load (Not at mid point)

Suppose




⇒ for left end

$$Pab^2/L^2 = 10(12)(8)^2/(20)^2$$

$$= 19.2 K \cdot ft$$

⇒ For Right End

$Pa^2b/L^2 = 10(12)^2(8)/(20)^2$

$= 28.8 \text{ k.ft}$

⇒ total moment at left End;

$19.2 + 66.67 = 85.87 \text{ k.ft}$

⇒ total Moment at right End

$28.8 + 66.67 = 95.47 \text{ k.ft}$

⇒ So $[ADL] = -85.87 + 15$

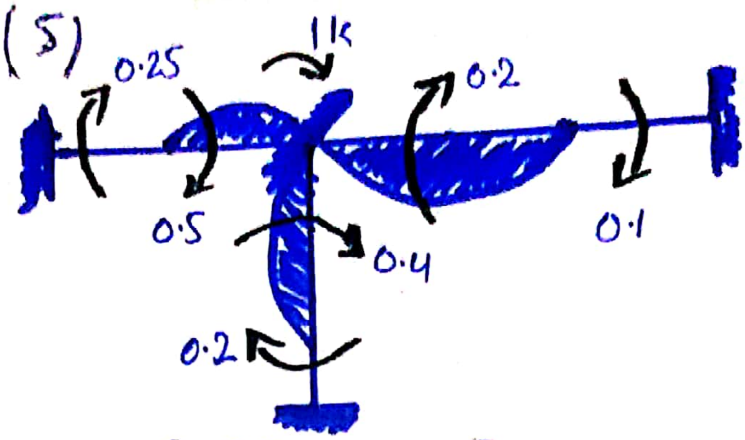
$= -70.87 \text{ k.ft}$

Step; (04)

Determine [S] Matrix:

$[S] = [S_{11}]$

Now:




D = 1K

$\Rightarrow \quad \frac{4EI}{8} = 0.5 \qquad \frac{2EI}{8} = 0.25$

$\Rightarrow \quad \frac{4EI}{20} = 0.2 \qquad \frac{2EI}{20} = 0.1$

$\Rightarrow \quad \frac{4EI}{10} = 0.4 \qquad \frac{2EI}{10} = 0.2$

$[S] = (0.5 + 0.4 + 0.2)\,EI$

$[S] = 1.1\,EI$

Step # (05)

Compute [D] Matrix

$[D] = [S]^{-1} \times [AD] - [ADL]$

$[D] = \frac{1}{1.1} \times [0] - [-70.87]$

$= \frac{70.87}{1.1}$

$\left( [D] = [64.42] / EI \right)$