Name: Sajid Ahmad Paragrame: BE(E)
ID: 12671 ① Subject: Differential Equat
Module: 11th Semester

Q1 (A) Estimate the general solution of

$$y' = (x+2)y^2$$

Sol:→

$$y' = (x+2)y^2$$

$$\frac{dy}{dx} = (x+2)y^2$$

$$\int \frac{1}{y^2}\,dy = \int (x+2)\,dx$$

$$\int y^{-2}\,dy = \int (x+2)\,dx$$

$$\frac{y^{-2+1}}{-2+1} = \frac{a^2}{2} + 2x + c$$

$$\frac{y^{-1}}{-1} = \frac{x^2}{2} + 2x + c$$

Xing both side by -1

$$y^{-1} = -\left(\frac{x^2}{2} + 2x + c\right)$$

$$\boxed{y = -\left(\frac{1}{\frac{x^2}{2} + 2x + c}\right)}$$ Ans.

P.T.O

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ID : 12671 (2) Subject: Differential

Q#2 (B) $y' = (y+9x)^2$

Sol:→

$$y' = (y+9x)^2$$

let suppose

$$y + 9x = u \quad \text{—— (1)}$$

$$\frac{dy}{dx} + 9 = \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{du}{dx} - 9$$

equation (1) become

$$\frac{du}{dx} - 9 = u^2$$

$$\frac{du}{dx} = u^2 + 9$$

$$\int dx = \int \frac{1}{u^2+9}\, du$$

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$$\int dx = \int \frac{1}{u^2+3^2} du$$

$$x + c_1 = \frac{1}{3} \tan^{-1}\left(\frac{u}{3}\right)$$

$$3x + 3c = \tan^{-1}\left(\frac{u}{3}\right)$$

$$\frac{u}{3} = \tan(3x + c)$$

$$u = 3\tan(3x + c)$$

$$y + 9x = 3\tan(3x + c)$$

$$\boxed{y = -9x + 3\tan(x + c)}$$

Ans.

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Q# 2 (A) Estimate the general solution of

$x^3 dx + y^3 dy = 0$

Sol:→

$$x^3 dx + y^3 dy = 0$$

$$\uparrow \qquad \uparrow$$

$$M dx + N dy = 0$$

$$M = x^3 , \quad N = y^3$$

$$\frac{\partial M}{\partial y} = \frac{\partial (x)^3}{\partial y} , \quad \frac{\partial N}{\partial x} = \frac{\partial (y^3)}{\partial x}$$

$$\frac{\partial M}{\partial y} = 0 \qquad \frac{\partial N}{\partial x} = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \longrightarrow \text{So exact}$$

$$u = \int M dx + k(y) \longrightarrow \text{Formula}$$

$$u = \int x^3 dx + k(y)$$

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$$u = \frac{x^4}{4} + k(y) \longrightarrow eq\ (i)$$

$$\frac{\partial u}{\partial y} = 0 + \frac{d}{dy} k(y)$$

Note:

$$\frac{\partial u}{\partial y} = \frac{d}{dy} k(y)$$

Since :-

$$\frac{\partial u}{\partial y} = N \Rightarrow y^3$$

$$y^3 = \frac{d}{dy} k(y)$$

$$\int d\, k(y) = \int y^2 dy$$

$$k(y) = \frac{y}{4} + C_1 \longrightarrow \cancel{put}$$

put in eq (i)

$$u = \frac{x^4}{4} + \frac{y^4}{4} + C_1$$

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$$C_2 = \frac{x^4}{4} + \frac{y^4}{4} + C_1$$

$$\frac{x^4}{4} + \frac{y^4}{4} = C_2 - C_1$$

$$\boxed{\frac{x^4}{4} + \frac{y^4}{4} = C}$$ Ans.

P.T.O

Name :

Parogram :

iD :

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Subject :

Q#3 (A) Find the general solution $4y'' - 20y + 25y = 0$

Sol:→

$$4y'' - 20y + 25y = 0$$

This is 2nd order homogenous differential equation with Constant coefficient.

$$ay'' + by' + cy = 0$$

and the solution for this is $y = e^{\lambda}$ —— (i)

General Solution.

$$y = C_1 e^{\lambda} + C_2 + e^{\lambda}$$

Now

$$4 \frac{d^2}{dx^2}(y) - 20 \frac{d}{dx}(y) + 25(-y) = 0$$

equation (A)

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put eq (i) in eq (A)

$$\Rightarrow 4\frac{d^2}{dx^2}(e^{\lambda x}) - 20\frac{d}{dx}(e^{\lambda x}) + 25e^{\lambda x} = 0$$

$$\Rightarrow \frac{d^2}{dx^2}e^{\lambda x} = \lambda^2 e^{\lambda x} \quad \text{——— (B)}$$

put eq (B) and eq (i) in eq ~~(B)~~ (A)

$$\Rightarrow 4\lambda^2 e^{\lambda x} - 20 e^{\lambda x} + 25 e^{\lambda x} = 0$$

$$\Rightarrow e^{\lambda x}(4\lambda^2 - 20\lambda + 25) = 0$$

$$\Rightarrow e^{\lambda x} \neq 0$$

$$\Rightarrow 4x^2 - 20\lambda + 25 = 0$$

$$\Rightarrow (2\lambda - 5) = 0$$

$$\lambda = \frac{5}{2} \quad \text{or} \quad \lambda = \frac{5}{2}$$

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$\Rightarrow y(x) = y_1(x) + y_2(x)$

$\Rightarrow \boxed{y(x) = C_1 e^{\frac{5}{2}x} + C_2 x e^{\frac{5}{2}x}}$

Ans.

Q#3 (B) Estimate general solution of

$4y - 6y - 7y = 0$

Sol:→

Assume $y(x) = e^{\lambda x}$

put in equation

$\Rightarrow 4 \cdot \frac{d^2 \cdot y(x)}{dx^2} - 6 \frac{d}{dx} y(x) - 7y(x) = 0$

$\Rightarrow 4 \cdot \frac{d^2}{dx^2}(e^{\lambda x}) - 6\frac{d}{dx}(e^{\lambda x}) - 7e^{\lambda x} = 0$ → eq (i)

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$$\Rightarrow \frac{d^2}{dx^2}(e^{\lambda x}) = \lambda^2 e^{\lambda x} \quad \text{— (A)}$$

$$\Rightarrow \frac{d}{dx}(e^{\lambda x}) = \lambda e^{\lambda x} \quad \text{— (B)}$$

put (A) and (B) in eq (i)

$$\Rightarrow 4\lambda^2 e^{\lambda x} - 6\lambda e^{\lambda x} - 7e^{\lambda x} = 0$$

$$\Rightarrow (4\lambda^2 - 6\lambda - 7)e^{\lambda x} = 0$$

$$\Rightarrow \lambda = \frac{3}{4} - \frac{\sqrt{37}}{4}$$

$$\Rightarrow \lambda = \frac{3}{4} + \frac{\sqrt{37}}{4}$$

$$\Rightarrow y(x) = y_1(x) + y_2(x)$$

$$\boxed{\Rightarrow y(x) = c_1 e^{\left(\frac{3}{4} - \sqrt{\frac{37}{4}}\right)x} + C_2 e^{\left(\frac{3}{4} + \sqrt{\frac{37}{4}}\right)x}}$$

Ans.

The end.