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Section "B"

Fourth Semester.

Sub: MOS II

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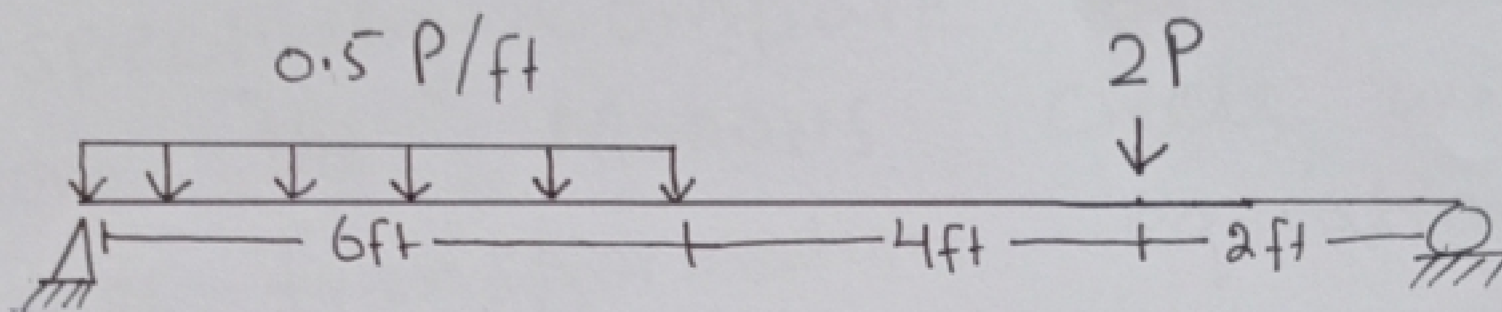
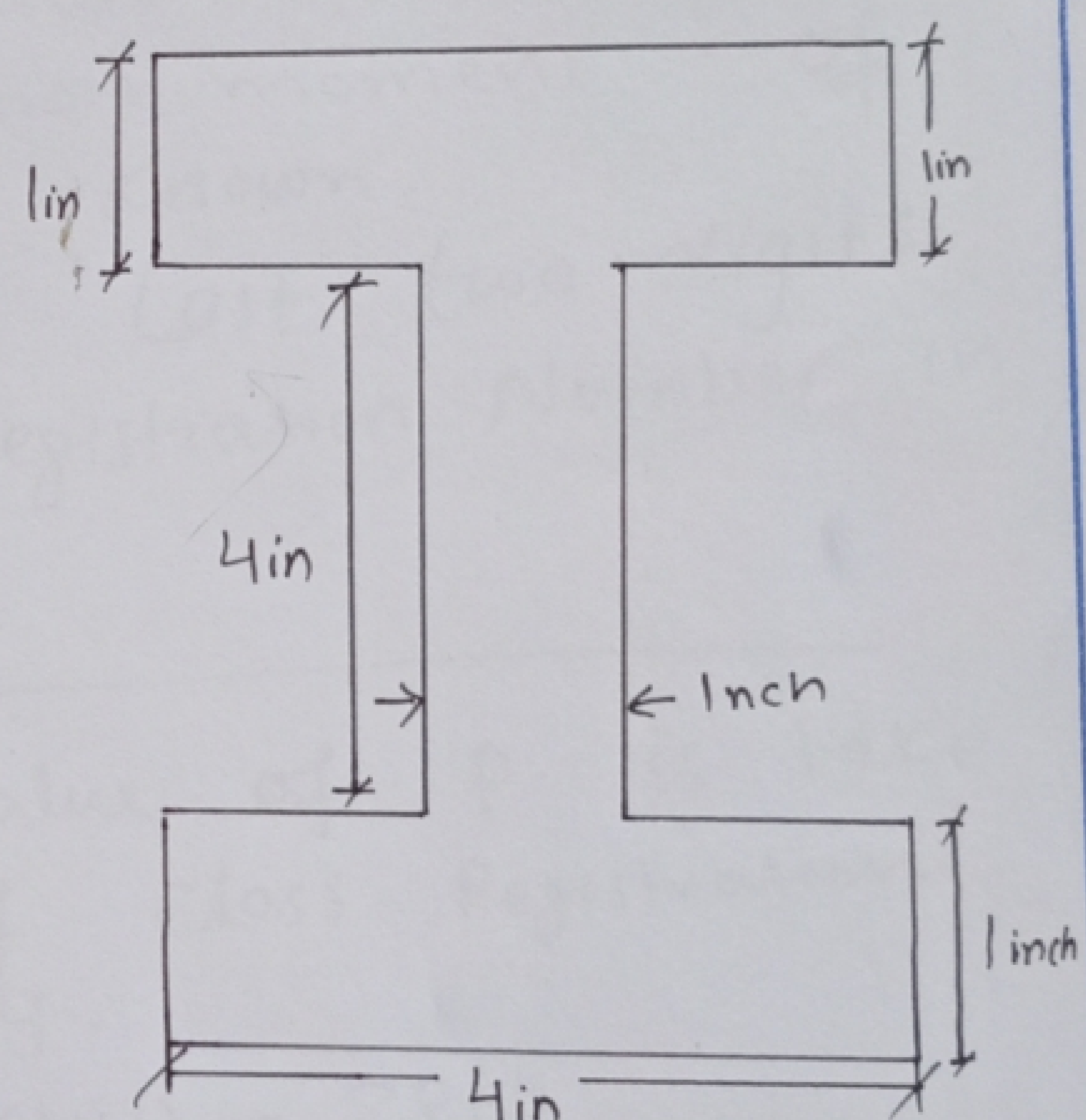
Sir.

Department of Civil Engineering

Question No#01





Construct the Mohar's Circle Diagrams and find the principle Stresses and and Maximum in plane shear stress for the Stress of a point "C" Located at the center of the uniformly distributed Load. And 1 inch below from the Top Fibre of the Beam. The cross-section of the Beam shown. However To Construct the Mohar's Circle. it is Necessary to draw the shear stress and flexure stress veriation diagram. for the Maximum shear force and Bending moment Respectively. Compare the Results obtain from the Mohar's circle with the stress transformation equation.

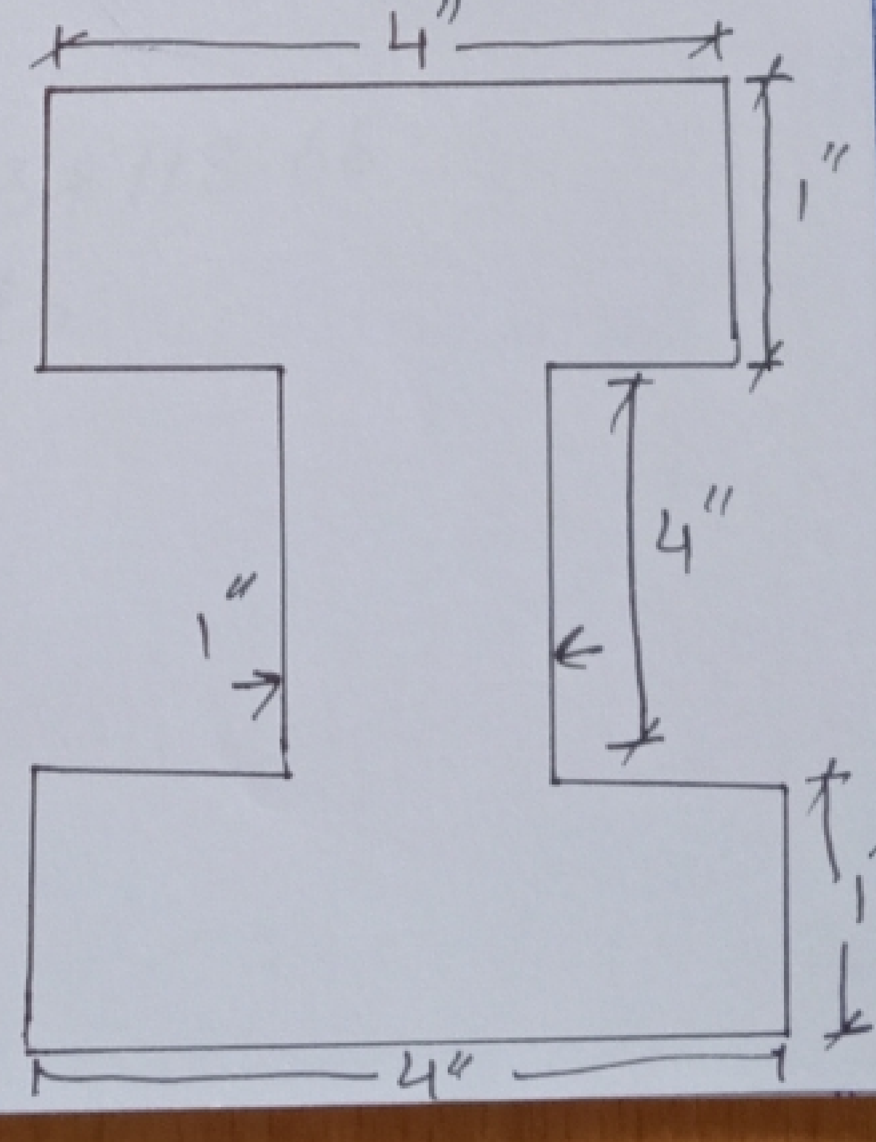
Hints :→ To calculate the stress in the Beam crossection moment of Inertia Must Be Known.

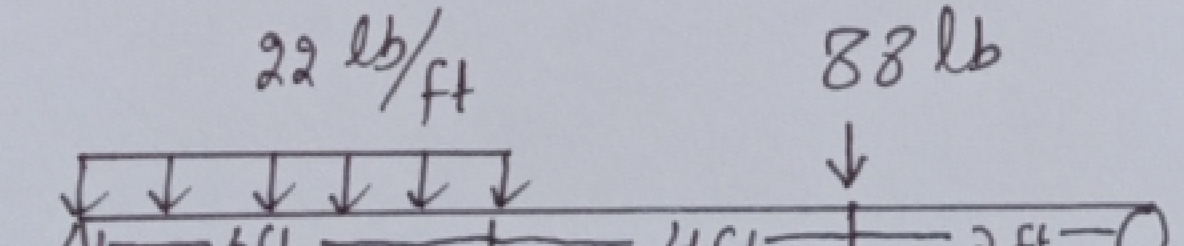
where P is the Last two digits of your class Registration Number in Pounds.

Solution :→ The value of P is take Last two digits of Class Registration Number. So 7944.

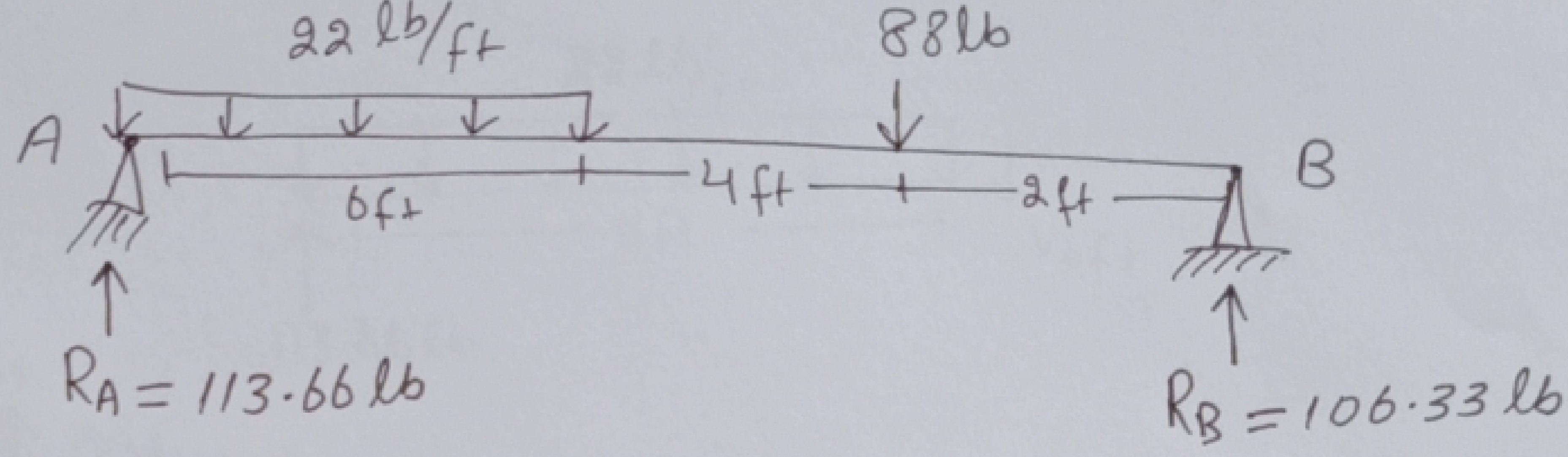
$$2P = 2(44) = 88\,lb$$

$$0.5P = 0.5(44) = 22\,lb.$$





First of all we find Reaction of Beam.



(+) $\Sigma M_B = 0$

$$R_A * 12 - (22*6)(6+3) - 88*2 = 0$$

$$R_A = 113.66 \text{ lb}$$

(+) $\Sigma M_A = 0$

$$-R_B * 12 + (88*10) + (22*6)*3 = 0$$

$$R_B = 106.33 \text{ lb}$$

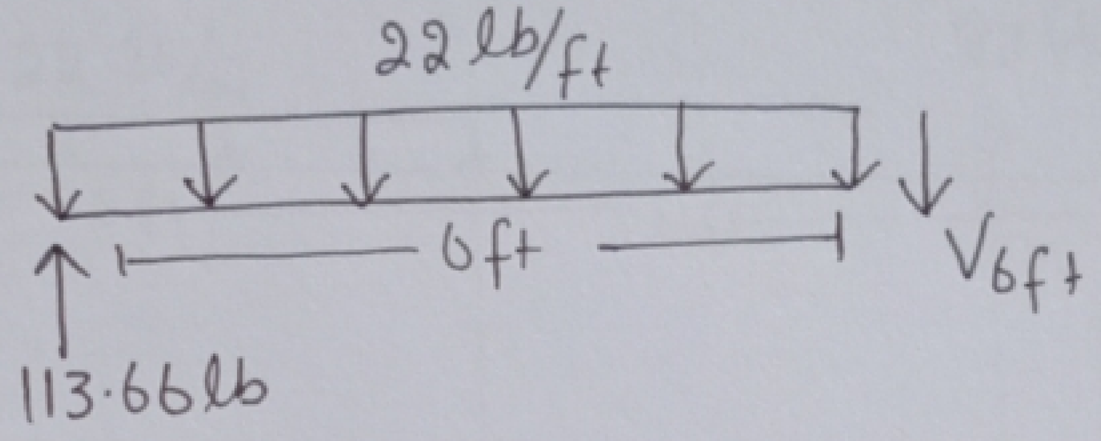
Now we check sum of upward and downward forces. Thier Reactions.

$$\Sigma F_y = 0$$

$$22*6 + 88 = 106.33 + 113.66$$

$$220 \text{ lb} = 220 \text{ lb}$$

Now we find shear force at change point.



$\overset{+}{\uparrow} \sum F_y = 0$

$$113.66 - 22 * 6 - V_{6ft} = 0$$

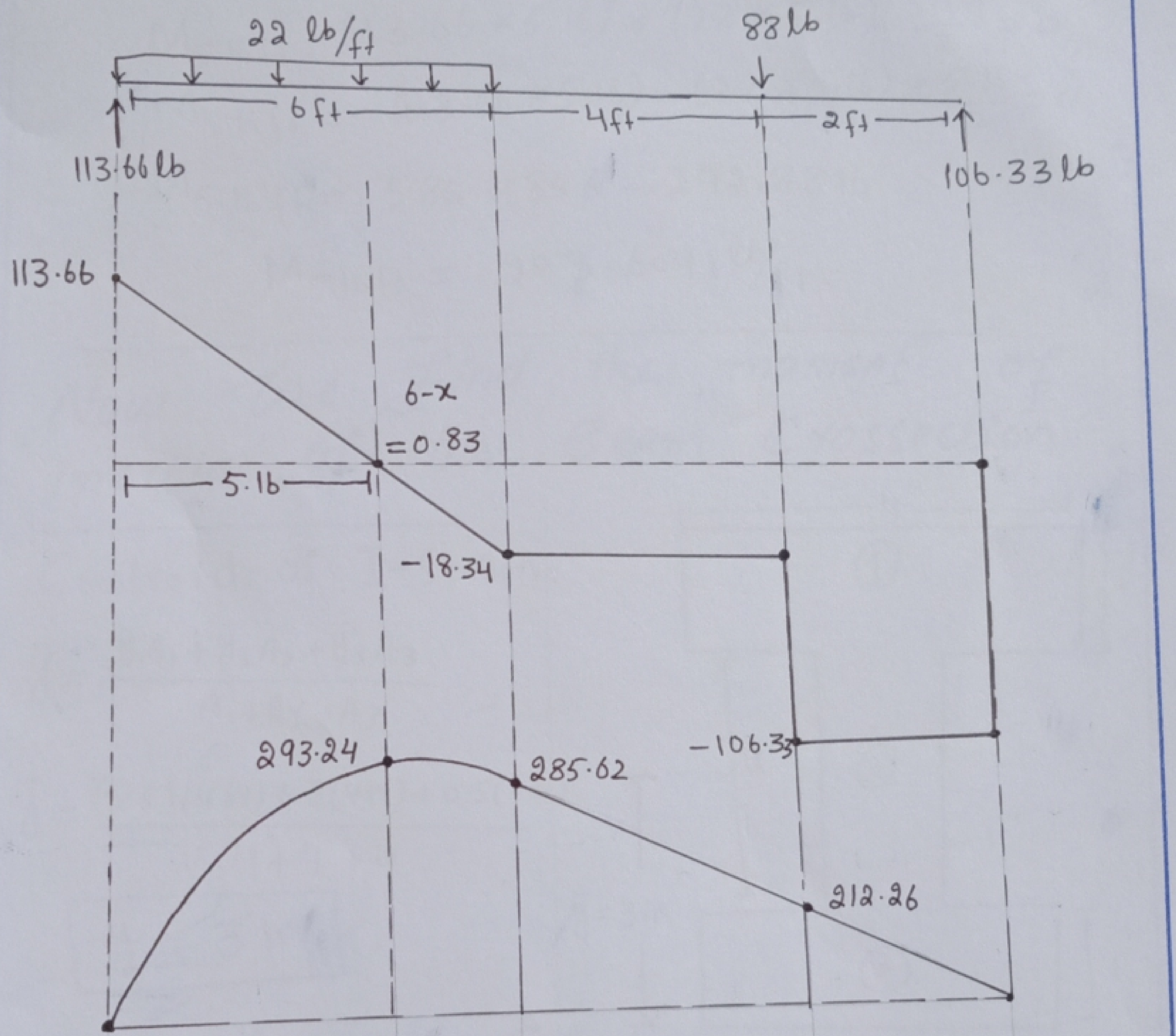
$$V_{6ft} = 113.66 - (22 * 6)$$

$$\boxed{V_{6ft} = -18.34 \text{ lb}}$$

$\overset{+}{\uparrow} \sum F_y = 0$

$$-V_{10ft} - 88 - (22 * 6) + 113.66 \text{ lb} = 0$$

$$V_{10ft} = -106.33 \text{ lb.}$$



x = distance = ?

$$\frac{113.66}{x} = \frac{18.34}{6-x}$$

$$x\,18.34 = 113.66(6-x)$$

$$x(18.34 + 113.66) = 681.6$$

$$x = \frac{681.6}{131.94}$$

$$\boxed{x = 5.16}$$

$$\Rightarrow 6 - 5.16 = \boxed{0.83}$$

BMD Detail: →

$$\Rightarrow \frac{5.16 * 113.66}{2} = \boxed{293.24}$$

$$\Rightarrow 293.24 - \frac{18.34 * 0.83}{2} = \boxed{285.62}$$

$$\Rightarrow 285.62 - (18.34 * 4) = \cancel{212.02}\ \boxed{212.25}$$

$$\Rightarrow 212.25 - (106.33 * 2) = 0$$

$$\overset{\curvearrowleft}{+} \sum M_{5.16ft} = 0$$

$$M_{5.16ft} - (113.66 * 5.16) + (22 * 5.16)\frac{5.16}{2} = 0$$

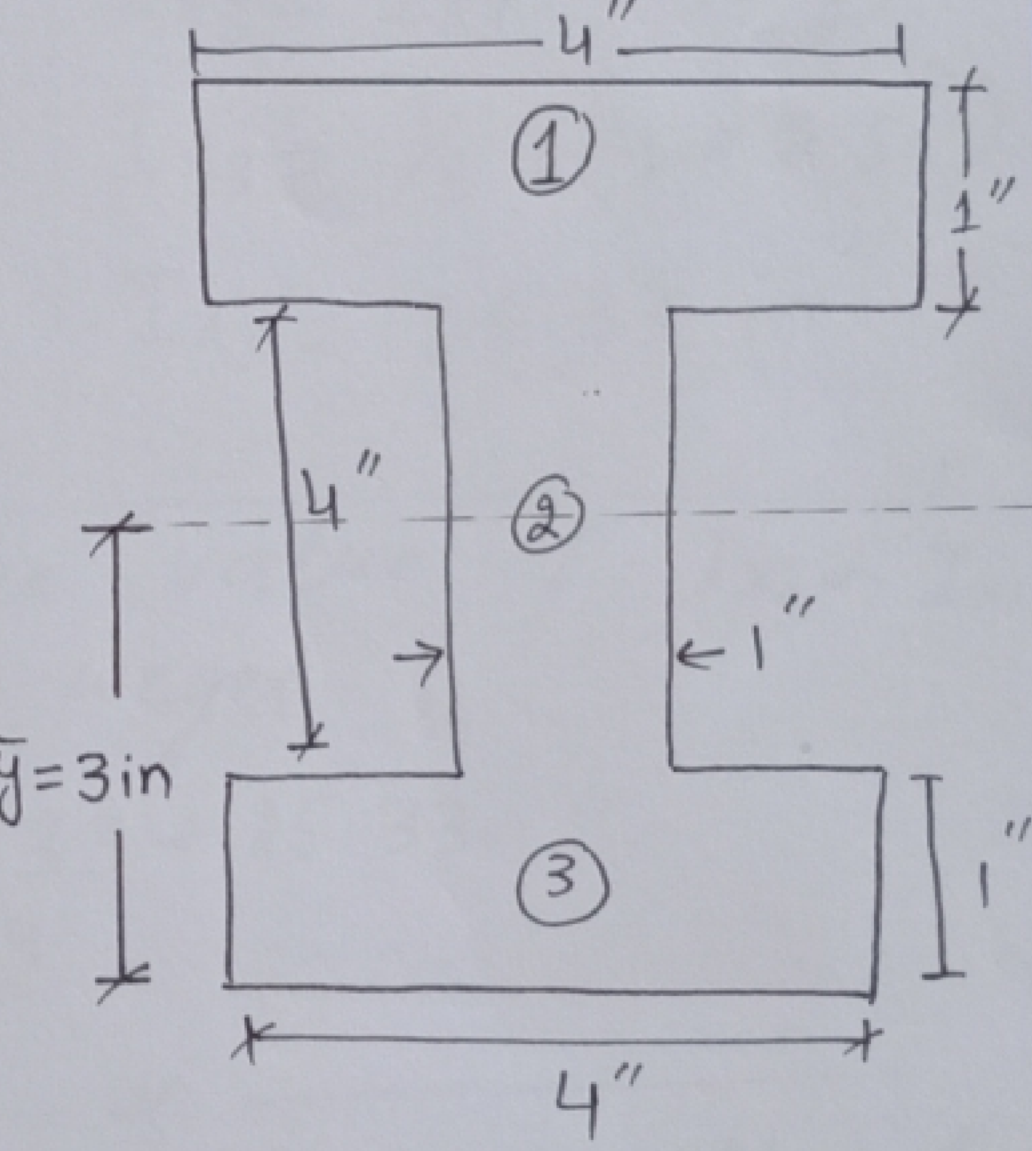
$$M_{5.16ft} = (113.66 * 5.16) - (22 * 5.16) * \frac{5.16}{2} = 0$$

$$M_{5.16ft} = 586.4856 - 292.8816$$

$$M_{5.16ft} = 293.604 \; lb/ft$$

Now we find the moment of Inertia of the Beam Crossection.

Centroid of I-Section:



$$\bar{y} = \frac{y_1A_1 + y_2A_2 + y_3A_3}{A_1 + A_2 + A_3}$$

$$\bar{y} = \frac{(5.5)(4*1) + 3(4*1) + 0.5(4*1)}{4+4+4}$$

$$\boxed{\bar{y} = 3 \text{ in}}$$

$$I_x = I_{x_1} + I_{x_2} + I_{x_3} \quad \text{— (A)}$$

For $I_{x_1} = I_1 + A_1 d_{y_1}^2$

$$A_1 = 4*1 = 4 \text{ in}^2$$

$$I_1 = \frac{bh^3}{12} = \frac{4(1)^3}{12}$$

$$I_1 = \frac{1}{3} \text{ in}^4$$

$$d_y = y - \bar{y} = 5.5 - 3$$

$$d_{y_1} = 2.5 \text{ in}$$

$$I_{x_1} = I_1 + A_1 d_{y_1}^2$$

$$I_{x_1} = \frac{1}{3} + 4 * (2.5)^2$$

$$I_{x_1} = 25.33$$

For I_{x2}

$A_2 = 4 * 1$

$A_2 = 4 \text{ in}^2$

$I_2 = \frac{bh^3}{12}$

$I_2 = \frac{1 * 4^3}{12}$

$I_2 = 16/3 \text{ in}^4$

$dy_2 = y - \bar{y}$

$= 3 - 0$

$dy_2 = 0$

$I_{x2} = I_2 + A_2 dy_2^2$

$I_{x2} = \frac{16}{3} + 0$

$I_{x2} = 5.333 \text{ in}^4$

For I_{x3}

$A_3 = 4 * 1 \quad A_3 = 4 \text{ in}^2$

$I_3 = \frac{bh^3}{12} = \frac{4 * 1^3}{12}$

$I_3 = \frac{1}{3}$

$dy_3 = \bar{y} - y$

$= 3 - 0.5$

$dy_3 = 2.5 \text{ in}$

$I_{x3} = I_{\cancel{x}3} - A_3 dy_3^2$

$I_{x3} = 1/3 + (4 * 2.5^2)$

$I_{x3} = 25.33 \text{ in}^4$

Now put the value of I_{x_1}, I_{x_2}, I_{x_3} in equation (A) weget

$$I_x = 25.33 + 5.33 + 25.33$$

$$I_x = 56 \text{ in}^4$$

*

Shear stress : As per the Question The maximum shear stress $\tau = \frac{VQ}{Ib}$ occurs where the maximum shear force lies. In this diagram the max shear force value is 113.66 lb.

Now to find the shear stress $\tau = \frac{VQ}{Ib}$ it is necessary to find "I" moment of inertia of the Given crossection which we already find in previous page.
As we evaluate the shear force and Bending moment degram and Moment of inertia of the section it is possible to calculate the shear stress and flexural stress at any point.

For shear stress.
Shear stress along the depth of the section. $\tau = \frac{VQ}{Ib}$

where $V_{max} = 113.66 \text{ lb}$

$I_x = 56 \text{ in}^4$

$b =$ Thickness / breath

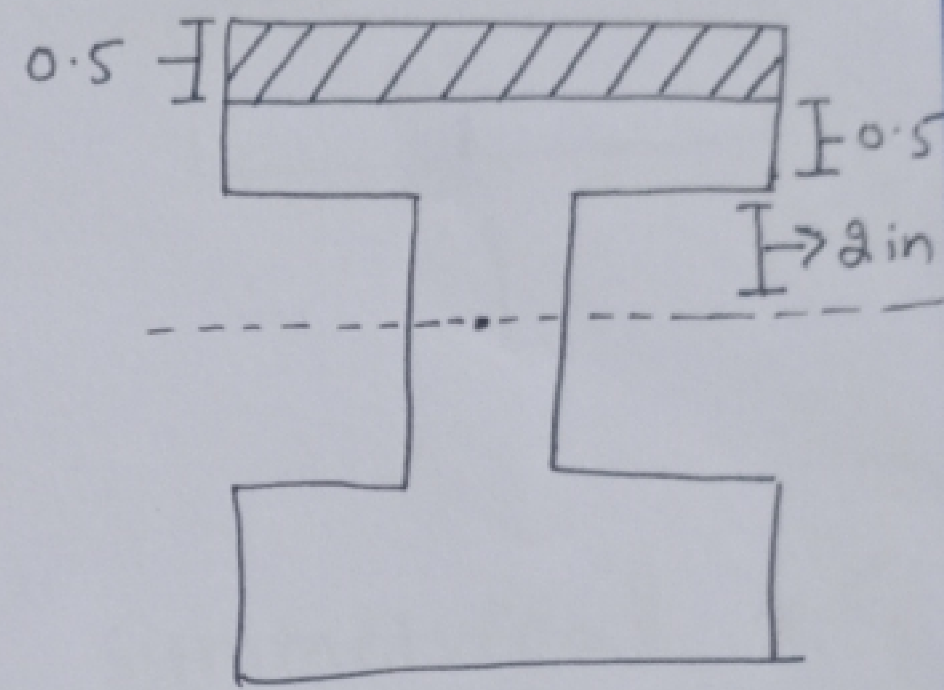
$Q = \bar{Y}A$

Case (1) τ at the top fibre

$\tau = Q = \bar{y} * 0 = 0$

$$\tau = \frac{113.66(0)}{56*1} = 0$$

Case (2) $\tau =$ in 0.5 in below top fibre.



$Q = \bar{y} A$

$Q = \left(2 + 0.5 + \frac{0.5}{2}\right)(0.5 * 4)$

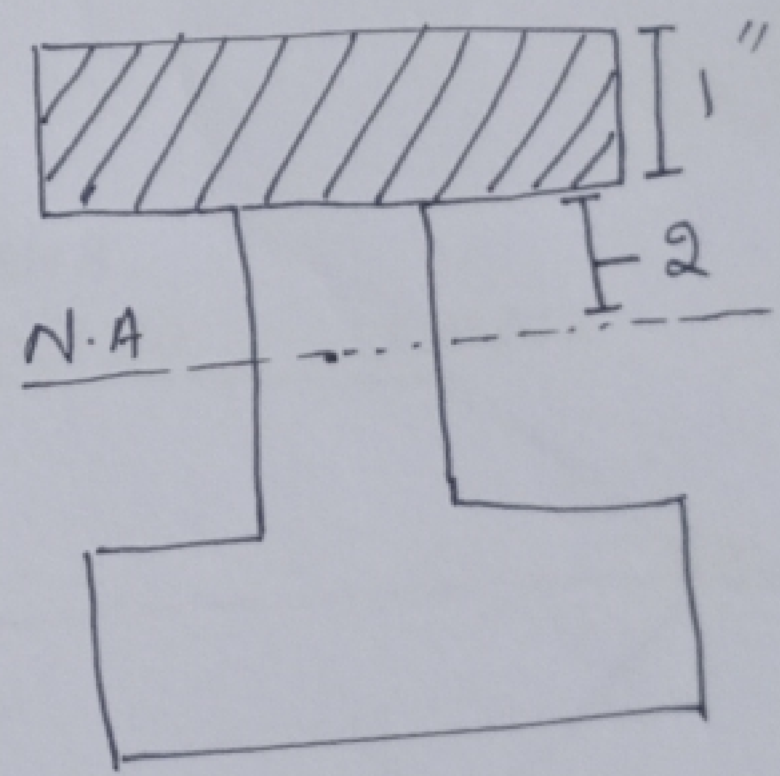
$Q = 5.5$

$$\tau = \frac{VQ}{Ib} = \frac{113.66(5.5)}{56*4}$$

$\boxed{\tau = 2.79 \text{ psi}}$

Case (3) τ 1 in below top fibre.

(A) $Q = \bar{y} A$, $b = 4''$



$Q = \left(2 + \frac{1}{2}\right)(1 * 4)$

$Q = 10$ in

$$\tau = \frac{VQ}{Ib} = \frac{113.66(10)}{56*4}$$

$\boxed{\tau = 5.07 \text{ psi}}$

(B) $Q = \bar{y} A$, $b = 1''$

$Q = 10$ in

$$\tau = \frac{113.66(10)}{56*1} = 20.2964 \quad \boxed{\tau = 20.2964 \text{ psi}}$$

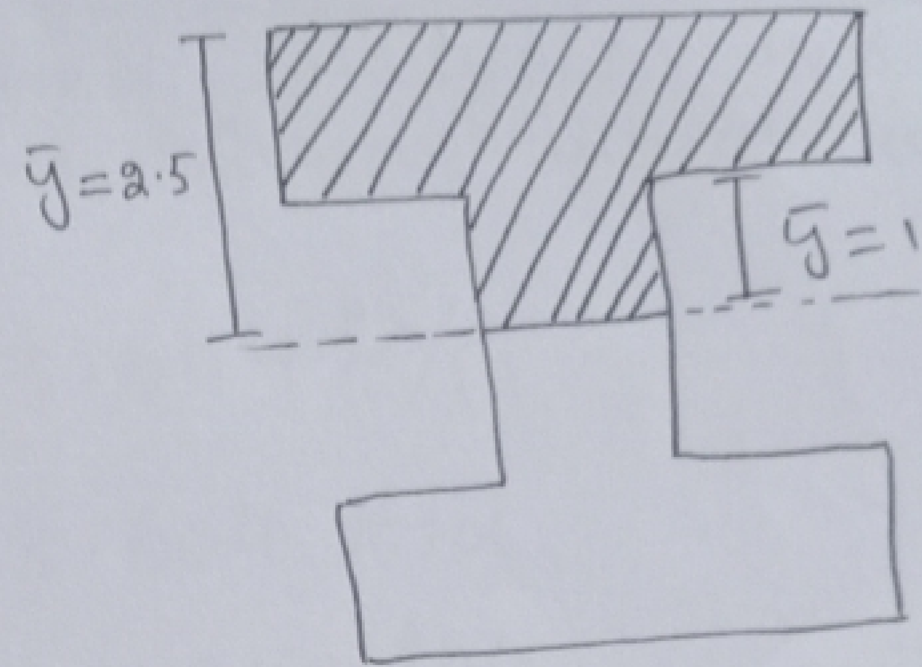
Case (4) τ at Centeroid.

$Q = Q_1 + Q_2$

$Q = \bar{y}_1 A_1 + \bar{y}_2 A_2$

$Q = 2.5 * (4*1) + 1(2*1)$

$Q = 12\ in^3$

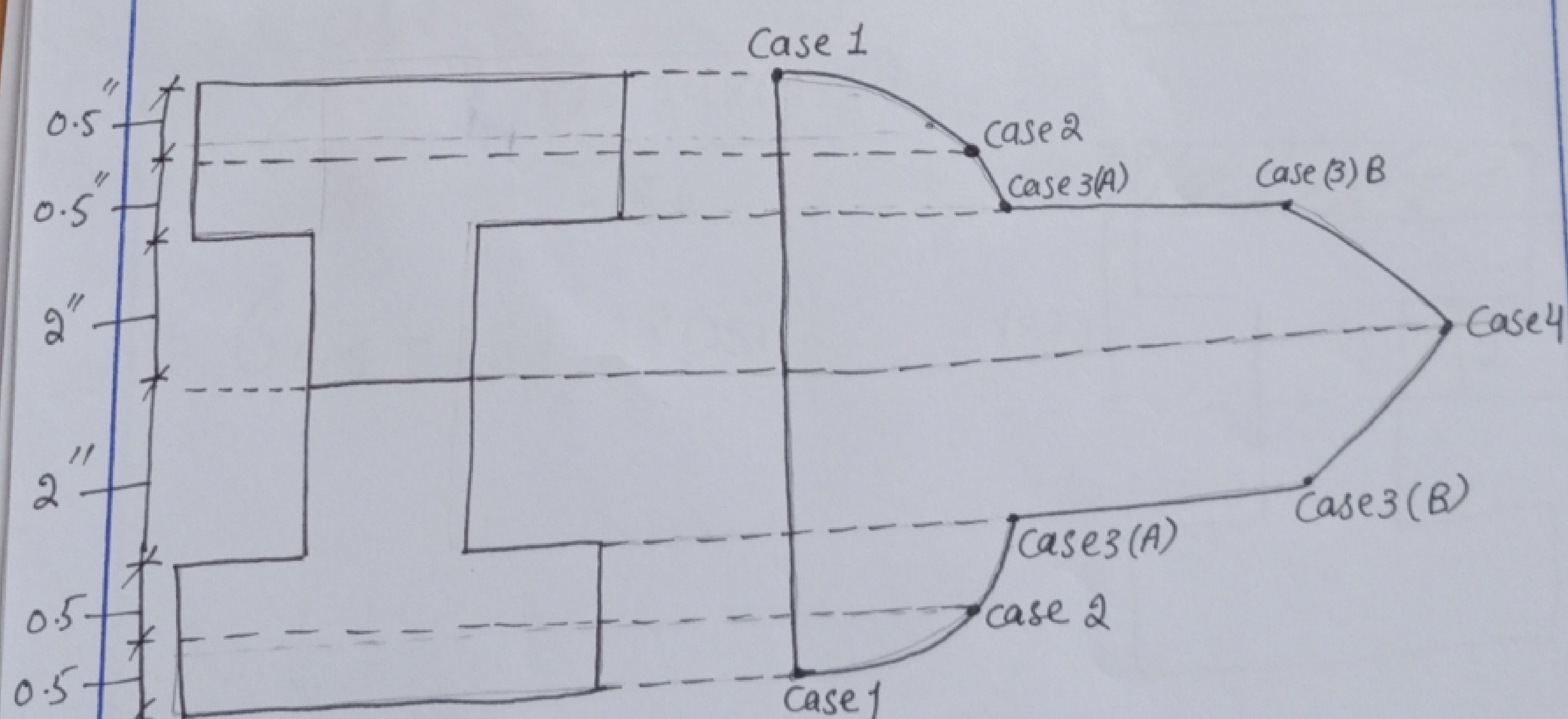


$\tau = \frac{VQ}{Ib} = \frac{113.66 * 12}{56 * 1}$

$\boxed{\tau = 24.355\ psi}$

As the I-section is symmetrical x & y axis will occur below of the centroidal axis as occured above.

Shear Stress veriation Diagram.



Flexural Stresses Analysis:

For The flexural stress analysis we Consider Maximum Moment which as given below

$$M_{max} = 293.604 \ lb/ft$$

$$M_{max} = 293.604 * 12$$

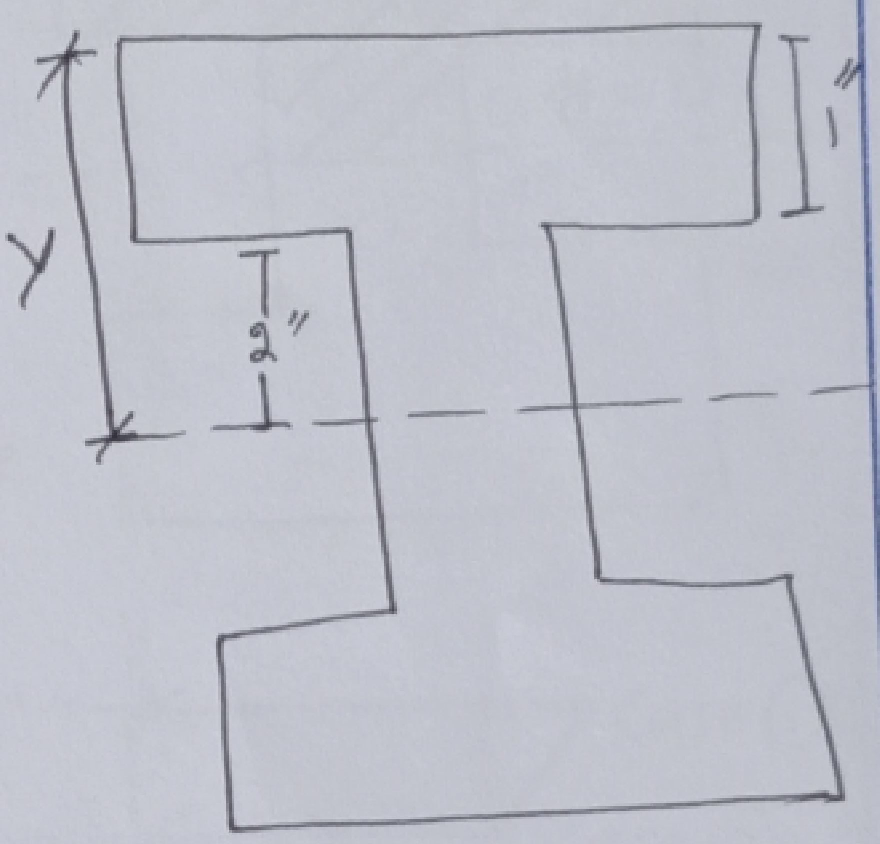
$$Max\,M = 3523.248 \ lb/in$$

& $\sigma = \frac{MY}{I}$

Case 1

$$\sigma_{Top} = \frac{3523.248(3)}{56}$$

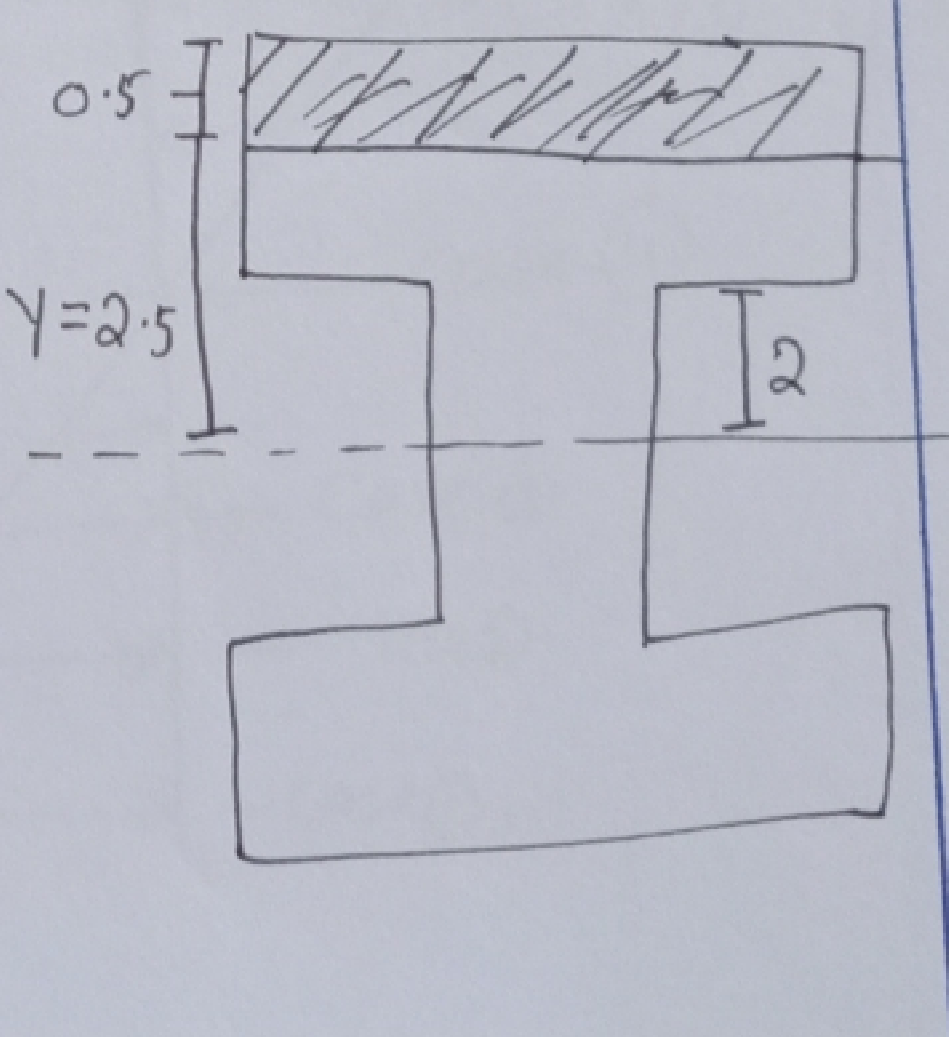
$$\sigma_{Top} = 188.94 \ lb/in^2$$



Case 2

$$\sigma_{0.5} = \frac{3523.248(2.5)}{56}$$

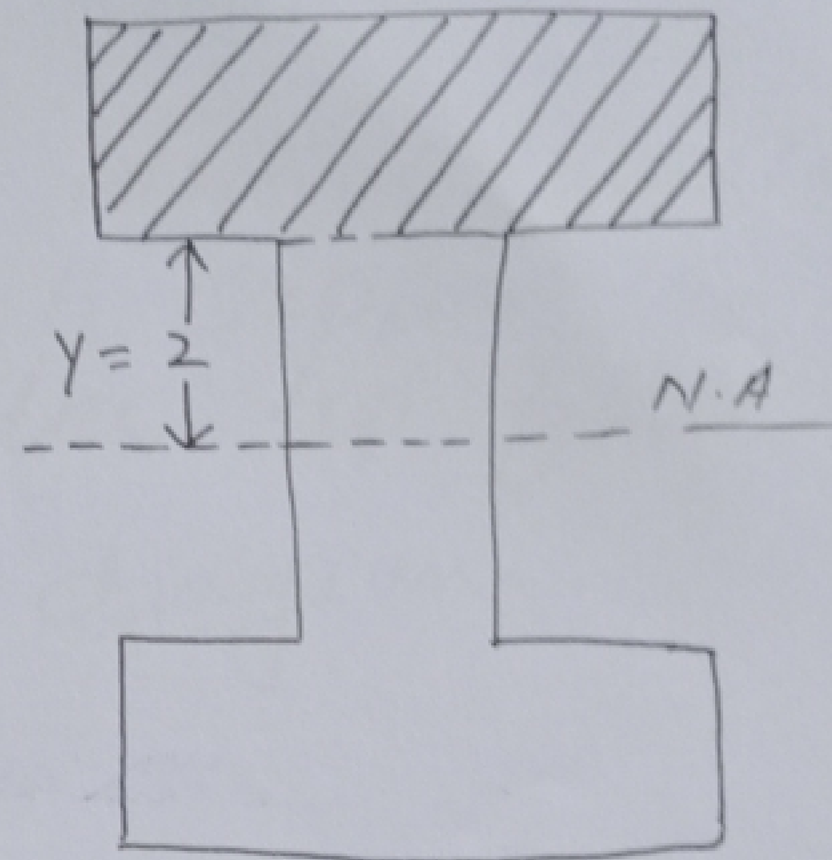
$$\sigma_{0.5} = 157.28 \, psi$$



Case $\underline{\underline{3}}$

$$\sigma_1'' = \frac{3523.24(2)}{56}$$

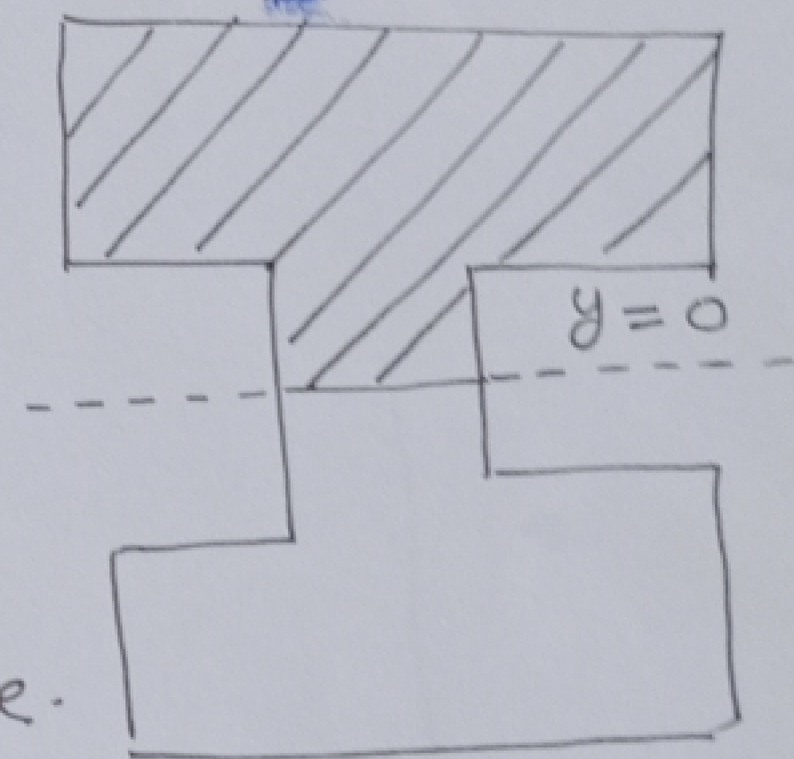
$$\sigma_1'' = 125.83 \text{ psi}$$



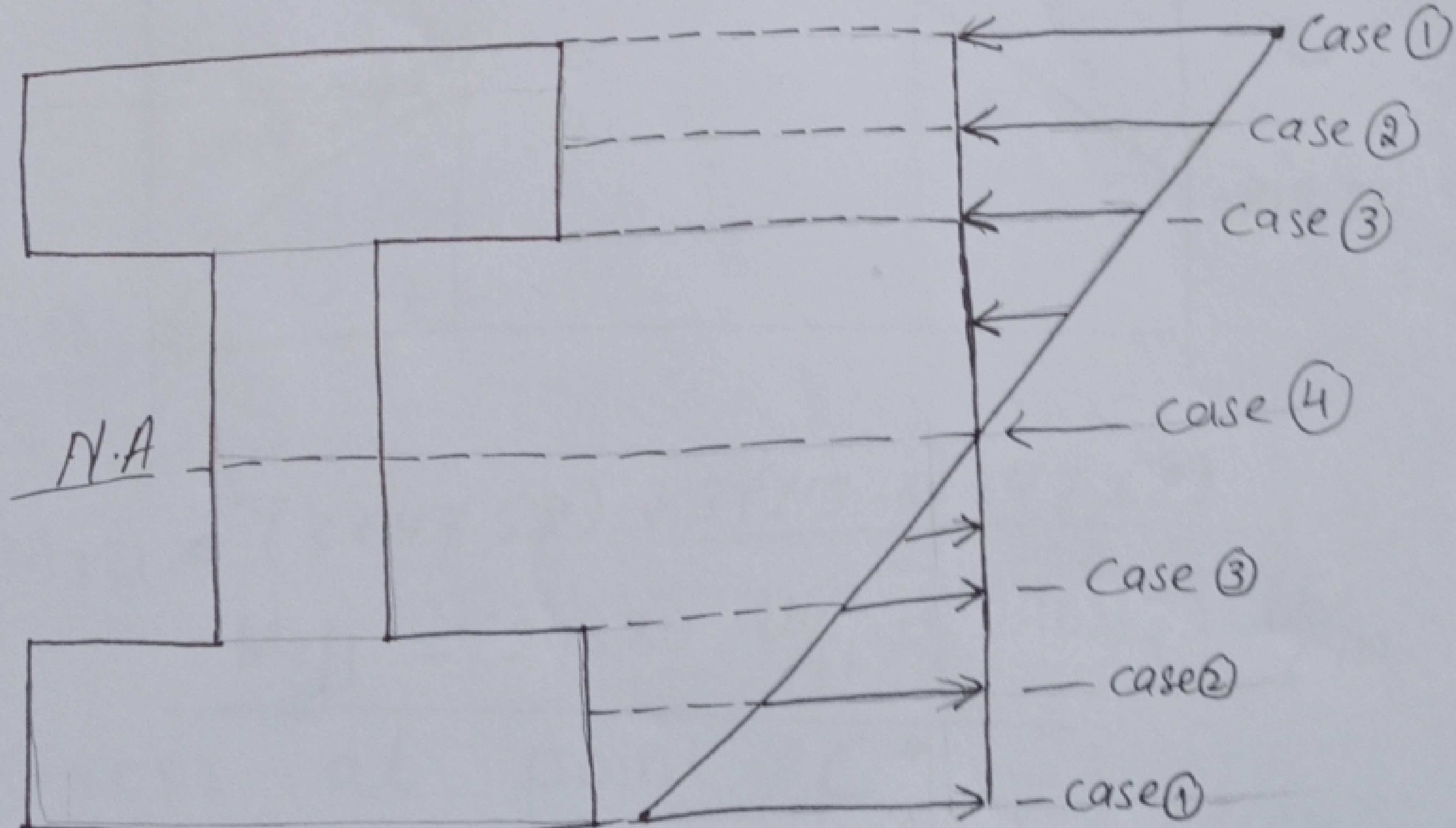
Case $\underline{\underline{4}}$

$$\sigma_{N.A} = \frac{3523.24(0)}{56}$$

$$\sigma_{N.A} = 0$$



Note :→ Because of the symmetrical shape of I-section Flexural Stresses below The N.A will be Same as above.



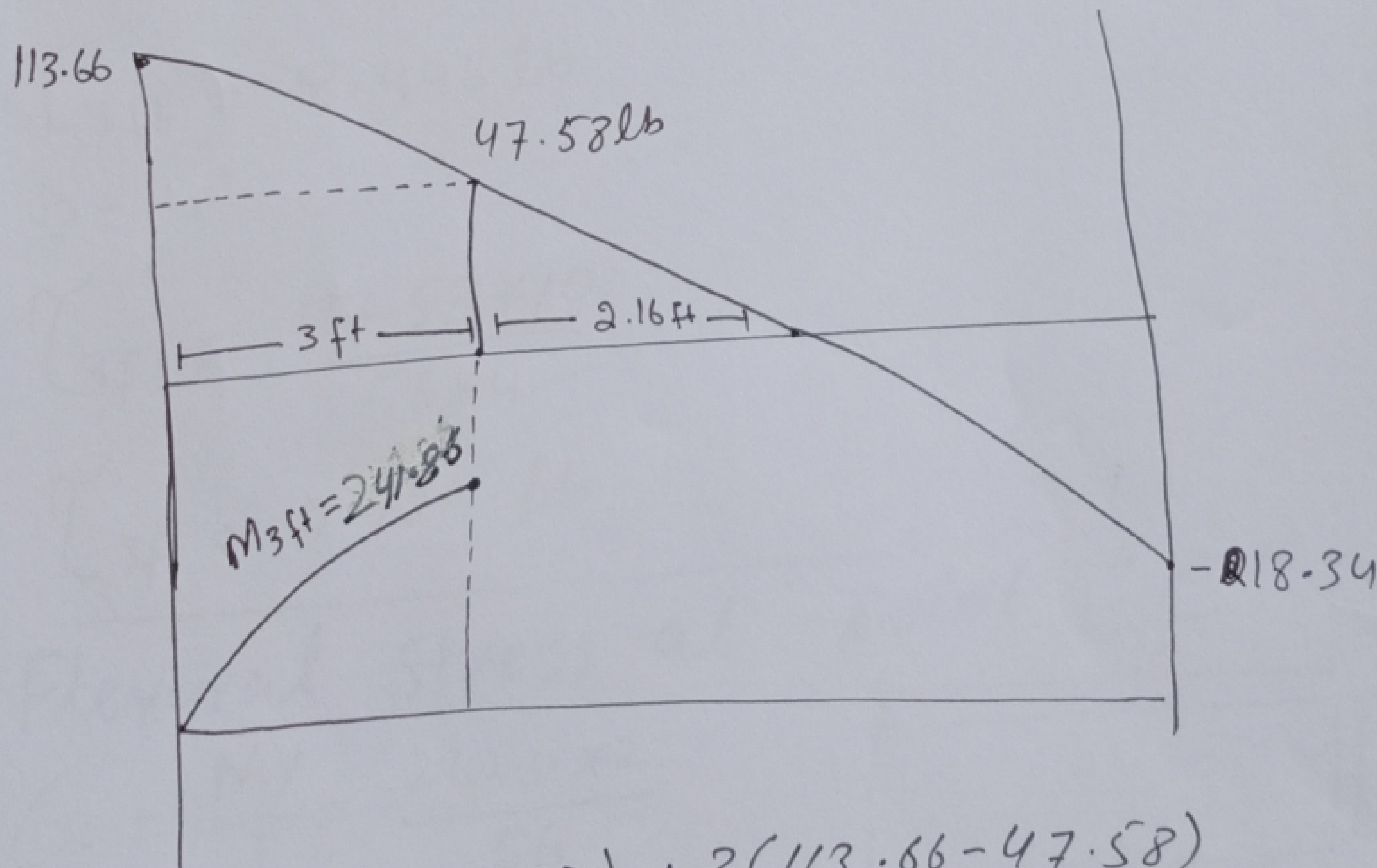
Stress State at point "C"

Stress at point "C" element located at 3 ft from left support & 1" below from the top fibre.

From shear force diagram.

$$\frac{V_{3ft}}{(5.16-3)} = \frac{2.16 * 113.66}{5.16}$$

$$V_{3ft} = 47.58 \text{ lb}$$



$$M_{3ft} = (3 * 47.58) + \frac{3(113.66 - 47.58)}{2}$$

$$M_{3ft} = 241.86 \text{ lb/ft} = 2902.32 \text{ lb/in}$$

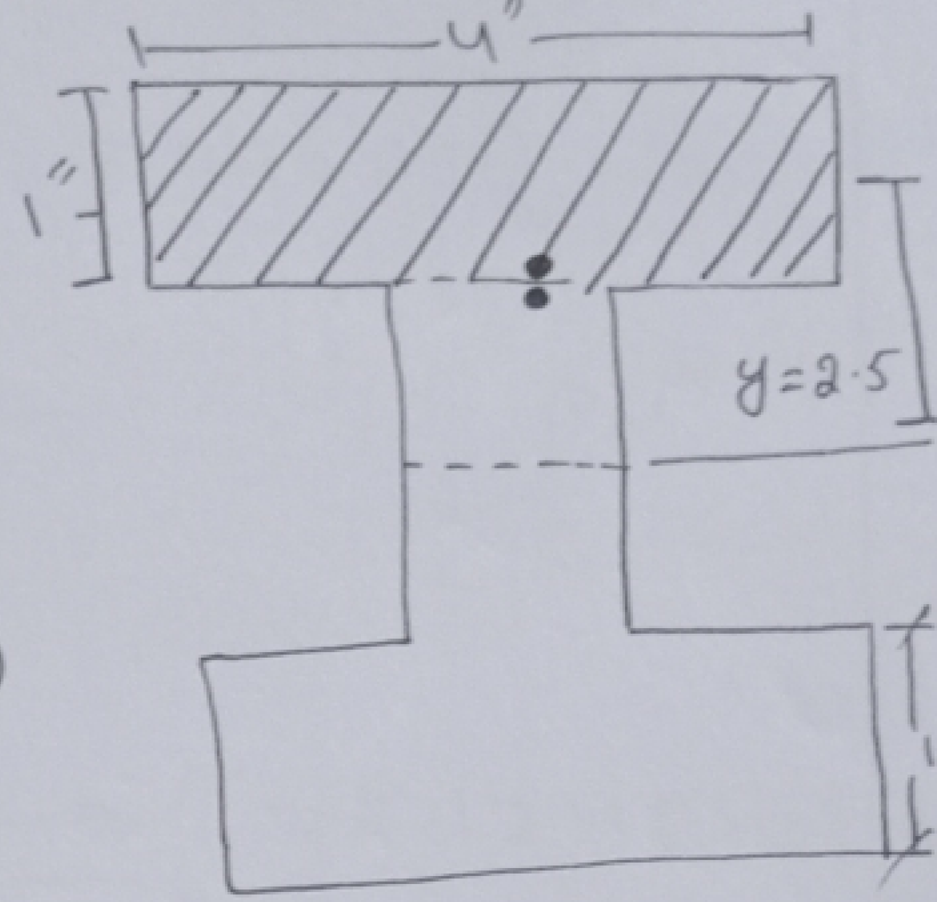
Stress at point "C"

As point C lies 1 in below top fibre so we will take two values i-e $b = 1''$, $b = 4''$

for $b = 1''$

$V = 47.58\ lb$

$I = 56\ in^4$



$$\tau_{3ft} = \frac{VQ}{Ib}$$

here $Q = \bar{y}A = (2.5)(4*1)$

$Q = 10\ in^3$

$$\tau_{3ft} = \frac{47.58*10}{56*1}$$

$$\tau_{3ft} = 8.496\ lb$$

$b = 4''$

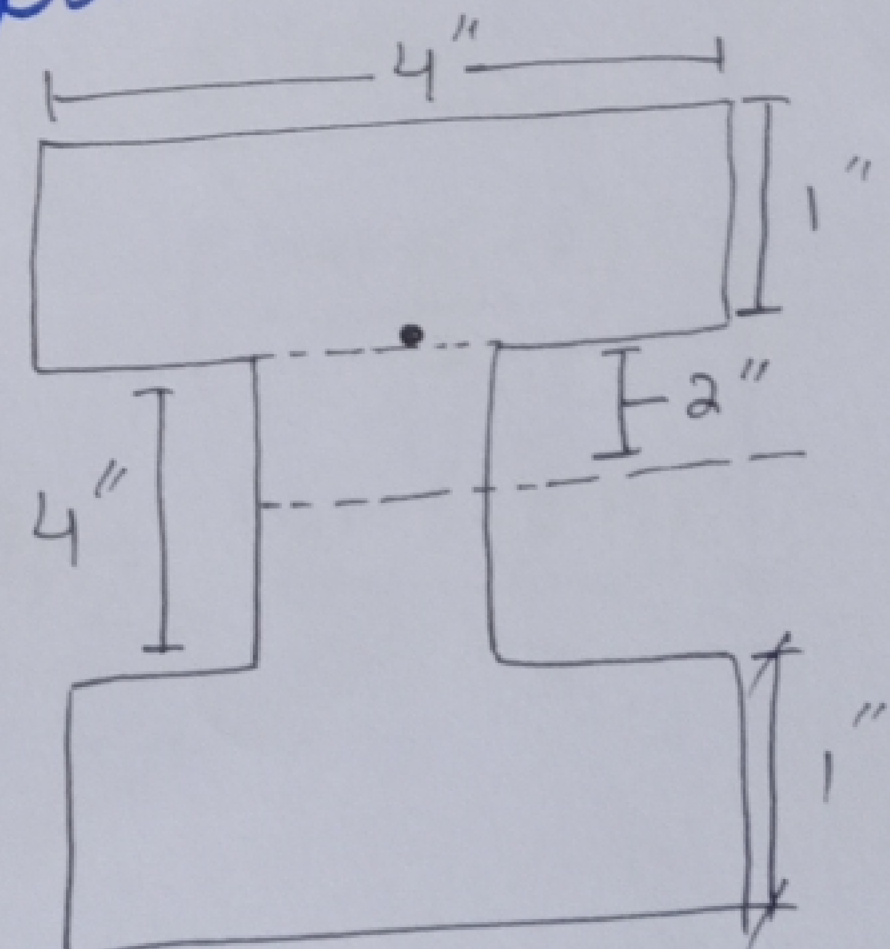
$$\tau_{3ft} = \frac{47.58*10}{56*4}$$

$$\tau_{3ft} = 2.124\ lb$$

Flexural Stress at point "C"

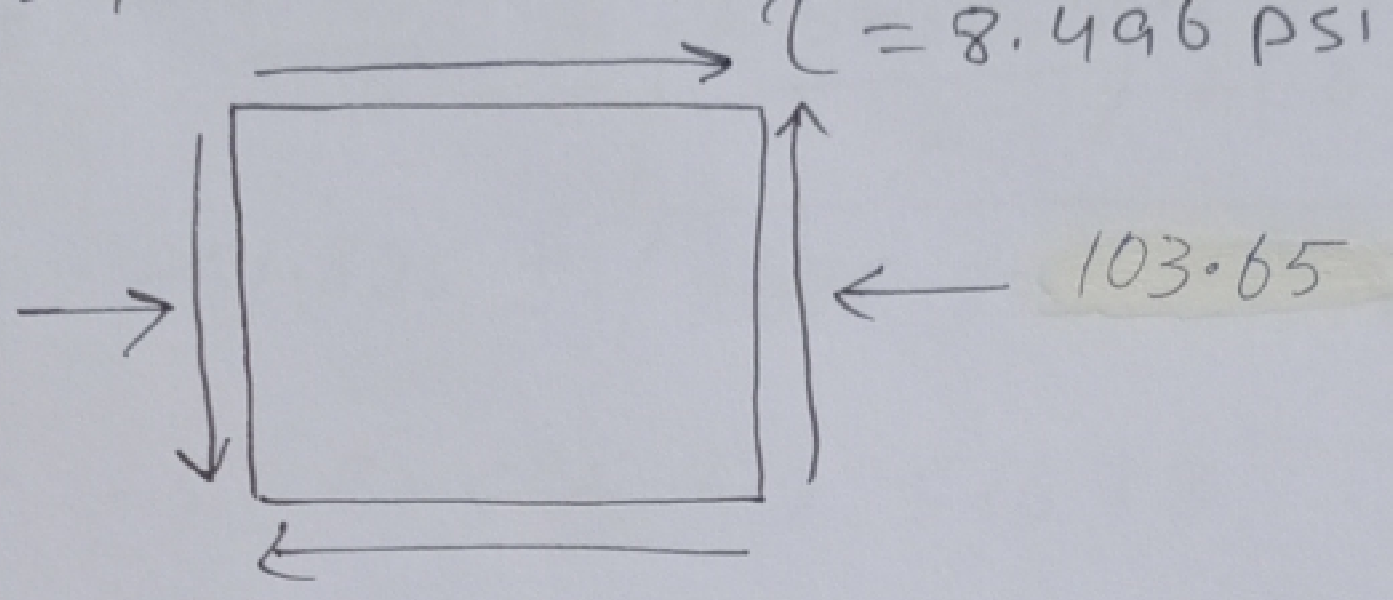
$$\sigma_c = \frac{MY}{I} = \frac{2902.32*2}{56}$$

$$\sigma_c = 103.65\ psi$$

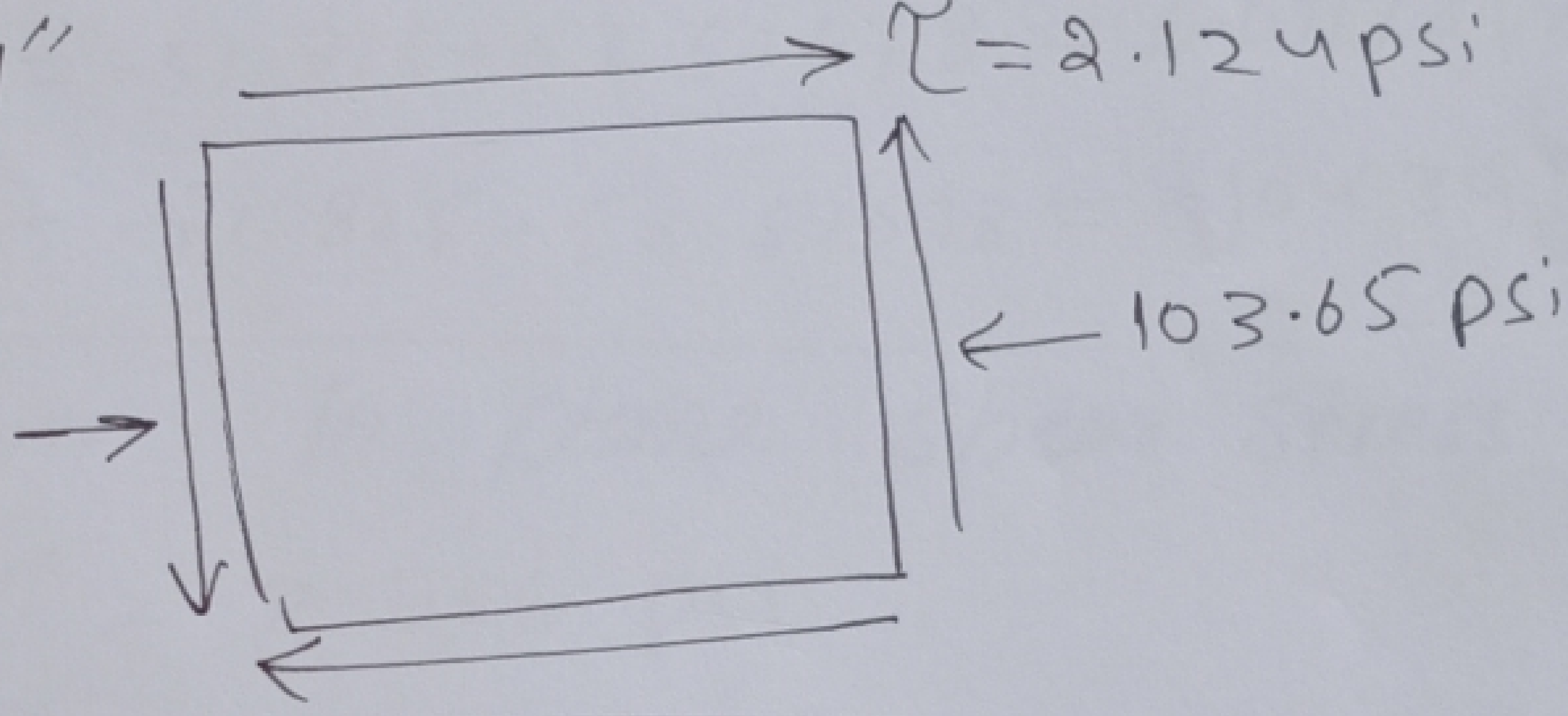


2D Representation of The point "C"

for $b = 1''$



for $b = 4''$



Principle Stresses:→

For $\tau = 8.496$ psi

$\sigma_x = -103.65$ psi

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)} = \frac{8.496}{\left(\frac{-103.65 - 0}{2}\right)}$$

$$\tan 2\theta_p = \frac{8.496}{-51.825} = \theta_p = \frac{\tan^{-1}(-0.163)}{2}$$

$$\boxed{\theta_p = -4.65}$$

$$\sigma_{1,2} = \frac{\sigma x + \sigma y}{2} \pm \sqrt{\left(\frac{\sigma x + \sigma y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{-103.65 + 0}{2} \pm \sqrt{\left(\frac{-103.65 - 0}{2}\right)^2 + (8.496)^2}$$

$$\sigma_{1,2} = -51.825 \pm \sqrt{2685.830625 + 72.18}$$

$$\sigma_{1,2} = -51.825 \pm 52.51678$$

$$\sigma y = \sigma_1 = -51.825 + 52.51678 = 0.69178 \text{ psi}$$

$$\sigma x = \sigma_2 = -51.825 - 52.51678 = -104.34 \text{ psi}$$

Maximum in plane shear stress.

$$\tau = 8.496 \text{ psi}$$

$$\sigma x = -103.65 \text{ psi}$$

$$\tan 2\theta_s = \frac{-(\sigma x - \sigma y/2)}{\tau xy}$$

$$= \frac{-(-103.65 - 0/2)}{8.496}$$

$$= \frac{51.825}{8.496} = \frac{\tan^{-1} 6.0}{2}$$

$$\boxed{\theta_s = 40.26}$$

$$\boxed{\theta_s = 40.26}$$

τ_{max} in plane

$$= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(-\frac{103.65 - 0}{2}\right)^2 + 8.496^2}$$

$$\tau_{\substack{max \\ \text{in plane}}} = 52.516 \text{ lb}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

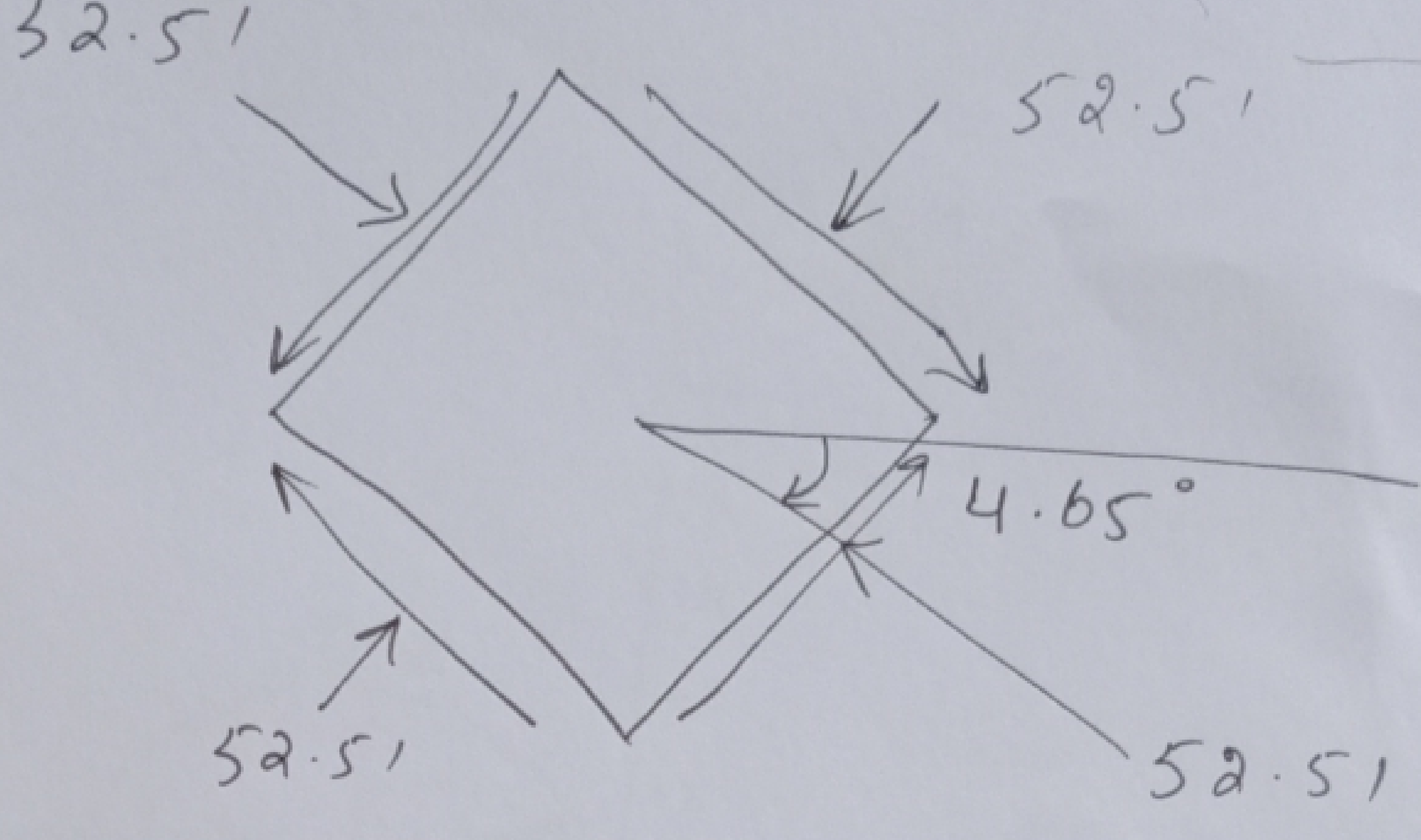
$$= \frac{-104.34 + 0.69178}{2}$$

$$= -52.53 \text{ lb}$$

& $\theta_s = -4.65$

$$4.65 + 40.26 = 45^\circ$$

The difference between principle shear stress is 45° degree.



Mohar's Circle

$\sigma_x = -103.65$ psi

$\tau_{xy} = 8.496$ psi

So $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$R = \sqrt{\left(\frac{-103.65 - 0}{2}\right)^2 + 8.496^2}$

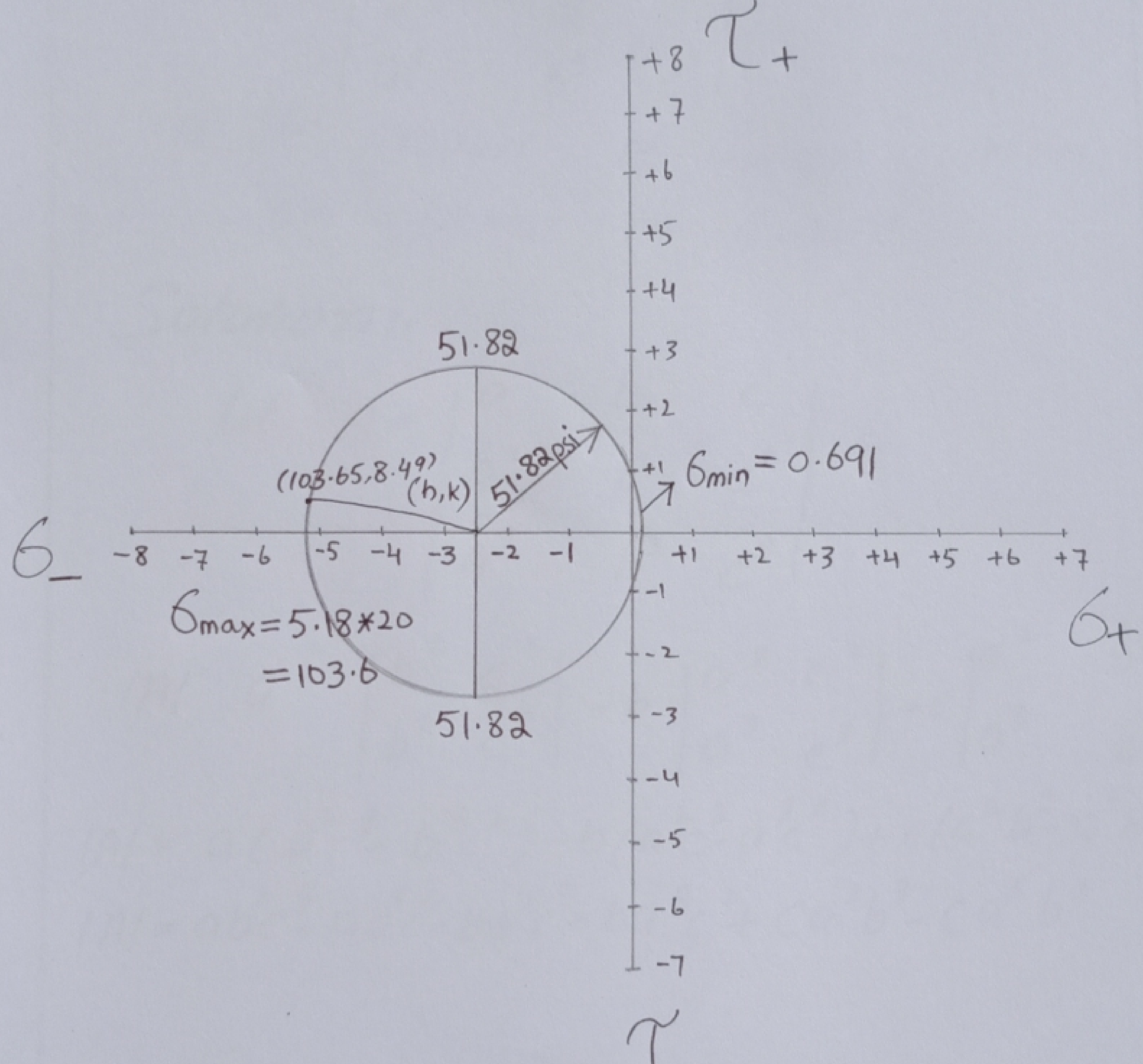
$R = 52.51$ psi

Center Co-orinates

$(h, k) = \left(\frac{\sigma_x + \sigma_y}{2}, 0\right) = \left(\frac{-103.65 + 0}{2}, 0\right)$

$(h, k) = (-51.82, 0)$

Assumption:
1 cm = 20 psi



Hance shear stress occur at 90° to the principle stresses in mohar's circle so here it is equal to radius of the circle which is $R = 51.82$ psi