BS-SE

PROGRAM

(SUBMITE BY) [MUHAMMAD FAROOQUE]

SUBJECT

CALCULUS AND ANALYTICAL GEOMETRY

SUBMITTED TO :SIR MUHAMMAD ABRAR KHAN

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IQRA NATIONAL UNIVERSITY PEAHAWAR

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Matrices and Determinants:

Exercise (9.1)

Q1: Write the following matrices in tabular form:

(i) $A = [a_{ij}]$, where $i = 1, 2, 3$ and $j = 1, 2, 3, 4$

Sol:- $i = [1, 2, 3]$ (j) $[1, 2, 3, 4]$

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \text{ Ans}$$

(ii) $B = [b_{ij}]$, where $i = 1$ and $j = 1, 2, 3, 4$

Sol:. $i = [1]$ $j = [1, 2, 3, 4]$

$$\Rightarrow \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \end{bmatrix}$$

(P # 2)

(iii) $C = [c_{jk}]$, where $j = 1, 2, 3$ and $k = 1$

Sol:- $j = [1 \ 2 \ 3]$ $k = [1]$

$$\Rightarrow \begin{bmatrix} c_{11} \\ c_{21} \\ c_{31} \end{bmatrix} \text{ Ans}$$

Q No:2 = write each sum as a single matrix:

(i) $\Rightarrow \begin{bmatrix} 2 & 1 & 4 \\ 3 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 3 & 0 \\ -2 & 1 & 0 \end{bmatrix}$

Sol:- $\Rightarrow \begin{bmatrix} 2+6 & 1+3 & 4+0 \\ 3-2 & -1+1 & 0+0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 8 & 4 & 4 \\ 1 & 0 & 0 \end{bmatrix} \text{ Ans}$$

(ii) $\Rightarrow [1 \ 3 \ 5 \ 6] + [0 \ -2 \ 1 \ 3]$

Sol:- $\Rightarrow [1+0 \ \ 3-2 \ \ 5+1 \ \ 6+3]$

$$[\Rightarrow 1 \ 1 \ 6 \ 9] \text{ Ans}$$

(P # 3)

(iii) $\Rightarrow \begin{bmatrix} 4 \\ 3 \\ -1 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \\ -2 \end{bmatrix}$

Sol:- $\Rightarrow \begin{bmatrix} 4+6 \\ 3+0 \\ -1-2 \end{bmatrix} \Rightarrow \begin{bmatrix} 10 \\ 3 \\ -3 \end{bmatrix}$ Ans

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(iv) $\Rightarrow \begin{bmatrix} 2 & 3 & 4 \\ -1 & 6 & 2 \\ 1 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Sol:- $\Rightarrow \begin{bmatrix} 2+0 & 3+0 & 4+0 \\ -1+0 & 6+0 & 2+0 \\ 1+0 & 0+0 & 3+0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 3 & 4 \\ -1 & 6 & 2 \\ 1 & 0 & 3 \end{bmatrix}$ Ans

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(P # 4)

(v) $\Rightarrow 2\begin{bmatrix} 6 & 1 \\ 0 & -3 \\ -1 & 2 \end{bmatrix} - 3\begin{bmatrix} 4 & 2 \\ 0 & 1 \\ -5 & -1 \end{bmatrix}$

Sol:- $\Rightarrow \begin{bmatrix} 12 & 2 \\ 0 & -6 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} -12 & 6 \\ 0 & -3 \\ 15 & 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & 8 \\ 0 & -9 \\ 13 & 7 \end{bmatrix}$ Ans

Q3: Show that $\begin{bmatrix} b_{11} - a_{11} & b_{12} - a_{12} \\ b_{21} - a_{21} & b_{22} - a_{22} \end{bmatrix}$ **is a solution of the matrix equation** $X + A = B$**, where** $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ **and** $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

Sol:- let $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Subtract $\Rightarrow$ "B" to "A"

$\Rightarrow B - A = \begin{bmatrix} b_{11} - a_{11} & b_{12} - a_{12} \\ b_{21} - a_{21} & b_{22} - a_{22} \end{bmatrix}$

$\Rightarrow X + A = B$ let $X = \begin{bmatrix} b_{11} - a_{11} & b_{12} - a_{12} \\ b_{21} - a_{21} & b_{22} - a_{22} \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$\Rightarrow (X + A) = \begin{bmatrix} b_{11} - \cancel{a_{11}} + \cancel{a_{11}} & b_{11} - \cancel{a_{12}} + \cancel{a_{12}} \\ b_{21} - \cancel{a_{21}} + \cancel{a_{21}} & b_{22} - \cancel{a_{22}} + \cancel{a_{22}} \end{bmatrix}$ $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

Ans

(P= 5)

Q4: Solve each of the following matrix equation:

(i) ⇒ $X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$

Sol:- let $X = \begin{bmatrix} 2 & 2 \\ -5 & -1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 2 \\ -5 & -1 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$ Ans

(ii) ⇒ $X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$

Sol:- R-H-S $\Rightarrow \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2-4 & 6-8 \\ 1-2 & 5+0 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix}$

L-H-S $\Rightarrow \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -1-1 & 0-2 \\ 0-1 & 2+3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix}$ Ans

Q4(iii) (P# 6)

$$3x + \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 4 & -1 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 3 & 1 \\ -1 & -2 & 0 \\ 0 & 1 & 5 \end{bmatrix}$$

Sol:- $$3x = \begin{bmatrix} -2 & 3 & 1 \\ -1 & -2 & 0 \\ 0 & 1 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 4 & -1 & 5 \end{bmatrix}$$

$$3x = \begin{bmatrix} -3 & 3 & -1 \\ -3 & -3 & -3 \\ -4 & 2 & 0 \end{bmatrix}$$

$$x = \begin{bmatrix} -1 & 1 & -1/3 \\ -1 & -1 & -1 \\ \frac{-4}{4} & 2/3 & 0 \end{bmatrix}$$ An

(P = 7)

Q4(iv) $X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$

Sol:- $X + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow X + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

det $X = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$X + 2I \Rightarrow \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$ Ans

Q5: Write each product as a single matrix:

(i) $\Rightarrow \begin{bmatrix} 3 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 1 & 0 \end{bmatrix}$

Sol:- $\begin{bmatrix} 3+0+(-1) & -3+2+0 \\ 0+0+2 & 0-2+0 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 2 & -2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & -1 \\ 2 & -2 \end{bmatrix}$ Ans

(P # 8)

Q 5(ii)

$$\begin{bmatrix} 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

Sol:- $\begin{bmatrix} 3 & -4 & -4 \end{bmatrix} = \begin{bmatrix} -5 \end{bmatrix}$

Q 5(iii) $\Rightarrow$

$$\begin{bmatrix} 2 & -2 & -1 \\ 1 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & -2 & 5 \\ -1 & -1 & 3 \\ -1 & -2 & 4 \end{bmatrix}$$

Sol:- $\begin{bmatrix} -2+2+1 & -4+2+2 & 10-6-4 \\ -1-1+2 & -2-1+4 & 5+3-8 \\ -1+0+1 & -2-0+2 & 5+0-4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Ans

(P#9)

Q5 (iv)

$$\Rightarrow \begin{bmatrix} -1 & -2 & 5 \\ -1 & -1 & 3 \\ -1 & -2 & 4 \end{bmatrix} \begin{bmatrix} 2 & -2 & -1 \\ 1 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

Sol:-

$$\begin{bmatrix} -2-2+5 & 2-2+0 & 1+4-5 \\ -2-1+3 & 2-1+0 & 1+2-3 \\ -2-2+4 & 2-2+0 & 1+4-4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ Ans

P# 10

Q6: If $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$,

Find $A^2 + BC$.

Sol:- $A^2 = A \cdot A + B \cdot C$

$$\Rightarrow \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+8 & 4+4 \\ 2+2 & 8+1 \end{bmatrix} + \begin{bmatrix} -3+0 & 0+4 \\ 4+0 & 0+2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9-3 & 8+4 \\ 4+4 & 9+2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6 & 12 \\ 8 & 11 \end{bmatrix}$$ Ans

Q7: show that if $A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$

then:

(a) $(A+B)(A+B) \neq A^2 + 2ab + B^2$

(b) $(A+B)(A-B) \neq A^2 - B^2$.

Sol:- Taking L-H-S

$$A+B = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}$$

$$(A+B)(A+B) = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 0-2 & 0+6 \\ 0-3 & -2+9 \end{bmatrix}$$

$$\Rightarrow (A+B)(A+B) = \begin{bmatrix} -2 & 6 \\ -3 & 7 \end{bmatrix}$$

Taking R-H-S

$$A^2 = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1+0 & -2+2 \\ 0+0 & 0+1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A^2$$

$$2AB = \begin{bmatrix} -2 & 4 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2-4 & 0+8 \\ 0-2 & 0+4 \end{bmatrix} = \begin{bmatrix} -6 & 8 \\ -2 & 4 \end{bmatrix}$$

(P# 12)

$$B^2 = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+0 & 0+0 \\ -1-2 & 0+4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix}$$

$$A^2 + 2ab + B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -6 & 8 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1-6+ & 0+8+0 \\ 0-2-3 & 1+4+4 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ -5 & 9 \end{bmatrix}$$

so L-H-S = R-H-S

$$\begin{bmatrix} -2 & 6 \\ -3 & 7 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ -5 & 9 \end{bmatrix}$$ Ans

P# 13

Q8: show that:

(i) $$\begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \\ 3 & 5 & -1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -a+2b+3c \\ 2a+b \\ 3a+5b-c \end{bmatrix}$$

(ii) $$\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ +\sin\theta & 0 & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & 0 & +\sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(i) Sol: $\Rightarrow$ L.H.S $= \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \\ 3 & 5 & -1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} -a+2b+3c \\ 2a+b \\ 3a+5b-c \end{bmatrix} \rightarrow \text{R.H.S}$$

Proved.

(ii) $= \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & 0 & +\sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} \cos^2\theta+\sin^2\theta & 0+0+0 & \sin\theta\cos\theta+0-\cos\theta\sin\theta \\ 0+0+0 & 0+1+0 & 0+0+0 \\ \sin\theta\cos\theta+0-\cos\theta\sin\theta+0+0+0 & & \sin^2\theta+0+\cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \text{R.H.S}$$

proved

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Exercise 9.2

Q1 Expand the determinants.

(i) $\begin{vmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \\ -2 & 1 & 3 \end{vmatrix}$

Sol:- Expand by R_1 = $\begin{vmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \\ -2 & 1 & 3 \end{vmatrix}$

$\Rightarrow 1(-3-4) - 2(9+8) + 0$

$\Rightarrow -7 - 2(17) \quad \Rightarrow -7-34$

$\Rightarrow -41$

(ii) $\begin{vmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix}$ Expand by R_1

Sol:-

$\begin{vmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix} \Rightarrow x(x^2-0) - 0(0-0) + 0(0-0)$

$\Rightarrow x^3 - 0 - 0 + 0$

$\Rightarrow x^3$ ← Ans

Q2: Without expansion, verify that:

(i) $\begin{vmatrix} -2 & 1 & 0 \\ 3 & 4 & 1 \\ -4 & 2 & 0 \end{vmatrix}$ Sol:- $\begin{vmatrix} -2 & 1 & 0 \\ 3 & 4 & 1 \\ -4 & 2 & 0 \end{vmatrix}$

$$\Rightarrow (2)\ R_3 \quad \begin{vmatrix} -2 & 1 & 0 \\ 3 & 4 & 1 \\ -4 & 2 & 0 \end{vmatrix} = 0$$

Because R_1 and R_3 are same by the properties of determinants.

Q3 Show that:

$$\Rightarrow \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1x+d_1 & c_2x+d_2 & c_3x+d_3 \end{vmatrix} = x \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix}$$

Taking L-H.S

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1x & c_2x & c_3x \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix}$$

$$x \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix}$$

L-H.S = R-H-S. Ans

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Q5: Show that:

(i) $\begin{vmatrix} l & a & a \\ a & l & a \\ a & a & l \end{vmatrix} = (2a+l)(l-a)^2$

Sol:- $\begin{vmatrix} l & a & a \\ a & l & a \\ a & a & l \end{vmatrix}$ Take L-H-S

Expand by R_1

$\Rightarrow l(l^2-a^2) - a(al-a^2) + a(a^2-al)$

$\Rightarrow l^3 - a^2l - a^2l - \cancel{a^3} + \cancel{a^3} - a^2l$

$\Rightarrow l^3 - a^2l - a^2l - a^2l$

$\Rightarrow l(l^2-a^2) - 2a^2l$

$\Rightarrow l(l-a)^2 - 2a^2l$

$\Rightarrow (2a+l)(l-a)^2$ L.H.S = R.H.S Ans

Q6: Prove that $\begin{vmatrix} a & b+c & a+b \\ b & c+a & b+c \\ c & a+b & c+a \end{vmatrix} = a^3+b^3+c^3-3abc$

Expend by: $a[(c^2+ac+aca^2)-(ba+b^2+ca+cb)-b(bc+ba+c^2+ca)$

$\Rightarrow -(ca+cb+ba+b^2))] + c((b^2+bc+cb+c^2)-(ac+a^2+bc+ba))$

$\Rightarrow (ac^2+a^2c+a^2c+^3-ba^2-ab^2-ca^2-cba)$

$\Rightarrow (ac^2+a^2c+a^2c+^3-ba^2-ab^2-ca^2-cba$

$- b^2c - b^2a - bc^2\ ba + cb^2 + b^2a\ b^3 + b^2c + bc^2 + c^2b - ac^2$

$- a^2c\ bc^2 - bac)$

$\Rightarrow a^3+b^3+c^3-3abc(ac^2+a^2c+a^2c-ba^2-ab^2-ca^2+b^2c$

$-b^2a-bc^2+cb^2+b^2a+bc^2+c^2b-ac^2-a^2c-bc^2)$

$\Rightarrow a^3+b^3+c^3-3abc$

L-H-S = R-HS. Ans

Q7: Find values of x if

$$\begin{vmatrix} 3 & 1 & x \\ -1 & 3 & 4 \\ x & 1 & 0 \end{vmatrix} = -30$$

Sol:- Expend by R_3

$x(4-3x) - (12+x) + 0(9+1) = -30$

$\Rightarrow (4x - 3x^2) - (12+x) + 0$

$\Rightarrow 4x - 3x^2 - 12 - x$

$\Rightarrow -3x^2 + 3x - 12 = -30$

$\Rightarrow -3(x^2 - x) = -30 + 12$

$\Rightarrow -3x(x-1) = 18$

$\Rightarrow -3x = -18$ OR $x - 1 = -18$

$\Rightarrow x = \frac{-18}{-3}$ $\qquad$ $x = -18 + 1$

$\Rightarrow x = 6$ $\qquad$ $x = -17$

Ans

Q8: Use Cramer's rule to salue the following system of equation:

(i) $x - y = 2$
$x + 4y = 5$

Hence the determinants of the cofficient is:

$\Rightarrow |A| = \begin{vmatrix} 1 & -1 \\ 1 & 4 \end{vmatrix} = 4+1 = 5$

$\Rightarrow |A| = 5$ For $|A_x|$ replace the first column of $|A|$ with corresponding constant 2, 5, we have $|A| = \begin{vmatrix} 2 & -1 \\ 5 & 4 \end{vmatrix}$

$\Rightarrow 8+5 = 13$

$\Rightarrow |A_x| = 13$ Similarly $|A_y| = \begin{vmatrix} 1 & 2 \\ 1 & 5 \end{vmatrix}$

$5 - 2 = 3$

$|A_y| = 3$

Hence $x = \dfrac{|A_x|}{|A|}$ put value

$x = 13/5$

$y = \dfrac{|A_y|}{|A|} = 3/5$

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(iii) $x - 2y + z = -1$
$3x + y - 2z = 4$
$y - z = 1$

Sol:- Hence the determinants of the cofficient is:

$$|A| = \begin{vmatrix} 1 & -2 & 1 \\ 3 & 1 & -2 \\ 0 & 1 & -1 \end{vmatrix} \quad \text{Expend by } C_1$$

$\Rightarrow 1(-1+2) - 3(-2-1) + 0(4-1)$

$\Rightarrow 1 + 9 + 0$

$\Rightarrow |A| = 10$

For $|A_x|$ Replace the column of $|A|$ with corresponding constant $-1, 4, 1$ we have

$$\Rightarrow |A_x| = \begin{vmatrix} -1 & -2 & 1 \\ 4 & 1 & -2 \\ 1 & 1 & -1 \end{vmatrix} \quad \text{expend by } C_1$$

$\Rightarrow -1(-1+2) - 4(2-1) + 1(4-1)$

$\Rightarrow -1(1) - 4(1) + (3)$

$\Rightarrow -1 - 4 + 3$

$\Rightarrow |A_x| = -2$

similarly $|A_y| = \begin{vmatrix} 1 & -1 & 1 \\ 3 & 4 & -2 \\ 0 & 1 & -1 \end{vmatrix}$

Expend by C_1

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$\Rightarrow \quad 1(-4+1) - 3(1-1) + 0(2-4)$

$\Rightarrow \quad -2 - 0 + 0$

$\Rightarrow \quad |A_y| = -2$ similarly

$$\Rightarrow \quad |A_z| = \begin{vmatrix} 1 & -2 & -1 \\ 3 & 1 & 4 \\ 0 & 1 & 1 \end{vmatrix}$$

expend by C_1

$\Rightarrow \quad 1(1-4) - 3(-1+2) + 0(-1+2)$

$\Rightarrow \quad -3 - 3 + 0$

$\Rightarrow \quad |A_z| = -6$

Hence $x = \dfrac{|A_x|}{|A|} = \dfrac{\cancel{-2}}{\cancel{10}} = -1/5$

$y = \dfrac{|A_y|}{|A|} = \cancel{-2}/\cancel{10} = -1/5$

$z = \dfrac{|A_z|}{|A|} = \dfrac{-6}{10} = -3/5$

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(v) $x + y + z = 0$

$2x - y - 4z = 15$

$x - 2y - z = 7$

Sol:. The determinants of cofficient as:

$$\Rightarrow |A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & -4 \\ 1 & -2 & 1 \end{vmatrix} \text{ expend by } R_1$$

$$\Rightarrow 1(-1-8) - 1(2+4) + 1(-4+1)$$

$$\Rightarrow -9 - 6 - 3$$

$$\Rightarrow |A| = -18$$

we can find $|Ax|$ determinants and put $0, 15, 7$ we have

$$\Rightarrow |Ax| = \begin{vmatrix} 0 & 1 & 1 \\ 15 & -1 & -4 \\ 7 & -2 & 1 \end{vmatrix} \text{ expend by } R_1$$

$$\Rightarrow 0(-1-4) - 1(15+28) + 1(-30+7)$$

$$\Rightarrow 0 - 43 - 23$$

$$\Rightarrow |Ax| = -66$$

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Similarly

$$|A_y| = \begin{vmatrix} 1 & 0 & 1 \\ 2 & 15 & -4 \\ 1 & 7 & 1 \end{vmatrix}$$ Expend by R_1

$\Rightarrow 1(15+28) - 0(2+4) + 1(14-15)$

$\Rightarrow 43 - 0 - 1$

$|A_y| = 42$

Similarly $|A_z| = \begin{vmatrix} 1 & 1 & 0 \\ 2 & -1 & 15 \\ 1 & -2 & 7 \end{vmatrix}$ expend by R_1

$\Rightarrow 1(-7+30) - 1(14-15) + 0(-4+1)$

$\Rightarrow 23 + 1 + 0$

$\Rightarrow |A_z| = 24$ Hence

$x = \dfrac{|A_x|}{|A|} = \dfrac{-66}{-18} = \dfrac{-11}{-3}$ $x = 11/3$

$y = \dfrac{|A_y|}{|A|} = \dfrac{42}{-18} = \dfrac{7}{-3}$ $y = -7/3$

$z = \dfrac{|A_z|}{|A|} = \dfrac{24}{-18} = -4/3$ $z = -4/3$

The solution set as Ans.

Exercise 9.3

Q1: Which of the following matrices are singular or non-singular.

(i) $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 0 & 1 & -1 \end{bmatrix}$ Sol: so if $|A| = 0$ then A is called singular

Hence $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 0 & 1 & -1 \end{bmatrix}$ expend by C_1

$\Rightarrow 1(-1+2) - 3(1+2) + 0(1+4)$

$\Rightarrow 1 - 9 + 0$

$\Rightarrow -8$ so A is non singular

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(ii) $\begin{bmatrix} 1 & 2 & -1 \\ -3 & 4 & 5 \\ -4 & 2 & 6 \end{bmatrix}$ Sol:- If $|A|=0$ Then A is called singuler otherwis A is non-singular

$\Rightarrow |A| = \begin{bmatrix} 1 & 2 & -1 \\ -3 & 4 & 5 \\ -4 & 2 & 6 \end{bmatrix}$ expend by R_1

$\Rightarrow 1(24-10)-2(-18+20)-1(-6+16)$

$\Rightarrow 14-2(-2)-1(10)$

$\Rightarrow 14-4-10$

$\Rightarrow |A| = 0$

So A is singuler matrix.

Q2: Which of the following matrices are symetric and skew-symetric.

(i) $\begin{bmatrix} 2 & 6 & 7 \\ 6 & -2 & 3 \\ 7 & 3 & 0 \end{bmatrix}$ Sol:- If $A^t = A$ Then A is symetrix matrix.

$A = \begin{bmatrix} 2 & 6 & 7 \\ 6 & -2 & 3 \\ 7 & 3 & 0 \end{bmatrix}$ $A^t = \begin{bmatrix} 2 & 6 & 7 \\ 6 & -2 & 3 \\ 7 & 3 & 0 \end{bmatrix}$ $A = A^t$

so A is symetrix

(ii) $\begin{bmatrix} 0 & 3 & -5 \\ -3 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$ Sol:- If $A = A^t$ Then A is called skew symetrics

$A = \begin{bmatrix} 0 & 3 & -5 \\ -3 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$ $A^t = \begin{bmatrix} 0 & -3 & 5 \\ 3 & -0 & 6 \\ -5 & 6 & 0 \end{bmatrix}$

$-A^t = \begin{bmatrix} 0 & 3 & -5 \\ -3 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$ $A = -A^t$ so is skew symetric.

Ans

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Q3: Find k such that the following matrix are singuler.

(i) $\begin{vmatrix} k & 6 \\ 4 & 3 \end{vmatrix}$ Sol: $\begin{vmatrix} k & 6 \\ 4 & 3 \end{vmatrix}$

$\Rightarrow 3k - 24 = 0$

$\Rightarrow 3k = 24 \quad \Rightarrow k = \cancel{24}^{8} / \cancel{3}_{1} \Rightarrow k = 8$ Ans

(ii) $\begin{vmatrix} 1 & 2 & -1 \\ -3 & 4 & k \\ -4 & 2 & 6 \end{vmatrix}$ expend by R_1

Sol:-

$\Rightarrow 1(24 - 2k) - 2(-18 + 4k) - 1(-6 + 16)$

$\Rightarrow 24 - 2k + 36 - 8k + 6 - 16 = 0$

$\Rightarrow 10k + 50 = 0$

$\Rightarrow -10k = 50 \quad \Rightarrow k = \cancel{50} / \cancel{10} \quad k = 5$ Ans

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Q4: Find the inverse if it exists, of the following matrices.

(i) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ Sol:- Let $A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

$\Rightarrow A^{-1} = \frac{1}{|A|} \text{ adj } A.$

$\Rightarrow |A| = \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} \Rightarrow |A| = -1-6 \Rightarrow |A| = -7$

adj of $A = \begin{bmatrix} -1 & -3 \\ 2 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{-7} \begin{bmatrix} -1 & -3 \\ 2 & 1 \end{bmatrix}$ $A^{-1} = \begin{pmatrix} +1/7 & 3/7 \\ 2/7 & 7/1 \end{pmatrix}$

$A^{-1} =$ exist. Ans

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(iii) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 0 & 2 & 2 \end{bmatrix}$ sol:. let $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 0 & 2 & 2 \end{bmatrix}$

we know that

$A^{-1} = \frac{1}{|A|}$ adj A $\Rightarrow |A| = \begin{vmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 0 & 2 & 2 \end{vmatrix}$ expand by C_1

$\Rightarrow 1(0-8) + 1(4-6) + 0(0-8)$

$\Rightarrow -8 - 2 + 0 \Rightarrow |A| = -10$

$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 0 & 2 & 2 \end{bmatrix}$ adj A

a_{11} $(0-8) = -8$
a_{12} $(-2-0) = -2$
a_{13} $3(-2-0) = -2$
a_{21} $-1(6-4) = -2$
a_{22} $0-2 = -2$
a_{23} $4(3-2) = 4$
a_{31} $(0-8) = -8$
a_{32} $2(4+3) = -7$
a_{33} $(0+2) = 2$

adj $A = \begin{bmatrix} -8 & 2 & -2 \\ -2 & -2 & -1 \\ -8 & -1 & 2 \end{bmatrix}$ Adj $A^t = \begin{bmatrix} -8 & -2 & -2 \\ 2 & -2 & -7 \\ -2 & -1 & 2 \end{bmatrix}$

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$$\Rightarrow A^{-1} = \frac{1}{|A|} \text{ adj of } A \quad \text{p.t.v}$$

$$\Rightarrow \frac{1}{-10} \begin{bmatrix} -8 & -2 & -8 \\ 2 & -2 & -7 \\ -2 & -1 & 2 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 4/5 & -1/5 & 0 \\ 2/10 & 0 & 7/10 \\ 1/5 & 1/10 & -1/5 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 4/5 & -1/5 & -4/5 \\ -1/5 & 1/5 & 7/10 \\ 1/5 & 1/5 & -1/5 \end{bmatrix}$$ Ans

Short Questions

Q1: Define row and column vectors.

Anss. In liner algebra a column vector or column matrix is an $m \times 1$ that is a matrix consisting of a Single column of m element.

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

Similarly: A row vector or row matrix is a $1 \times m$ consisting of Single row of m element.
Throughout, boldface is used for the row and column vector.
The Transpose (indicated by T) of a row vector is a column vector

$$\begin{bmatrix} x_1 & x_2 & \cdots & x_m \end{bmatrix}^T = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$$

Q2: Define identity matrix:

Ans:- **Identity Matrix:** I_n is a $n \times n$ square matrix with the main diagonal of 1's and all other elements are 0's.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If A is a $m \times n$ matrix, then $I_m A = A$ and $A I_n = A$.

If A is a $n \times n$ matrix then $A I_n A = A$

Q3: Symmetric M:-

A squer matrix A is called a symmetric matrix, if $A^t = A$

A square matrix A is called a skew-symmatric if $A^T = -A$. Any Square matrix can be expend as the sum of a symmetric and a skew-symmetric matrix. $A = \frac{A+A}{2} + \frac{A-A^T}{2}$ where $(A+A^T)$ is symmatric matrix and $(A-A^T)$ is skew-symmetric matrix.

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Q4: Diagonal Matrix:- A square matrix is called diagonal matrix if non-diagonal entries are all zero the main diagonal can be constants or zero. A diagonal matrix must fit the following:

$$D = \begin{bmatrix} d_{11} & 0 & 0 & \cdots & 0 \\ 0 & d_{22} & 0 & \cdots & 0 \\ 0 & 0 & d_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & d_{nn} \end{bmatrix}$$

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objective types:

(1) The order of the matrix $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ is:

Ans:- (c) 3×1

(2) The order of matrix $[1\ 2\ 3]$ is:

Ans:- (a) 1×3

(3) The matrix $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is called:

Ans: {d} Null

(4) Two matrix A and B are conformable for multiplication if:

Ans: {A} No of column in A = No of rows in B

(5) if The order of the matrix A is $p \times q$ and order of B is $q \times r$, Then order of AB will be: Ans: {c} $p \times r$

(6) In an Identity matrix all the diagonal elements are: Ans: {c} (1)

(7) The value of determinants $\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$ is

Ans: {a} 6

(8) if two row of a determinant are identical then its value is: Ans:- {b} Zero

(9) if $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$ is a matrix, then cofactor of 4 is: Ans: {a} -2

(10) if all the elements of a row or a column are zero, then value of the determinant is! Ans:- {c} zero

(11) value of m for which matrix $\begin{bmatrix} 2 & 3 \\ 6 & m \end{bmatrix}$ is singular: Ans: {d} 9

(12) if $[a_{ij}]$ and $[b_{ij}]$ are the same order and $a_{ij} = b_{ij}$ the matrix will be!

Ans:- {d} equal.

(13) Matrix $[a_{ij}]_{m \times n}$ is a row matrix if:

Ans:- {c} $m=1$

(14) Matrix $[c_{ij}]_{m \times n}$ is a rectangular if:

Ans:- {d} $m-n \neq 0$

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(15) if $A = [a_{ij}]_{m \times n}$ is a scalar matrix if:

Ans:- {d} (a) and (b)

(16) Matrix $A = [a_{ij}]_{m \times n}$ is an identity matrix if:

Ans:- {d} both (a) and (c)

(17) Which matrix can be rectangular matrix.

Ans:- {d} None

(18) If $A = [a_{ij}]_{m \times n}$ then order kA is:

Ans:- {a} $m \times n$

(19) $(A-B)^2 = A^2 - 2AB + B^2$, if and only if:

Ans:- {b} $AB - BA = 0$

(20) if A and B are symmetric then $AB =$

Ans:- {d} (a) and (c)

or $A^t B^t$