Department of Electrical Engineering
Assignment
Date: 13/04/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th
Instructor: **Total Marks:** 30

Student Details

Name: Hamza ishaq **Student ID:** 13748

Q1.	(a)	Consider the following analog signal $$x_a(t) = 3cos100\pi t + 4sin200\pi t$$ i. Determine the minimum sampling rate required to avoid aliasing. ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal. iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?	Marks 5 CLO 1
	(b)	Consider a discrete time signal which is given by $$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$ This is signal is sampled at the rate $F_s = 2Hz$. i. Draw the sampled signal. ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i . iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.	Marks 5 CLO 1
Q2.	(a)	Determine the response of the system to the following input signal with given impulse response $$x[n] = \left\{2, \underset{\uparrow}{1}, -2, 3, -4\right\} \quad , h[n] = \left\{\underset{\uparrow}{3}, 1, 2, 1, 4\right\}$$	Marks 5 CLO 2
	(b)	Compute the convolution y(n) of the following signal $$x(n) = \begin{cases} \alpha^{n+1}, & -3 \leq n \leq 5 \\ 0, & elsewhere \end{cases}$$ $$h(n) = \begin{cases} 2^n, & 0 \leq n \leq 4 \\ 0, & elsewhere \end{cases}$$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i. $$x(n) = \begin{cases} (\frac{1}{4})^n, & n \geq 0 \\ (\frac{1}{3})^{-n}, & n < 0 \end{cases}$$ ii. $$x(n) = \begin{cases} (\frac{1}{2})^n - 3^n, & n \geq 0 \\ 0, & elsewhere \end{cases}$$	Marks 10 CLO 2

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Q1

a) Consider the following analog signals
$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$

1) Determine minimum sampling rate required to avoid aliasing

$F_s \geq 2F_{max}$ $\qquad f = \frac{\omega}{2\pi}$

$f_1 = \frac{100\pi}{2\pi}$ $\qquad f_2 = \frac{200\pi}{2\pi}$

$f_1 = 50\ Hz$ $\qquad f_2 = 100\ Hz$

So

f_2 is max (greater then f_1)
$f_s \geq 2 \times 100\ Hz$ sample frequency to avoid aliasing
$f_s = 100\ Hz$

f_1 becomes

$f_1' = \frac{f_1}{f_s} = \frac{180}{100} = 0.5\ Hz$

f_2 becomes

$f_2 = \frac{f_2}{100} = \frac{100}{100} = 1\ Hz$

So $\omega_1' = 2\pi f_1'$ $\qquad \omega_2' = 2\pi f_2'$
$\omega_1' = 2\pi \times 0.5$ $\qquad \omega_2' = 2\pi \times 1$
$\omega_1' = \pi$ $\qquad \omega_2' = 2\pi$

(2)

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$x[n] = 3\cos 100\pi n + 4\sin 200\pi n$

The signal are
$x[n] = 3\cos \pi n + 4\sin 2\pi n$

The effect of sampling rate on the newly generated discrete time signal is that there will be no phenomenon means there will not be present unwanted component in the reconstruction of the signal. The reconstruct original signal

$w_1 = 100\pi$ $\quad$ $f_2 = 200\pi$

$f_1 = \frac{100\pi}{2\pi}$ $\quad$ $f_2 = 100\pi$

$\boxed{f_1 = 50}$

iii) What is the analog system signal $y_a(t)$ we can reconstructed from the sampling if we used ideal interpolation

Sol. folding frequency of the sampled signal is:

folding frequency $= \frac{F_s}{2} \Rightarrow \frac{100}{2}$

$= 50\,Hz$

we have frequency of the original signals

$f_1 = 50\,Hz$, $f_2 = 100\,Hz$

Both the frequency are either equal or greater

The folding frequency

Hence for the ideal interpolation we can construct the original signal

$x_a(t) = 3\cos 100\pi + 4\sin 200\pi t$

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(3)

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The original signal is constructed because we are sampling frequency at Nyquist rate. We can also reconstruct the signal for sampling frequency above the Nyquist rate.

Q1

(b) Consider a discrete time signal which is given by

$$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0. & n < 0 \end{cases}$$

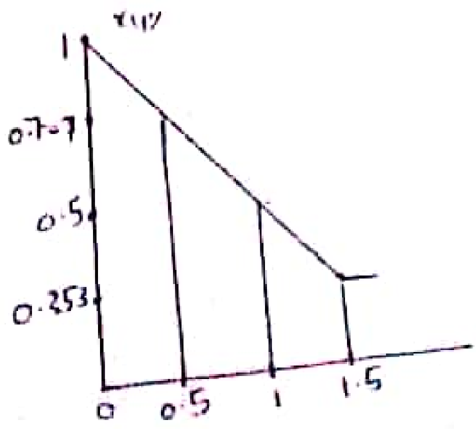
$$F_s = 2\,Hz$$

$$F_s = 1/T \Rightarrow T = 1/F_s = \frac{1}{2}$$

$$= 0.5 sec$$

(i) Draw the sampled signal

x_2	$= 0.5^n$
0	1
0.5	0.707
1	0.5
1.5	0.353



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ii)

Sol

$L = 2^n$

$n =$ bits

$= 3$

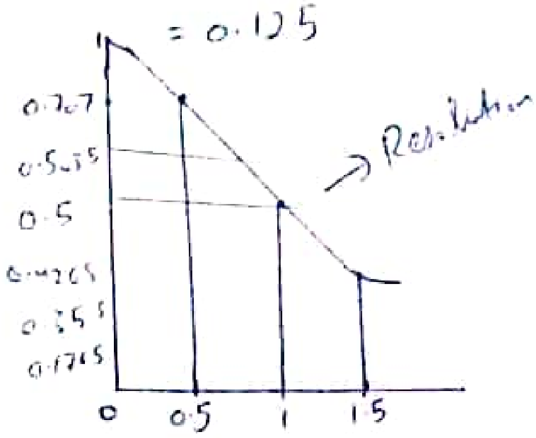
$L = 2^3$

$= 8$ levels

$$\text{Resolution} = \frac{x_{max} - x_{min}}{L}$$

$$= \frac{1-0}{8}$$

$$= 0.125$$



iii)

	Discrete Signal	Truncation	Reading	Error
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	-0.1
2	0.707	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.1	0.2	-0.1

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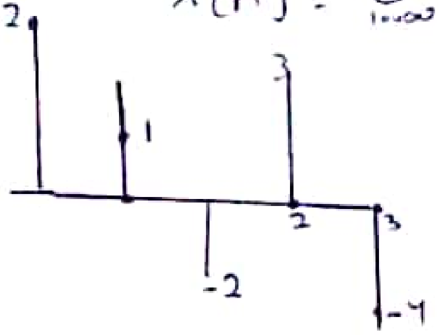
Q2 a)

Determine the response of the system to the following input signal with given impulse response

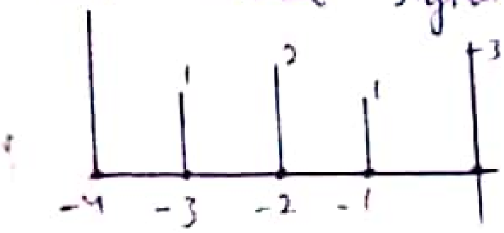
$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, 4\}$, $h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$

Sol:

$$x[n] = \sum_{-\infty}^{\infty} x[k]\, h(n-k)$$



(-K) folded signal



$$y(0) = \sum_{k=1}^{0} x(-1)h(-1) + x(0)h(0)$$

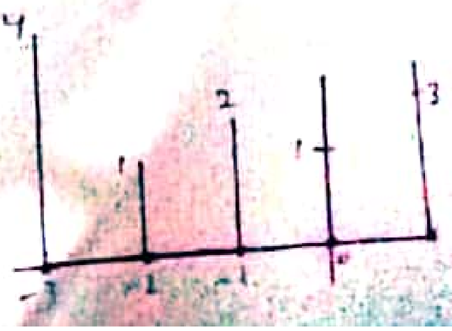
$y(0) = (2)(1) + (1)(3)$

$= 2+3$

$= 5$

for $n=1$

$h(1-k)$



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$$y(1) = \sum_{k=1}^{1} x(n)\ h(1-k)$$

$= x(-1)h(-1) + x(0)h(0) + 0\ x(1)h(1)$

$+ (x)\ h(1) + x(2)\ x(2) + 0(x(3)\ h(3))$

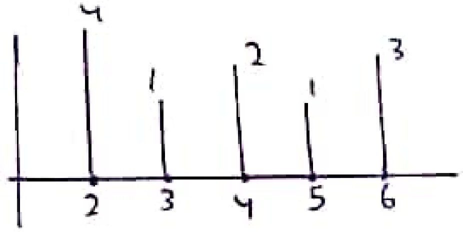
$y(2) = (2)(4) + (1)(1) + (-2)(2) + (3)(1) + (-4)(3)$

$= 8 + 1 - 4 + 3 - 12$

$= -4$

$n = 3$

~~n(3-k)~~ $n(3-k)$



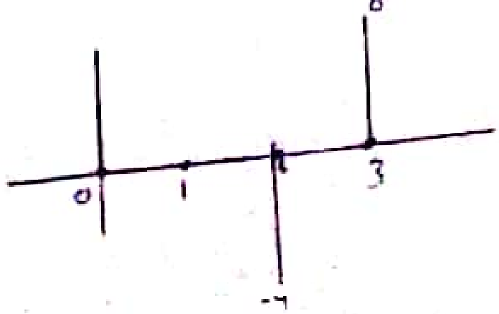
$$y(3) = \sum_{k=2}^{3} x(k)\ h(n-k)$$

$= x(2)\ h(2) + x(3)\ h(3)$

$(3)(4) + (-4)(1)$

$= 12 - 4$

$= 8$



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i) $x(n) = \begin{cases} \left(\frac{1}{4}\right)^n, & n \geq 0 \\ \left(\frac{1}{3}\right) & n < 0 \end{cases}$

Sol:

$$x(n) = \begin{cases} \left(\frac{1}{4}\right)^n, & n \geq 0 \\ \left(\frac{1}{3}\right)^{-n} & n < 0 \end{cases}$$

Writing in the form of z-transform

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n z^{-n} + \sum_{n=-\infty}^{0} \left(\frac{1}{3}\right)^n z^{-n} - 1$$

Using geometric series

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{n} - 1$$

$$= \frac{1 - \frac{1}{\frac{1}{4}z^{-1}} + 1 - \frac{1}{1} - 1}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)}$$

$$= \frac{1-\frac{1}{3}z + 1 - \frac{1}{4}z^{-1} - \left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

$$= \frac{1-\frac{1}{3}z + 1 - \frac{1}{4}z^{-1}\left(1+\frac{1}{3}z - \frac{1}{4}z^{-1} + \frac{1}{12}\right)}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

$$= \frac{1-\cancel{\frac{1}{3}z} + 1 - \cancel{\frac{1}{4}z^{-1}} - 1 + \cancel{\frac{1}{3}z} + \cancel{\frac{1}{4}z^{-1}} - \frac{1}{12}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z\right)}$$

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$$= \frac{1 + \frac{1}{12}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{13/12}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

Hence the ROC is $1/4 < |z| < 3$

(Q3 ii)

$$x(n) = \begin{cases} (1/2)^n - 3^n, & n \geq 0 \\ 0, & \text{else where} \end{cases}$$

Sol:-

In the form of Z-transform

$$X(z) = \sum_{n=0}^{\infty} (1/2)^n z^{-n} - \sum_{n=0}^{\infty} 3^n z^{-n}$$

Using geometric series

$$= \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-3z^{-1}}$$

$$= \frac{\cancel{1} - 3z^{-1} + \frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})}$$

$$= \frac{-\frac{5}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})}$$

The ROC is $|z| > 3$