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Subject Structural Analysis II

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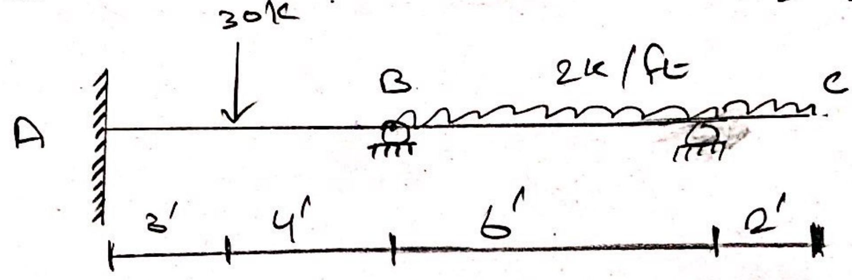
Summer Exam

FINAL Term

Q1) Analyze the beam shown in Fig 1 by stiffness method. Assume EI is constant.

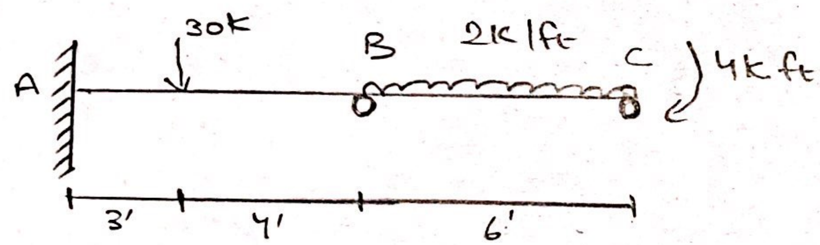


Solution :-

Determining Kinematic Indeterminacy

K.I = 5°

So we have to reduce the extended portion



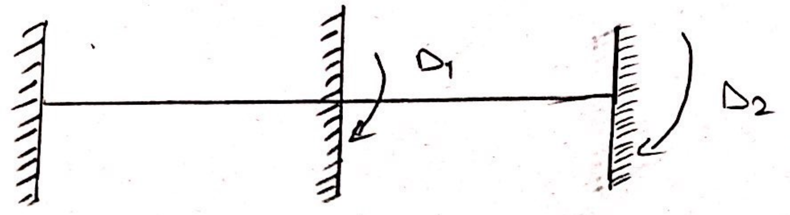
$$\Rightarrow \frac{2(2)}{1} = 4k.ft$$

Now

K.I = 2°

Step # 2

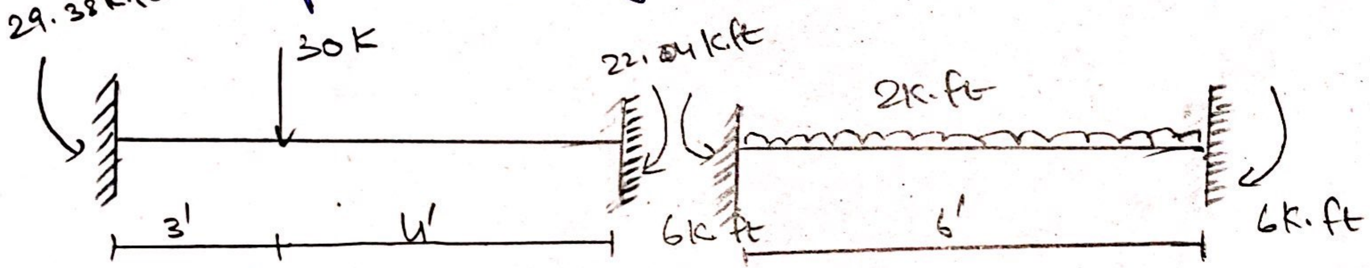
Determine Unknown Joint displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \qquad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step # 3

Compute [ADL] Matrix



=> For pointed load (not at mid)

=> For left hand/end

$$= \frac{Pab^2}{L^2} = \frac{(30)(3)(4)^2}{(7)^2} = 29.38\ k.ft$$

=> For Right end:-

$$\frac{Pa^2b}{L^2} = \frac{(30)(3)^2(4)}{(7)^2} = 22.04\ k.ft$$

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$\Rightarrow$ For UDL:-

$$\frac{wL^2}{12} \Rightarrow \frac{(2)(6)^2}{12} = \boxed{6k.ft}$$

$$ADL1 = +22.04 - 6 = 16.04 k.ft$$

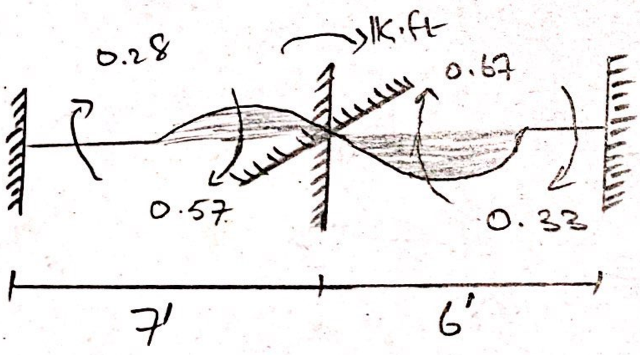
$$ADL_2 = 6k.ft$$

Step #4:-

Compute [S] Matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

a) $D_1 = 1K$, $D_2 = 0$



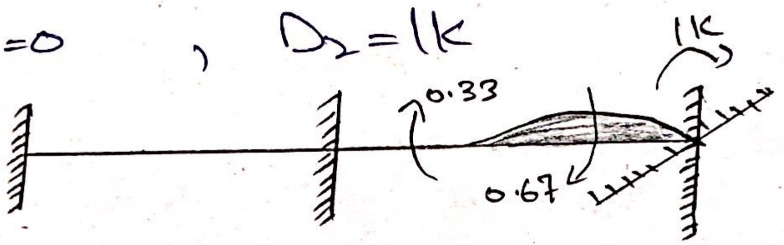
$$= \frac{4EI}{7} = 0.57 \qquad \frac{2EI}{6} = 0.33$$

$$\frac{4EI}{6} = 0.67 \qquad \frac{2EI}{7} = 0.28$$

$$S_{11} = 0.57 + 0.67$$
$$= 1.24\ EA$$

$$S_{21} = 0.33\ EA$$

b). $D_1 = 0$, $D_2 = 1k$



$$\frac{4EI}{} = 0.67$$

$$\frac{2EI}{6} = 0.33$$

$$S_{12} = 0.33$$

$$S_{22} = 0.67$$

$$S = \begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$

Step # 5:-

Compute [D] matrix

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}$$

$$= \frac{1}{\begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}} \times \text{Adj } A \ \alpha \begin{bmatrix} 0 - 16.04 \\ 4 - 6 \end{bmatrix} \begin{bmatrix} -16.04 \\ -2 \end{bmatrix} E$$

$$|S| = (1.24 \times 0.67) - (0.33 \times 0.33)$$

$$= 0.8308 - 0.1089$$

$$|S| = 0.7219$$

$$\text{Adj } A = \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}$$

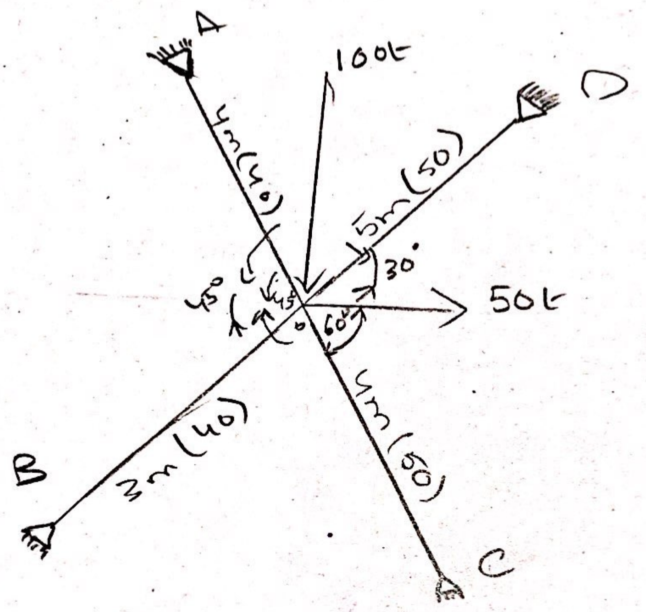
Now,

$$\begin{bmatrix} AD_1 & -ADL_1 \\ AD_2 & -ADL_2 \end{bmatrix} = \begin{bmatrix} 0 - 16.04 \\ 4 - 6 \end{bmatrix} = \begin{bmatrix} -16.04 \\ -2 \end{bmatrix} E$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{\begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}}{0.7219} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} -13.97 \\ 3.8902 \end{bmatrix}$$

Q2) Analyze the Pin-Jointed frame shown by stiffness method. Length of the members in "m" and cross sectional are of the members in cm^2 shown in Fig(3) $E = 2000\ t/cm^2$



Fig(3)

Solution :-

For A:-

$$\sin 45^\circ = \frac{P}{h} = \frac{P}{4}$$

$$\Rightarrow P = 2.828m$$

$$\cos 45^\circ = \frac{b}{4}$$

$$\Rightarrow b = 2.828m$$

For B:-

$$\sin 45^\circ = \frac{P}{3}$$

$$\Rightarrow P = 2.12m$$

$$\cos 45^\circ = \frac{b}{h}$$

$$\Rightarrow b = 2.12m$$

For D:-

$$\sin 30^\circ = \frac{P}{h} \quad , \quad h = 5$$

$$\Rightarrow P \Rightarrow 2.5m$$

$$\cos 30^\circ = \frac{b}{5}$$

$$\Rightarrow b = 4.33m$$

Now

$$EA(A) = 2000 \times 40 = 80{,}000\,t$$

$$EA(B) = 2000 \times 40 = 80{,}000\,t$$

$$EA(C) = 2000 \times 50 = 100{,}000\,t$$

$$EA(D) = 2000 \times 50 = 100{,}000\,t$$

Step # 1

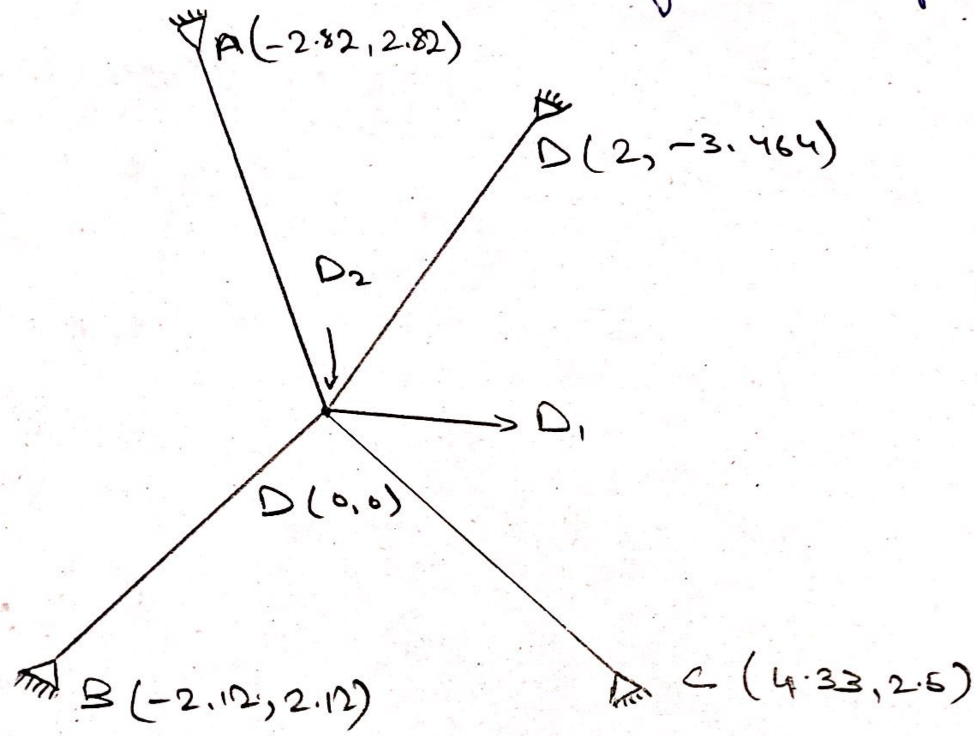
K.I

$$K.I = 2j - r$$

$$= 2(5) - 8 = 2^\circ$$

Step # 2

Select Unknown joint displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad \begin{bmatrix} AD \\ AD_1 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

Step # 3

$$[AMD]_{4\times 2} \quad \& \quad [S]_{2\times 2}$$

(i) $D_1 = 1 \, , \, D_2 = 0$

$$AMD = \frac{EA}{L^2}(X_k - X_j)$$

$$AMD_{11} = \frac{80,000}{(400)^2} \times (0 + 282) = 141$$

$$AMD_{21} = \frac{80,000}{(300)^2} \times (0 + 212) = 188.44$$

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$$AMD_{31} = \frac{100{,}000}{(500)^2} \times (0-4.33) = -173.2$$

$$AMD_{41} = \frac{100{,}000}{(400)^2} \times (0-200) = -125$$

Now $S_{11} = \sum_{i=1}^{m} \frac{EA}{L^3} (x_k - x_j)^2$

$$= \frac{80{,}000}{(400)^3} \times (282)^2 + \frac{80{,}000}{(300)^3} \times (212)^2 + \frac{100{,}000}{(500)^3} \times (-433)^2$$

$$+ \frac{100{,}000}{(400)^3} \times (-200)^2$$

$S_{11} = 99.405 + 133.107 + 149.991 + 62.5$

$$\boxed{S_{11} = 445.063}$$

$$S_{12} = S_{21} = \sum_{i=1}^{m} \frac{EA}{L^3} \times (x_k - x_j)(y_k - y_j)$$

$$= \frac{80{,}000}{(400)^3} \times (282)(-282) + \frac{80{,}000}{(300)^3} \times (212)(212)$$

$$+ \frac{100{,}000}{(500)^3} \times (-433)(0-250) + \frac{100{,}000}{(400)^3} \times (-200)(0+346)$$

$$\boxed{S_{12} = S_{21} = 12.237}$$

ii) $D_1 = 0$, $D_1 = 1k'$

$$AMD = \frac{EA}{L^2}(Y_k - Y_j)$$

$$AMD_{12} = \frac{80{,}000}{400^2}(-282) = -141$$

$$AMD_{22} = \frac{80{,}000}{300^2}(212) = 188.44$$

$$AMD_{32} = \frac{10{,}000}{500^2}(-250) = -100$$

$$AMD_{42} = \frac{100{,}000}{400^2}(346) = 216.25$$

Now, $S_{22} = \sum_{i=1}^{m} \frac{EA}{L^3}(Y_k - Y_j)^2$

$$= \frac{80{,}000}{400^3}(-282)^2 + \frac{80{,}000}{300^3}(212)^2 + \frac{100{,}000}{500^3}(-250)^3$$

$$+ \frac{100{,}000}{400^3}(346)^2$$

$$\boxed{S_{22} = 469.628}$$

Step # 4

$$[D] = [S]^{-1} \times [AD]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.063 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

Step # 05

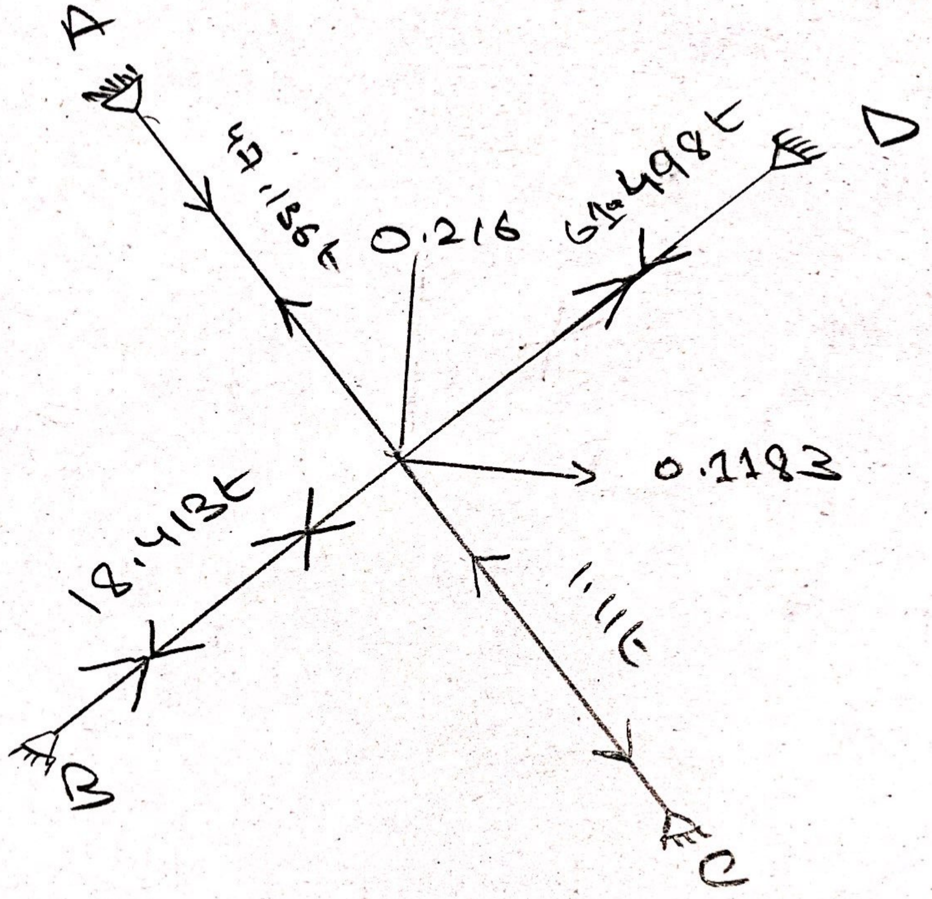
[AM]

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \propto \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

$$= \begin{bmatrix} 141 \times 0.1183 + (-141) \times (-0.216) \\ 188.44 \times 0.1183 + 188.44 \times (0.216) \\ -173.2 \times 0.1183 + (-100) \times (-0.216) \\ -125 \times 0.1183 + 216.25 \times (0.216) \end{bmatrix}$$

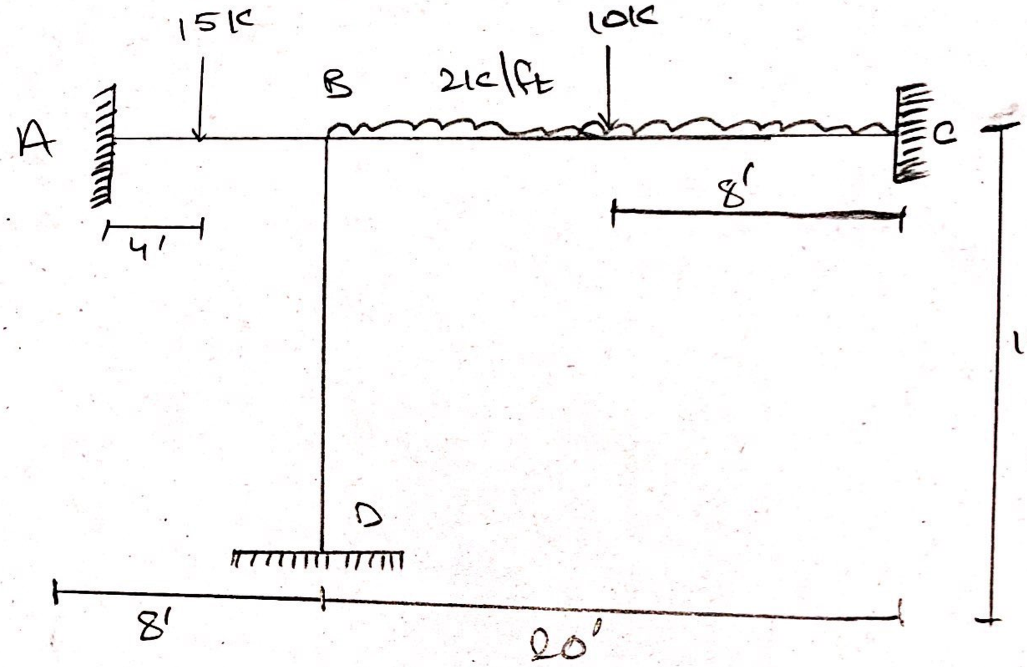
$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 16.68 + 30.46 \\ 22.29 - 40.70 \\ -20.49 + 21.6 \\ -14.79 - 46.71 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 47.136t \\ -18.413t \\ 1.11t \\ -61.498t \end{bmatrix}$$


A
D
B
C
47.186t
0.216
67.498t
0.1183
18.413t
1.11t

Q3)

Analyze the rigid Joint frame shown in Fig by stiffness method. Assum EI is constant



Solution:-

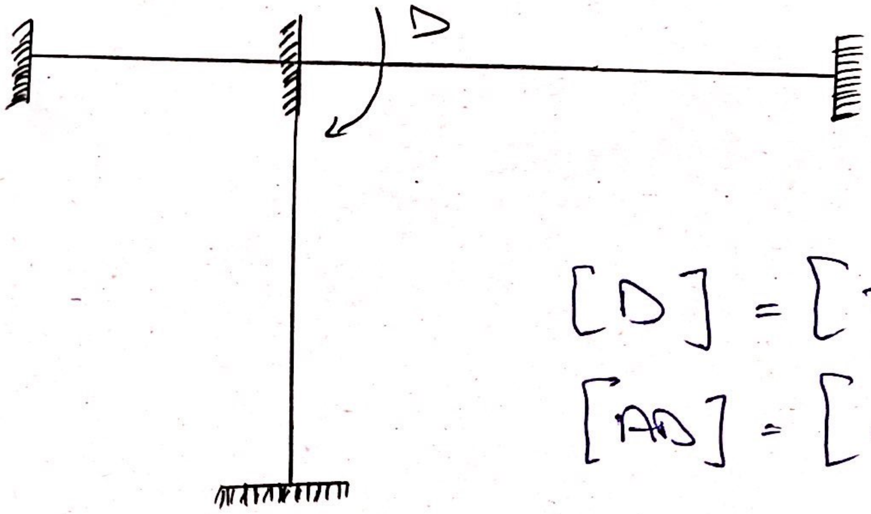
Step # 1

Determine Kinematic Indeterminancy

K.I = 1°

Step # 2 :-

Determine Unknown Joint Displacement

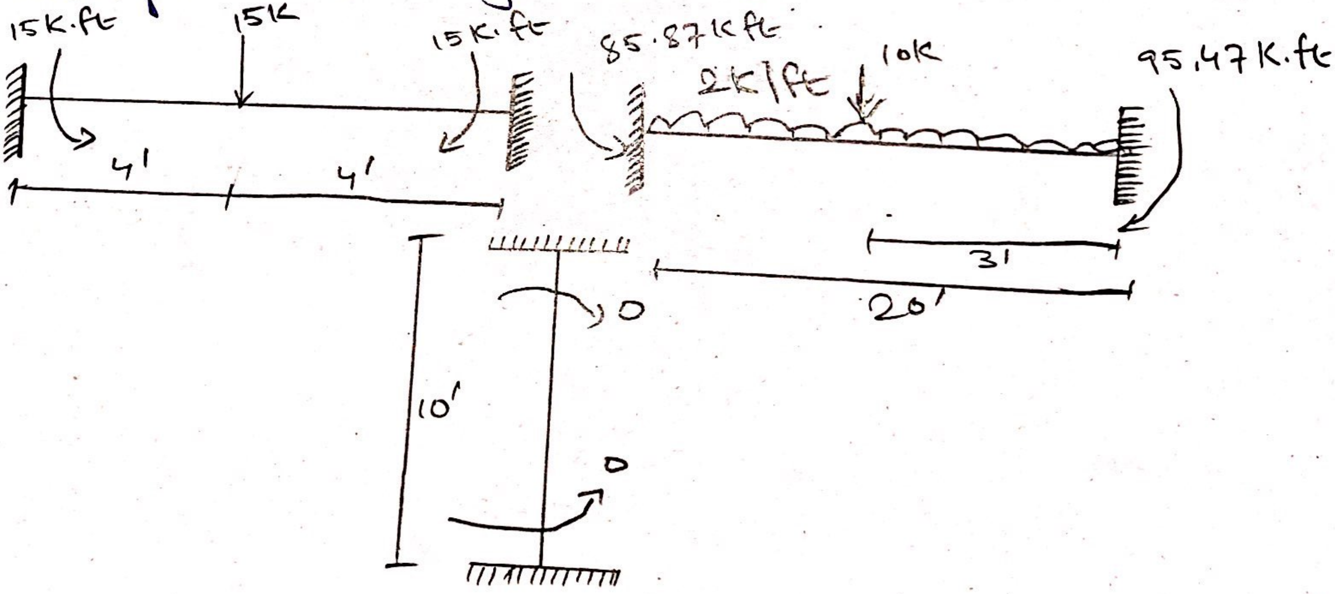


$[D] = [?]$

$[AD] = [0]$

Step # 3 :-

Compute [ADL] Matrix



$\Rightarrow$ Point load at Center:-

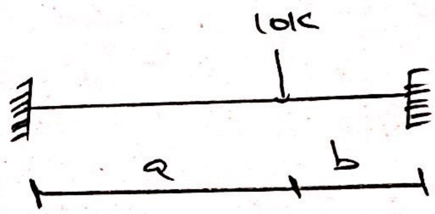
$$\frac{PL}{8} \Rightarrow \frac{(15)(8)}{8} = 15\text{kip.ft}$$

$\Rightarrow$ Uniformly Distributed load:-

$$\frac{WL^2}{12} \Rightarrow \frac{(2)(20)^2}{12} = 66.67\text{ k.ft}$$

=> Point load (Not at mid)

Suppose



For left end:-

$$\frac{Pab^2}{L^2} \Rightarrow \frac{(10)(12)(8)^2}{(20)^2} = 19.2\ k.ft$$

For Right End:-

$$\frac{Pa^2b}{L^2} = \frac{(10)(12)^2(8)}{(20)^2} = 28.8\ k.ft$$

So Total Moment at left end:-

$$19.2 + 66.67 = 85.87\ k.ft$$

Similarly at right end:-

$$28.8 + 66.67 = 95.47\ k.ft$$

So $[ADL]$ $-85.87 + 15 = -70.87\ k.ft$

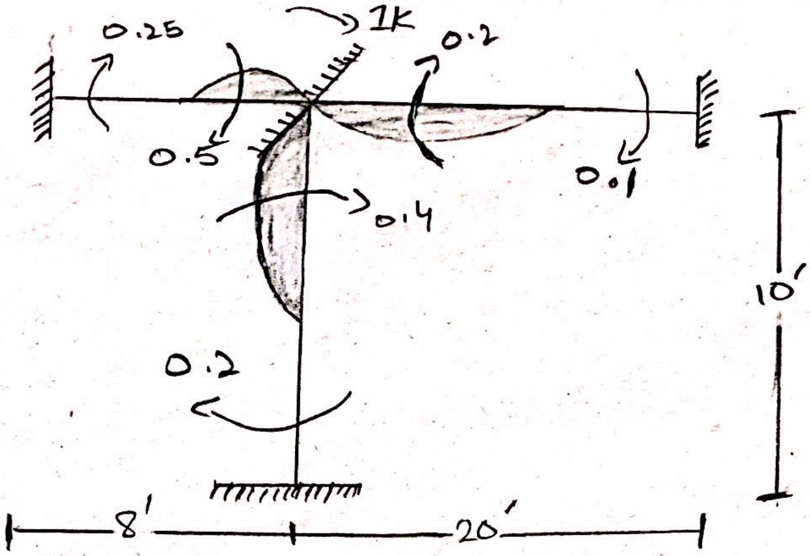
Step # 4 :-

Determine [S] Matrix

$$[S] = [S_{11}]$$

Now

$$D = 1k$$



$$\Rightarrow \frac{4EI}{8} = 0.5 \qquad \frac{2EI}{8} = 0.25$$

$$\Rightarrow \frac{4EI}{20} = 0.2 \qquad \frac{2EI}{20} = 0.1$$

$$\Rightarrow \frac{4EI}{10} = 0.4 \qquad \frac{2EI}{10} = 0.2$$

$$[S] = (0.5 + 0.4 + 0.2)EI$$

$$= 1.1EI$$

$$[S] = 1.1EI$$

Step # 5 :-

Compute [D] matrix

$$[D] = [S]^{-1} \times [AD] - [ADL]$$

$$[D] = \frac{1}{1.1} \times [0] - [-70.87]$$

$$= \frac{70.87}{1.1}$$

$$[D] = [64.42] \, \frac{1}{EI} \quad \text{Ans}$$