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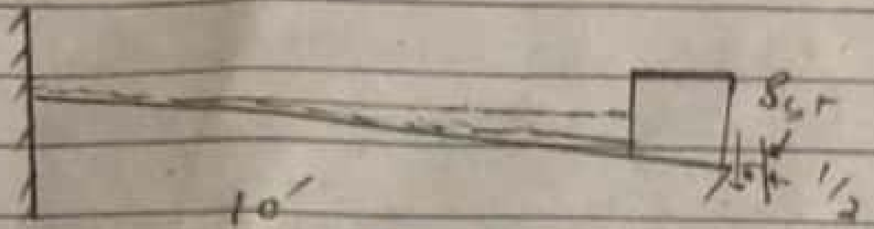
Section A

Subject

INTRO TO Structural Dynamic
& Earth QuaKE ENGG

# Q No # 01




Ans

**Given Data**

$E = 29{,}000\ ksi$

$I = 150\ in^4$

Deflection st = 7728

**Required:-**

Natural time period = ?

$W_n = ?$ $m = ?$

$T_n = ?$ $f_s(t) = ?$ $KU_o = ?$

1) Graph show to varition of displacement with time

2) Varation of equalent static force with Time.

**Solution:-**

As we know that:-

$$Ku + c\dot{u} + m\ddot{u}(\ ) = p(t)$$

In our case system is undamped ($c=0$) under going free vibration ($p(t)=0$)

Hence general EOM becomes

$$Ku + Mu = 0$$

$$K = 3EI/L^3$$

$$= \frac{3 \times 29000 \times (150)^4 in}{(10 \times 12in)^3}$$

$$= 7.55 \, K/in$$

In order to ~~determine~~ eliminate the chances of mistake during calculation, It is more appropriate to use fundemental units like lb, ft sec or kg m, sec.

$$K = 7.55 \, K/in = 90625 \, lb/ft$$

Now

$$w_n = \sqrt{\frac{k}{m}} \Rightarrow \sqrt{\frac{90625}{m}}$$

Now put the value of m in
W. formula.

$W_n \Rightarrow$ [illegible] $\quad w_n = \sqrt{\frac{90625}{240}}$

$W_n = 19.43$ rad/sec

$$T_n = \frac{2\pi}{w_n} = \frac{2\pi}{19.43}$$

$$\boxed{T_n = 0.323 \text{ sec}}$$

Substituting the corresponding value
in eq -1

$$90625\,u + 240\,\ddot{u} = 0$$

Where K is lb/ft and m is
in lb sec/ft²

Genral solution to the EOM
Undamped free Vibration is

$$u(t) = u(0)\cos(w_n t) + \frac{\dot{u}(0)}{w_n}\sin(w_n t)$$

$$u(0) = \tfrac{1}{2}'' = \frac{1}{24}\text{ ft} \quad \text{and} \quad \dot{u}(0) = 0$$

$u(t) = \left(\frac{1}{24}\right) \cos(19.43t) + 0 = \left(\frac{1}{24}\right) \cos(19.43t)$

Equivalent Static force at any time "t" is

$f_{(s)}(t) = K \cdot u(t) = \frac{90625 \cos(19.43t)}{24}$

$f_{(s)}(t) = 3776 \cos(19.43t)$

Amplitude of dynamic displacement $u_0$ for undamped free vibration is

$$u_0 = \sqrt{\left[u(0)^2 + \left(\dot{u}(0)/\omega_n\right)^2\right]}$$

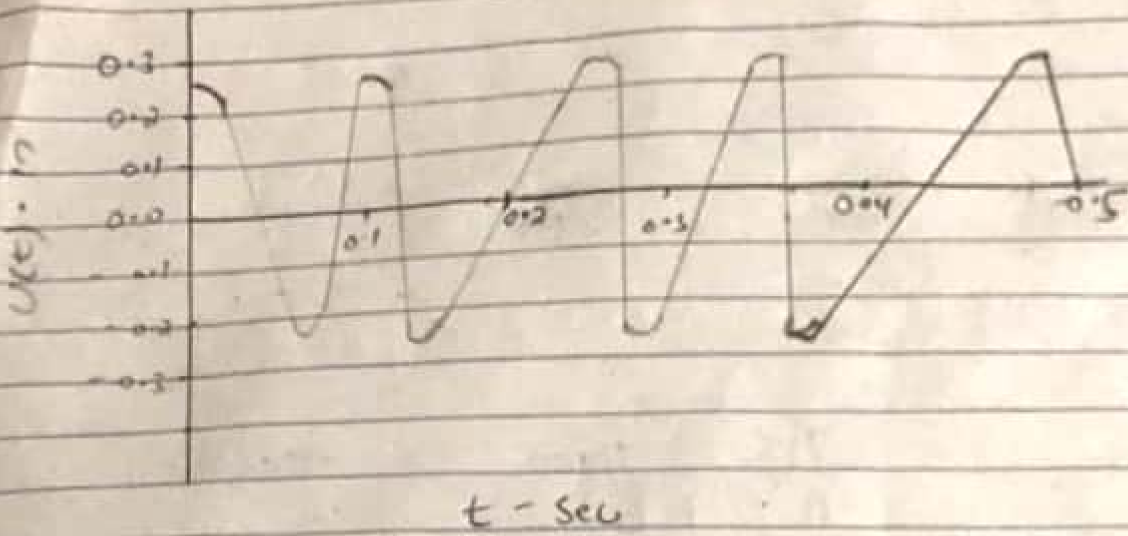
$$= \sqrt{\left(\left(\frac{1}{24}\right)^2 + 0\right)} = \frac{1}{24} \text{ ft}$$

Amplitude of equivalent satic force, $f_{so}$

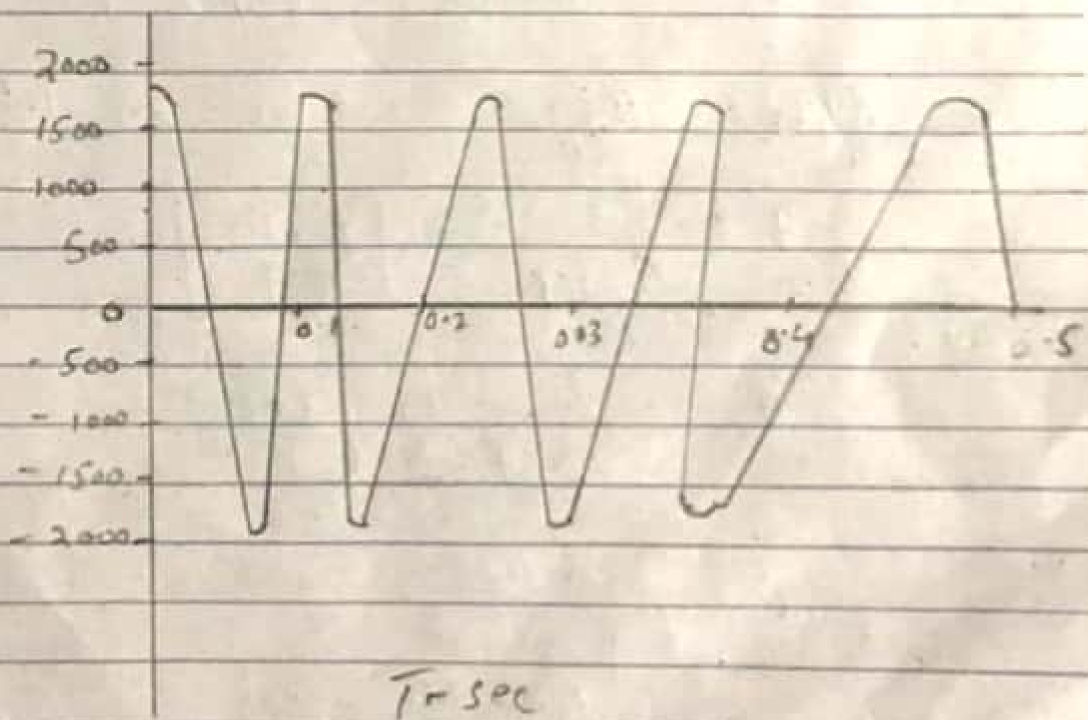
$Ku_0 = 90625 \times \frac{1}{24} = 3776 \text{ lb}$

$\boxed{Ku_0 = 3776 \text{ lb}}$

undamped free vibration




a) Variation of displacement with time




Variation of equivalent static forces with time

# Q NO 2

## Given Data:-

Damping ratio of reainforce concrete with considreble cracking = 3-5%
so we take $\zeta_0$ = 3%

other data are take from Question No 1

## Reauired Data:-

Devolp and solve the eauation of motion for vibration at free end = ?

Devolp an eauation showing variation in Equvalent static forces with time = ?

## Solution

As we know that:

EoM (eauation of motion) for damped free vibration is

$$ku + c\dot{u} + m\ddot{u} = 0 \quad \longrightarrow (1)$$

As we know from Q NO data.

$K = 90625 \text{ lb/ft}$

$m = 240 \text{ lb} \cdot \text{sec}^2/\text{feet}$

$w_n = 19.43 \text{ rad/sec}$

As we know that

$c = \zeta \times 2 m w_n$

$c = (0.03) \times 2(240) \times 19.43$

$$\boxed{c = 279.792 \text{ lb} \cdot \text{sec} \cdot \text{ft}}$$

By putting value in eq ① we get

$90625 u + 279.792 \dot{u} + 240 \ddot{u} = 0$

Solution to the EOM for damped free vibration is;

$$u(t) = e^{-\zeta w_n t}\left[u(0)\cos(w_D t) + \frac{1}{w_D}\left\{\dot{u}(0) + u(0)\zeta w_n\right\}.\sin(w_D t)\right]$$

Now

$w_D = 19.45 \text{ rad/sec}$

$$u(t) = e^{-10.03 \times 19.45 \times t} \left[ \frac{1}{24} \times \cos(19.45t) + \frac{1}{19.45} \left(0 + \frac{1}{24} \times 0.03 \times 19.45 \times \sin(19.45t)\right)\right.$$

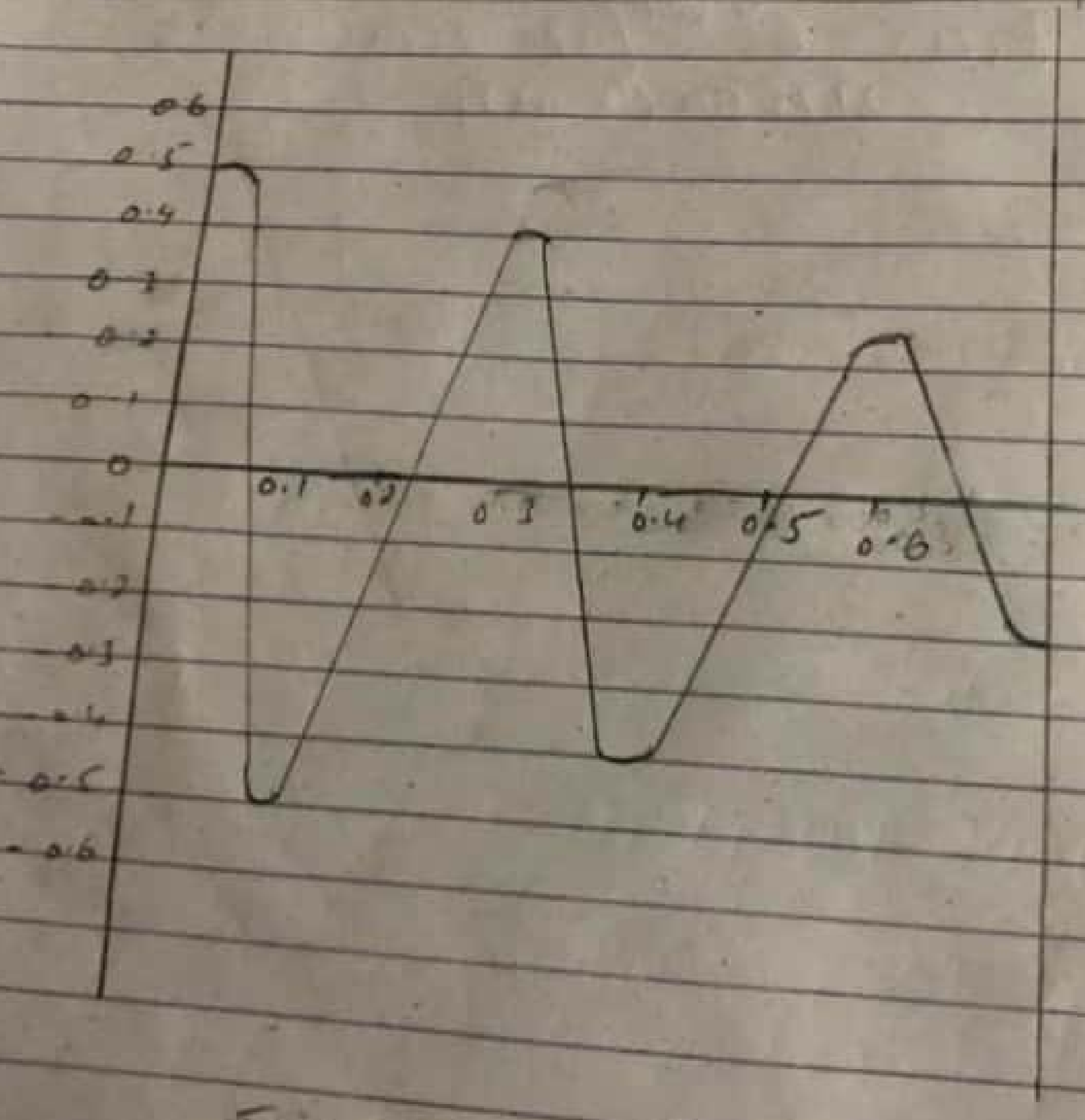
$$u(t) = e^{-0.584t} \left[ 0.042 \cos(19.45t) + 0.024 \sin(19.45t) \right]$$

$$f_{(s)}(t) = K u(t) = 90625 \times u(t)$$

$$f_s(t) = e^{-0.584t} \left[ 3806.25 \cos(19.45t) + 2175 \sin(19.45t) \right]$$

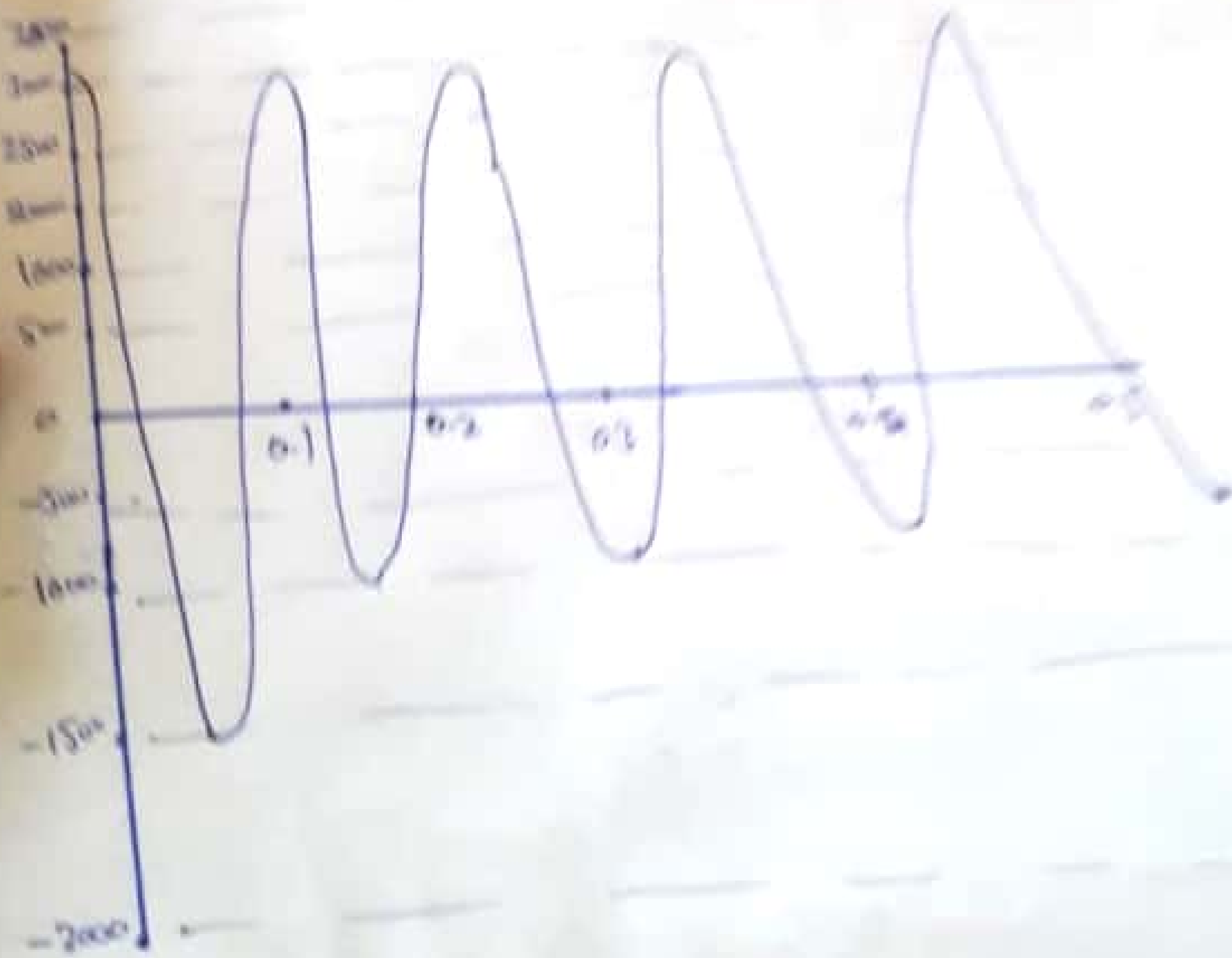
Graph:-

Show variation of displacement With time




Time = Sec

## ★ Damped free vibrations



# Q No 3

Solution:-

As given in question

$u_1 = 7.728$

After $J = 7$

$u_{J+1} = u_8 = 2.286 = 0.9$ in

a. $\xi$ = Damping ratio = ?

$$J = \frac{1}{2\pi\xi} \ln\left[\frac{u_1}{u_{J+1}}\right]$$

By putting values we get

$$7 = \frac{1}{2\pi\xi} \ln\left[\frac{7.728}{0.9}\right]$$

$$\xi = \frac{1}{2\pi \times 7} \ln\left[\frac{7.728}{0.9}\right]$$

$\xi = 0.0488$

$\xi = 4.88\,\%$ Ans

## b) Natural period of un-damped vibration

According to question

7 cycle of vibration are completed in 3.57 sec

Time period to complete one cycle

$$= \frac{3.57}{7}$$

$$\boxed{T_D = 0.51 sec}$$

As we know that.

$$\omega_D = \omega_n \sqrt{(1) - (\zeta)^2}$$

$$T_D = \frac{T_n}{\sqrt{(1) - (\zeta_0)^2}}$$

$$T_n = T_D \sqrt{(1) - (\zeta_0)^2}$$

By putting value we get.

$$T_n = 0.51 \sqrt{(1) - (0.0488)^2}$$

$$T_n = 0.5094$$

$$\boxed{T_n = 0.51 \text{ sec}}$$

C = Stiffness of Structure

As we know that :-

$$K = \frac{60 \cos(60)}{7.728}$$

$K = 3.881 \ K\,In$ $K = 46583.85$

$K = 46583.85$

d)

Weight of Tank = W = ?

As we know that

$$w_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{(W/g)}}$$

$$W_n = \sqrt{\frac{Kg}{W}}$$

$$W_n^2 = \frac{Kg}{W}$$

$$W = \frac{K \cdot g}{W_n^2} \quad — (1)$$

Also $W_n = \frac{2\pi}{T_n}$

$$W = \frac{K g \times T_n^2}{4\pi^2}$$

Scanned with CamScanner

putting values

$$W = \frac{46583.85 \times 32.2 \times (0.51)^2}{4 \times \pi^2}$$

$W = 9882.61$ lb

$\boxed{W = 9.882\ K}$

e) Damping coeffficent

As we know that

$$\zeta = \frac{c}{2 m \omega_n}$$

$$c = \frac{\zeta \times 4 \times \pi \times m}{T_n}$$

By puttin values

$$c = \frac{0.0488 \times 4 \times \pi \times \left(\frac{9.882.61}{32.2}\right)}{0.51}$$

$c = 369.041$ lb sec/ft

f. Number of cycles to reduces The displacement amplitude 0.5"

As we know that

$$j = \frac{1}{2\times \zeta} \ln \left[\frac{u_1}{u_{j+1}}\right]$$

$$j = \frac{1}{2\times \pi \times 0.0488} \ln \left[\frac{7.728}{0.5}\right]$$

$j = 8.929$ say at cycle

$$\boxed{j = 8.9 = 9 \text{ cycle}}$$