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Section

B

Subject

MOS 2

Exam

Mid term

department

Civil

Semister

CHH

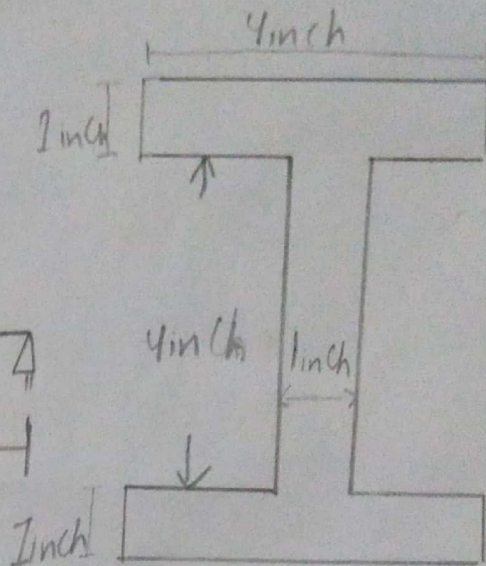
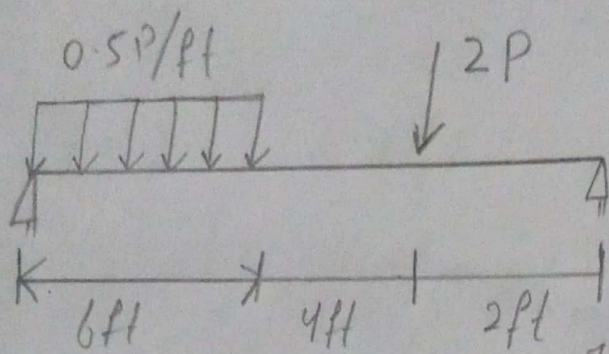


1)

# Question No 1

Ans :

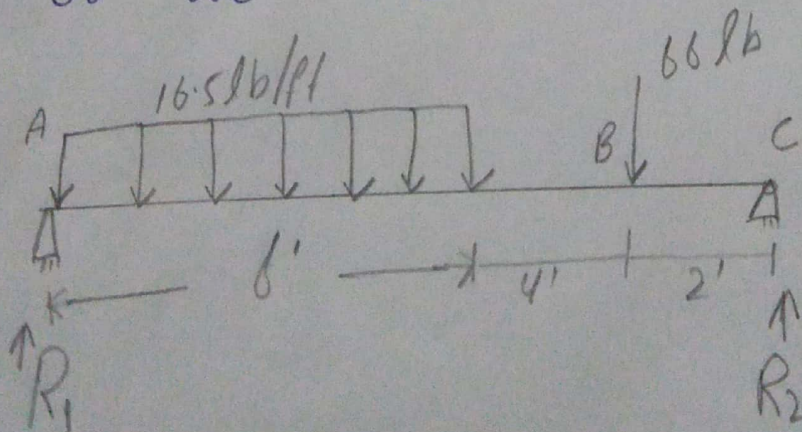
Given Beam



Note

Put the value of  $P = 33$

So we have





2)

First to find the unknown reaction at the support Apply equilibrium equation

$$\sum F_x = 0 \quad \text{i.e. } \boxed{R_3 = 0}$$

$$\sum F_y = 0 \quad \begin{array}{cc} +\uparrow & \downarrow- \end{array}$$

$$R_1 + R_2 = (16.5 \times 6) \text{ lb} + 66 \text{ lb}$$

$$R_1 + R_2 = 99 + 66$$

$$R_1 + R_2 = 165 \rightarrow \textcircled{1}$$

Next

$$\sum MA = 0 \quad \begin{array}{cc} (+\curvearrowright & -\curvearrowleft) \end{array}$$

$$R_2 \times 12 - 10 \times 66 - (16.5 \times 6) \times \frac{6}{2} = 0$$

$$\Rightarrow 12R_2 = 660 + 297$$

$$\Rightarrow 12R_2 = 957 \text{ lb-ft}$$



(3)

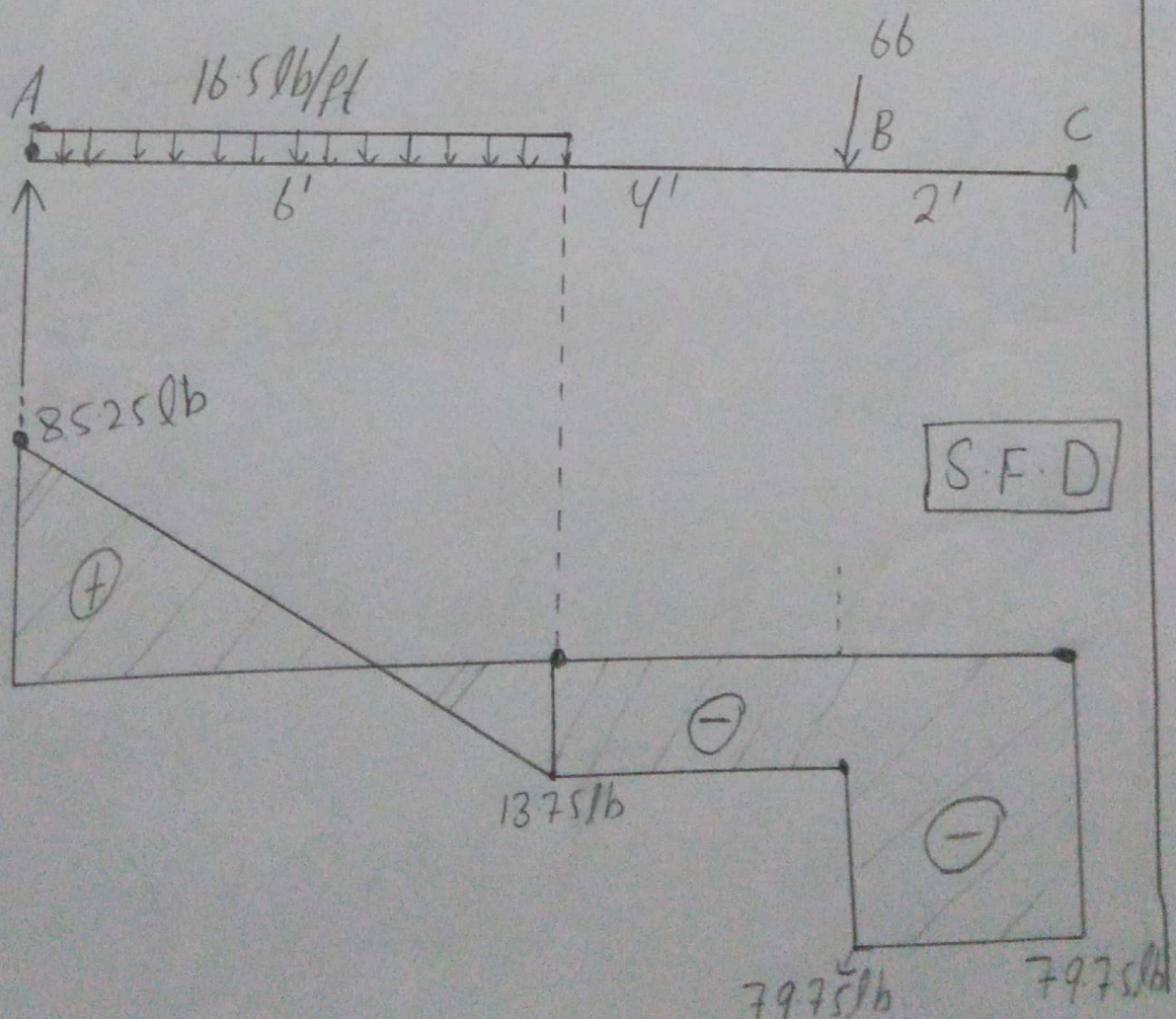
$$\Rightarrow R_2 = 79.75 \text{ lb}$$

$$\textcircled{1} R_1 + R_2 = 185$$

$$\Rightarrow R_1 = 185 - R_2$$

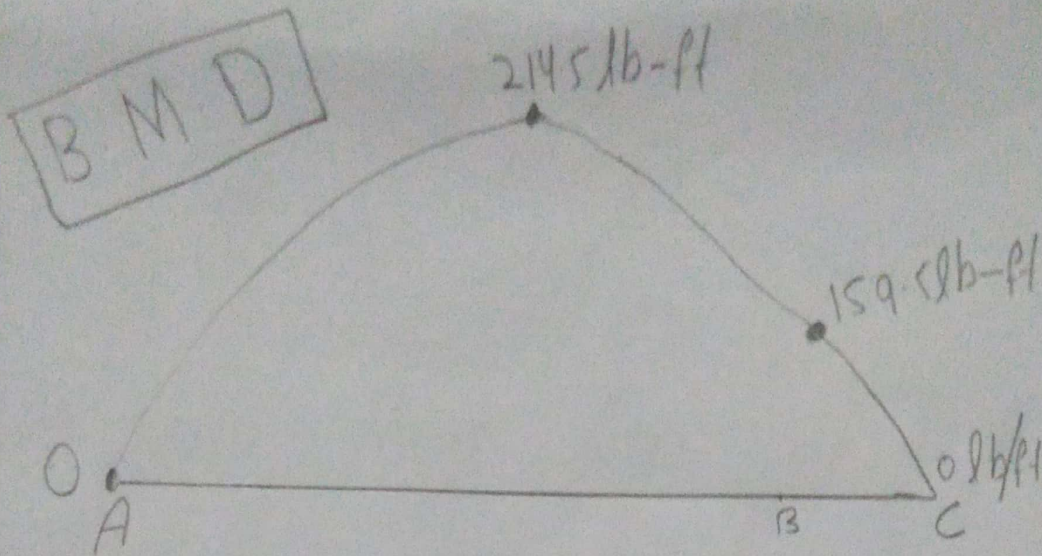
$$\Rightarrow R_1 = 185 - 79.75 \Rightarrow 85.25 \text{ lb}$$

Now draw Shear force and Bending moment diagram we have



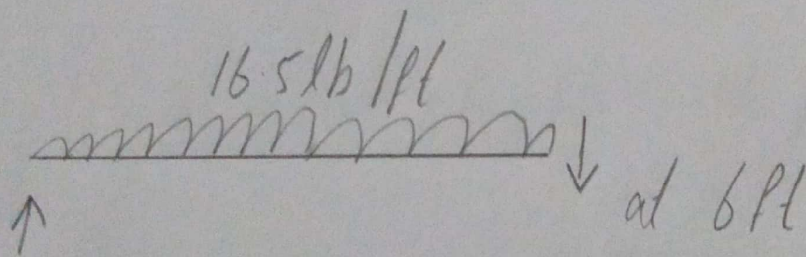


(4)



⇒

Now shear force at change  
point of Beam



Shear force at 6 ft from  
support

$$\sum F_y = 0 \quad + \uparrow \quad \downarrow -$$



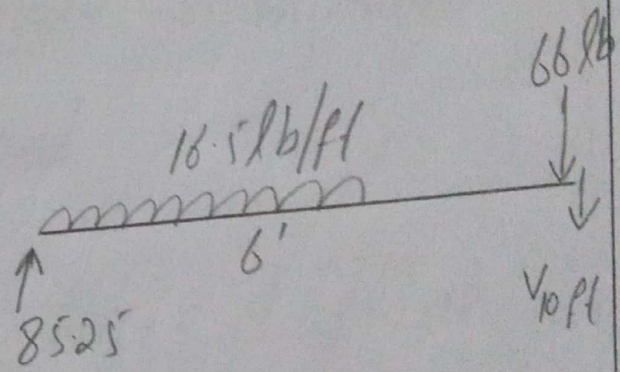
5

$$85.25 - 16.5 \times 6 - V_{6ft} = 0$$

$$\Rightarrow V_{6ft} = -13.5 \text{ lb}$$

⊖ Now Shear force at 10 ft

$$\sum F_y = \oplus \uparrow \quad \ominus \downarrow$$



$$85.25 - 16.5 \times 6 - 66 - V_{10ft} = 0$$

$$\Rightarrow V_{10ft} = -79.75 \text{ lb}$$

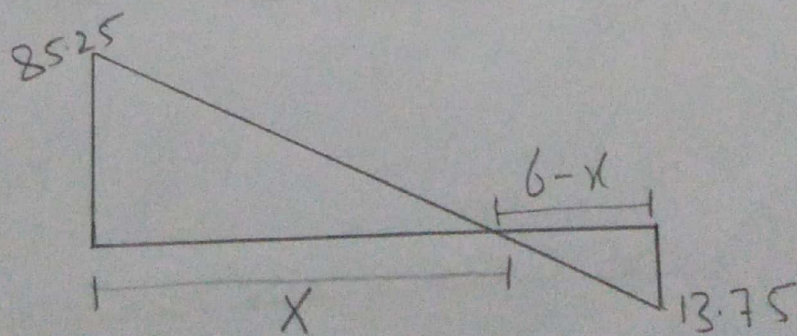


(6)

$\Rightarrow$  Point of maximum Bending moment

As we know that the point where shear force is ~~max~~ minimum the bending moment is maximum so from point of zero shear corresponding point will have maximum Bending moment

From shear force diagram on page 3 we have





(7)

We know that

$$\frac{85.25}{x} = \frac{13.75}{6-x}$$

$$\Rightarrow (6-x)85.25 = 13.75x$$

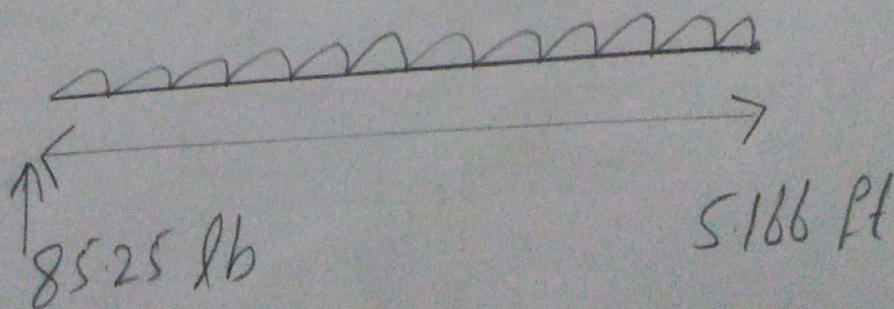
$$\Rightarrow 511.5 - 85.25x = 13.75x$$

$$\Rightarrow 511.5 = 13.75x + 85.25x$$

$$\Rightarrow 99x = 511.5$$

$$\Rightarrow \boxed{x = 5.166 \text{ ft}}$$

Now determine the value of moment at 5.166 ft





8)

$$M_{5.166} - 85.25 \times 5.166 + (16.5 \times 5.166) \times \left(\frac{5.166}{2}\right) = 0$$

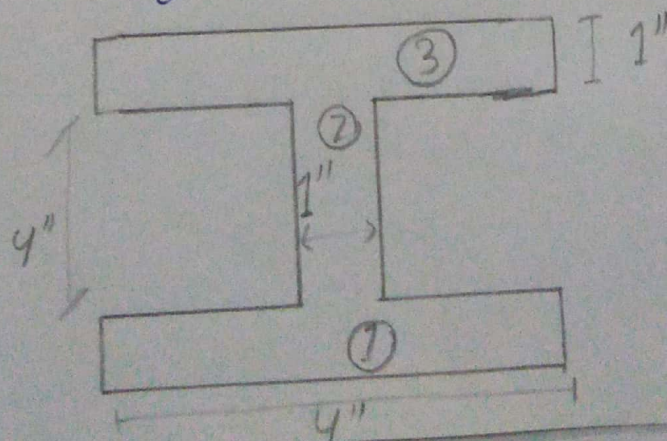
$$\Rightarrow M_{5.166} - 440.4015 + 215.50 = 0$$

$$\Rightarrow M_{5.166} \text{ ft} = 224.899 \text{ lb-ft}$$

For shear stress we have:

$$\tau = \frac{VQ}{Ib}$$

So first we determine moment of inertia  $I$  for the given section of beam





9

As the given figure is symmetrical along both the axes

$$\text{So } \bar{x} = \frac{4}{2} = 2 \text{ inch in } x$$

$$\bar{y} = \frac{6}{2} = 3 \text{ inch}$$

i.e.  $(\bar{x}, \bar{y}) = (2, 3)$  (centre of gravity)

extreme left and bottom

$$\text{Area of Point ①} = 4 \times 1 = 4 \text{ inch}^2$$

$$\text{Area of Point ②} = 4 \times 1 = 4 \text{ inch}^2$$

$$\text{Area of Point ③} = 4 \times 1 = 4 \text{ inch}^2$$

Moment of inertia about x-axis

(centroid I)  $I_{xx}$



10)

Determine distance b/w C.G.  
of the whole section and  
corresponding parts

Let

$G_1, G_2, G_3$  be in the centre  
of gravity of point ①, ②  
and ③ and  $k_1, k_2, k_3$  be  
the distance b/w  $\bar{y}$  and  
 $y_1, y_2, y_3$  respectively.

So

$$k_1 = \bar{y} - y_1 \Rightarrow 3 - 0.5 \Rightarrow 2.5 \text{ inch}$$

$$k_2 = \bar{y} - y_2 \Rightarrow 3 - 3 \Rightarrow 0 \text{ inch}$$

$$k_3 = \bar{y} - y_3 = 3 - 0.5 \Rightarrow 2.5 \text{ inch}$$



11  
So

$$I_{xx} = \frac{b_1 h_1^3}{12} + a_1 k_1^2 + \frac{b_2 h_2^3}{12} + a_2 k_2^2$$

$$+ \frac{b_3 h_3^3}{12} + a_3 k_3^2$$

$$I_{xx} = \frac{(4)(1)^3}{12} + 4(25) + \frac{(1)(4)^3}{12} + a_2(0) + \frac{4(1)^3}{12} + 4(25)$$

$$I_{xx} = \frac{4}{12} + 25 + \frac{64}{12} + \frac{4}{12} + 25$$

$$I_{xx} = \frac{4 + 12(25) + 64 + 4 + 12(25)}{12}$$

$$I_{xx} = 56 \text{ inch}^4$$

Now

$$I_{yy} = \frac{b_1^3 h_1}{12} + \frac{b_2^3 h_2}{12} + \frac{b_3^3 h_3}{12}$$

$$I_{yy} = \frac{(4)^3(1)}{12} + \frac{(1)^3(4)}{12} + \frac{(4)^3(1)}{12}$$



(12)

$$\bar{I}_{yy} = \frac{64}{12} + \frac{4}{12} + \frac{64}{12}$$

$$\bar{I}_{yy} = \frac{64 + 4 + 64}{12} = 11 \text{ inch}^4$$

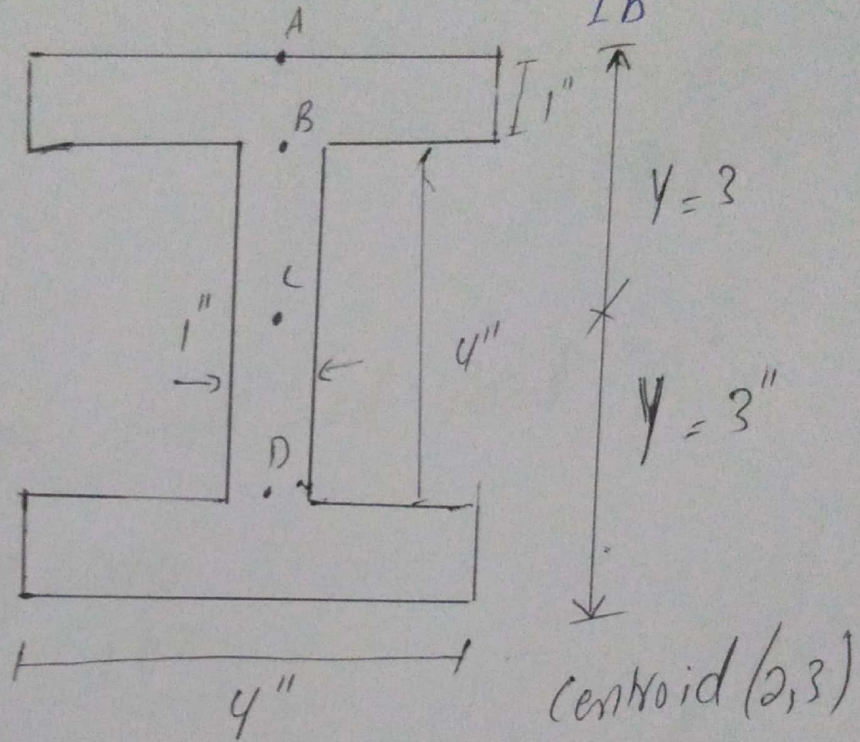


(13)

⇒ Now Find the Shear Stress

we know that

$$\tau = \frac{VQ}{Ib}$$



①

Shear stress at Point A

i.e. At the top of fibre

$$\tau = \frac{VQ}{Ib}$$

$$V_{\text{max}} = 79.75 \text{ lb}$$

$$I = 67 \text{ in}^4$$

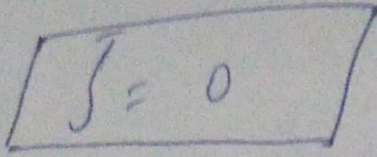
$$Q = A\bar{y}$$



(14)

So

$$\bar{J} = \frac{79.75 (0)}{67(4)}$$



Here  $A = 0$

Because no area of the section exist above

$$\text{ie } Q = A\bar{Y} =$$

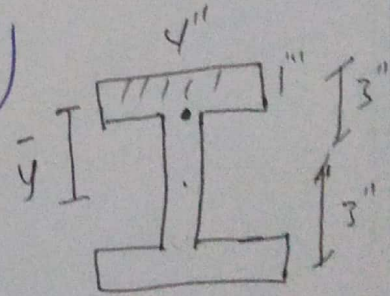
$$\Rightarrow Q(\bar{Y}) = 0$$

(11)

Shear stress at Point "B"

$$\tau = \frac{VQ}{Ib}$$

$$\tau = \frac{79.75 \times (4 \times 1) \times (3 - 0.5)}{67(4)}$$



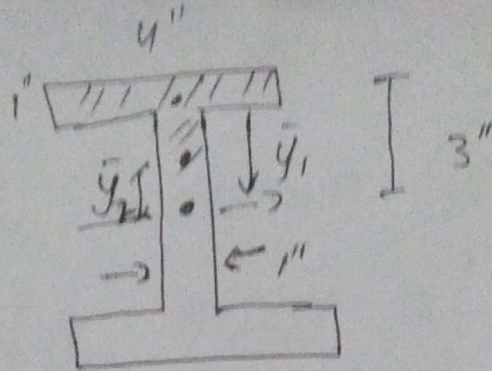
$$\tau = 2.97 \text{ lb/in}^2$$



(15)

(iii)

Now shear force at point "e"  
i.e. at N.A



$$\bar{J} = \frac{VQ}{It}$$

$$\bar{J} = \frac{79.75 \times [4 \times 1 \times (3 - 0.5) + (1 \times 2) \times (2 - 1)]}{67 \times 1}$$

$$\bar{J} = \frac{79.75 \times 12}{67}$$

$$\bar{J} = 14.28 \text{ lb/in}^2$$



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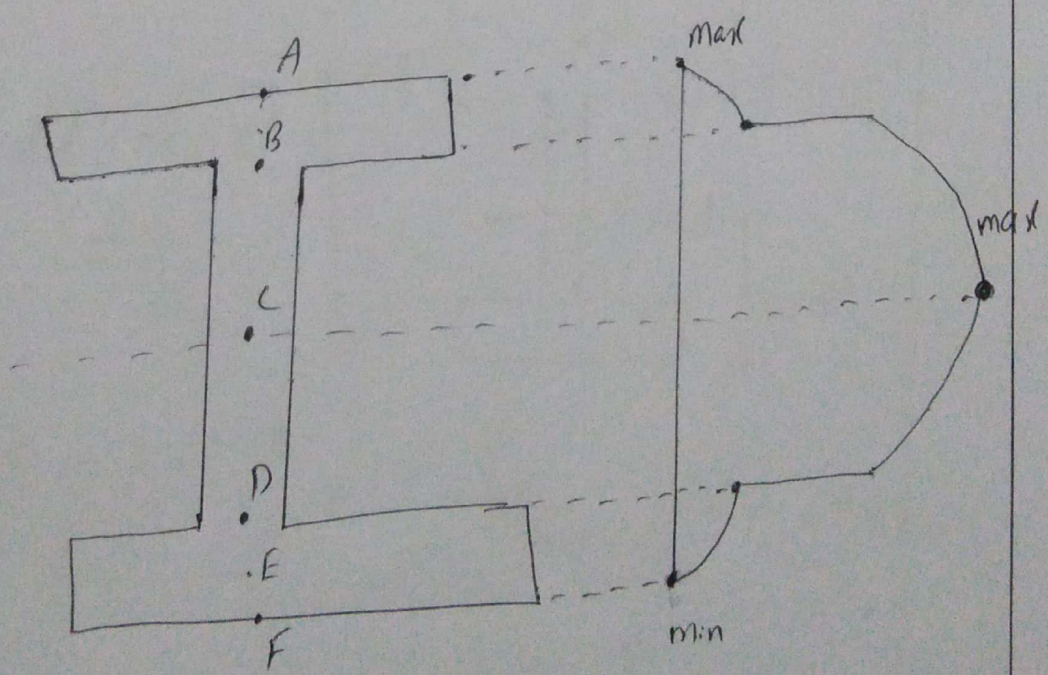
(iv)

Shear stress at point D and E will be the same.

Note:

The maximum shear stress value occur at the neutral axis and minimum value at the top of the section

Shear stress diagram





(17)

Flexural stress determination:

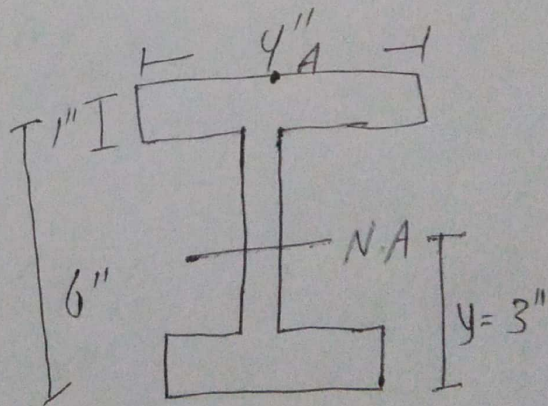
$$\sigma = \frac{M y}{I}$$

④

Flexural stress at the top  
at point "A"

$$\sigma = \frac{M y}{I}$$

$$\sigma = \frac{224.899(3)}{87}$$



$$\sigma = 10.07 \text{ lb/in}^2$$

$M_{\text{moment}}$

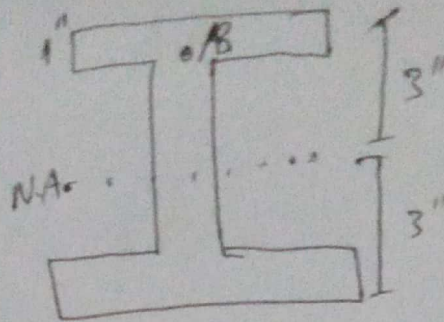
224.899



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(2)

Flexural Stress at Point B



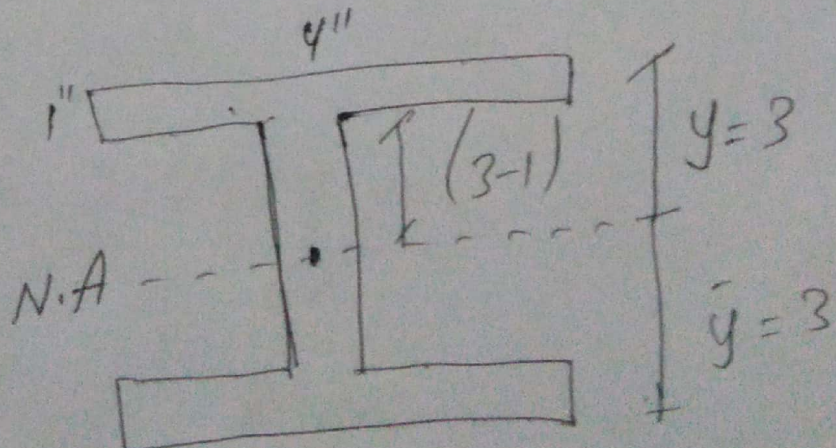
$$\sigma = \frac{My}{I}$$

$$\sigma = \frac{224.899 \times (3 - 0.5)}{67}$$

$$\sigma = 839 \text{ lb/in}^2$$

(III)

Flexure Stress at Point c





(19)

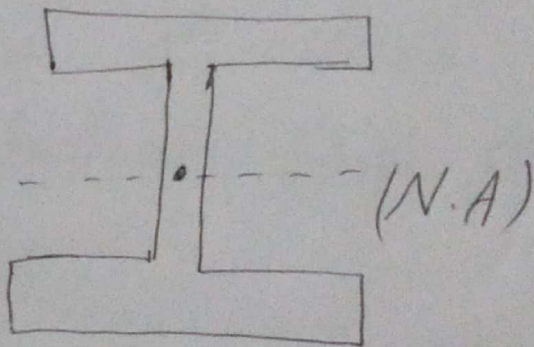
$$\sigma = \frac{My}{I}$$

$$\sigma = \frac{224.899 \times (3-1)}{67}$$

$$\sigma = 6.71 \text{ lb/in}^2$$

(iv)

Flexure stress at neutral axis  
(N.A)



$$\sigma = \frac{My}{I}$$

$$\sigma = \frac{224.899 \times 0}{67}$$

$$\sigma = 0 \text{ lb/in}^2$$



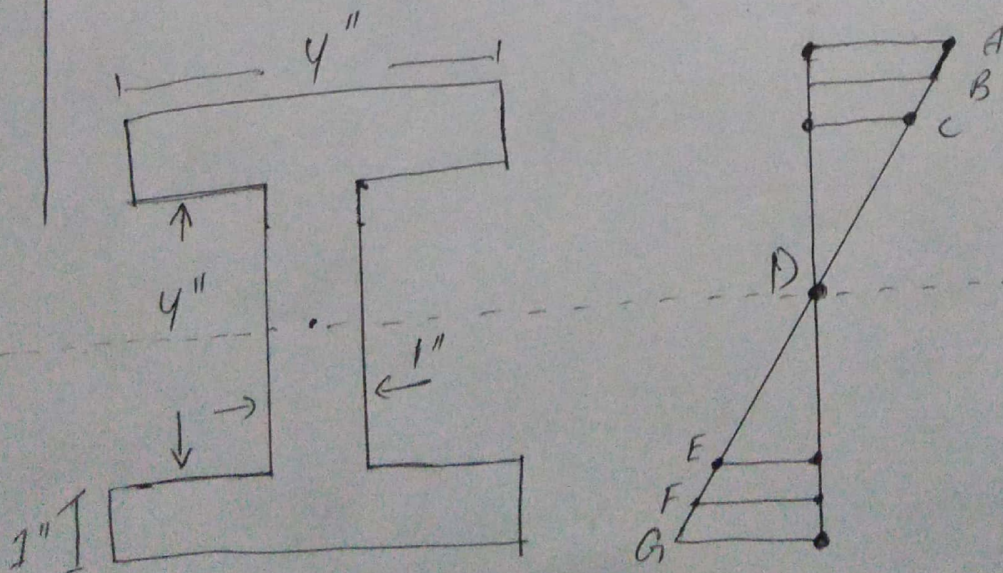
20

Flexure stress value at Point E, F and G Remain the same because of symmetry the upper portion above the N.A show tension and below the N.A shown compression.

Note

The flexure stress value is maximum at extreme top and bottom fibre at zero at N.A.

Flexure stress diagram.





## Stress State

Find stress state of a point  
C element located 3ft  
from left support and 2in  
below from top fibre.

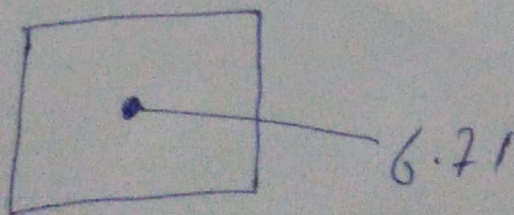
Flexural stress at point "C"

$$\sigma = 6.71 \text{ psi}$$

Shear stress at point "C"

$$\tau = 14.28 \text{ psi}$$

Consider point "C" is a  
planar element

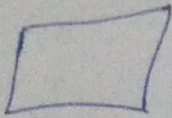


As the flexure  
stress is perpend-  
-icular to the  
cross section  
can be repre-  
-sented normal  
stress

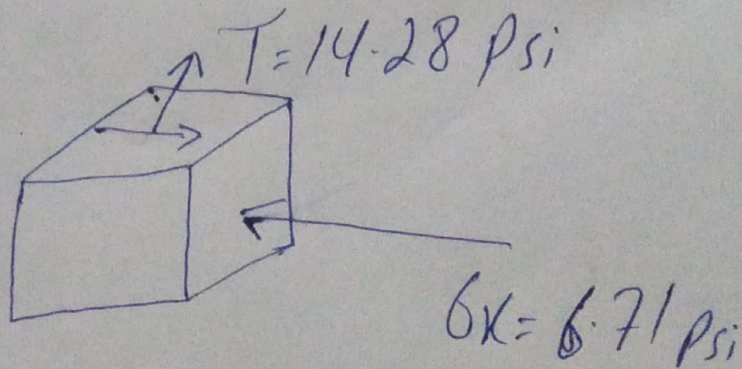
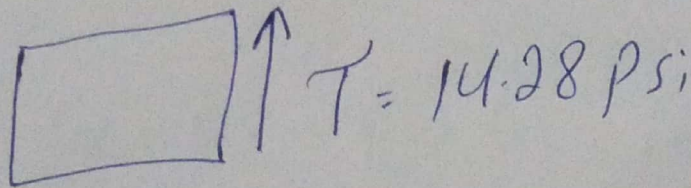


$\sigma = 14.28 \text{ psi}$  is compressive

Because point "C" lies in  
compression zone of beam cross  
section



If point C lies below the centroid  
then stress would be tensile.



Combine stress on 2D element.



21  
Find its principle stress

We have also find

~~600~~ =

$$\sigma_x = 6.71$$

$$\sigma_y = 0$$

$$\tau_{xy} = 14.28$$

Principle stress equation.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{-6.71 + 0}{2} \pm \sqrt{\left(\frac{-6.71 - 0}{2}\right)^2 + (14.28)^2}$$

$$\sigma_{1,2} = -3.355 \pm \sqrt{11.25 + 203.91}$$

$$\sigma_{1,2} = -3.355 \pm 14.66$$

$$\sigma_y = \sigma_1 = -3.355 + 14.66 = 11.305$$

$$\sigma_x = \sigma_2 = -3.355 - 14.66 = -18.015$$



or

First find  $\theta_p = ?$

$$\tan 2\theta_p = \frac{5 \times 4}{(6x - 6y)/2}$$

$$\tan 2\theta_p = \frac{14.28}{(-6.71 - 0)/2}$$

$$\tan 2\theta_p = -4.25$$

$$2\theta_p = \tan^{-1}(-4.25)$$

$$\theta_p = -38.37$$

Put in general equation

$$\sigma_{max} = \frac{-6.71 + 0}{2} + \frac{-6.71 + 0}{2} \cos 2(-38.37) + \frac{14.28}{2} \sin 2(-38.37)$$

$$\sigma_{p_{max}} = -3.355 - 3.355(0.2293)$$

$$\sigma_{p_{max}} = -3.355 - 0.7693 - 13.899$$

$$\sigma_{p_{max}} = -18.1243$$

$$\sigma_x = -6.71$$



22

Max in Plane Shear Stress

in this case

$$\tan 2\theta_s = \frac{-(6x - 6y)/2}{\tau_{xy}}$$

$$\tan 2\theta_s = \frac{-(-6.71 - 0)/2}{(14.28)}$$

$$\tan 2\theta_s = \text{---}$$

$$\theta_s = 6.67 \text{ Anti clock wise}$$

Put in these general equation

$$\sigma_{x'y'} = -\left[\frac{6x - 6y}{2}\right] \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{x'y'} = -\left[\frac{-6.71 - 0}{2}\right] \sin 2(6.67) +$$

$$14.28 \cos 2(6.67)$$



Scale:

$$\tau_{xy} = 14.66 \text{ Psi}$$

Max in Plane  
Shear Stress.

To Draw Mohr's circle:

Centre co-ordinate

$$(h, k) = \left( \frac{\sigma_x + \sigma_y}{2}, 0 \right)$$

$$\Rightarrow \left( \frac{-6.71 + 0}{2}, 0 \right)$$

$$\Rightarrow (-3.355, 0)$$

Radius of Mohr's circle.

$$r = \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} = \sqrt{\left( \frac{-6.71 - 0}{2} \right)^2 + (14.66)^2}$$

$$r = 215.1684$$



Scale

1 Psi = 1 cm

Mohr's Circle

