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Subject # learner Algebra

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Q no 1
(A part)

The non parallel vector

$$\overline{P_1 P_2} = (-3, 2, 3)$$

$$P_1 P_2 = (3, -1, 3)$$

$$\begin{aligned} & (-1, 0, 3) - (2, -2, 1) \\ & (-1, 0, 3) - 2, +2, -1 \\ & (-3, +2, 2) \end{aligned}$$

The Perpendicular vector is :-
 $m = \overline{P_1 P_2} \times P_1 P_2$

$$m = \begin{vmatrix} i & j & k \\ -3 & 2 & 2 \\ 3 & -1 & 3 \end{vmatrix}$$

$$\frac{\overline{P_1 P_2}}{P_1 P_2} = \frac{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}}{\sqrt{(-1-2)^2}}$$

$$\begin{aligned} m &= i(6+2) - j(-9-6) + k(3-6) \\ m &= 8i + 15j - 3k \\ m &= (2, 15, -3) \end{aligned}$$

Now $P_1(x_0, y_0, z_0) = (2, -3, 1)$

$$m(a, b, c) = 8, 15, -3$$

So equation of Plane is

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$
$$8(x-2) + 15(y+2) - 3(z-1) = 0$$

$$8x + 15y - 3z - 16 + 30 + 3 = 0$$

$$8x + 15y - 3z + 17 = 0$$

Q no (4)
(Part b)

$$\begin{aligned}x &= 2 - 3t \\y &= 3 + t \\z &= 2 - 4t\end{aligned}$$

Solution:-

$$z - 2 = -4t \Rightarrow t = \frac{z - 2}{-4}$$

$$y - 3 = t \Rightarrow t = \frac{y - 3}{1}$$

$$z - 2 = -4t \Rightarrow t = \frac{z - 2}{-4}$$

$$\text{So } \frac{x - 2}{-3} = \frac{y - 3}{1} = \frac{z - 2}{-4}$$

for 1st plane takes
1st and 2nd

$$\frac{x - 2}{-3} = \frac{y - 3}{1}$$

$$x - 2 = -3y + 9$$

$$x + 3y - 11 = 0$$

for 2nd plane take 1st and 3rd

$$\frac{x - 2}{-3} = \frac{z - 2}{-4}$$

$$-4x + 8 = -3z + 6$$

$$-4x + 3z + 2 = 0$$

or

$$4x - 3z - 2 = 0$$

Q NO (2)

$$L(x, y) = (x+1, y, x+y)$$

Solution:

$$L(x, y) = (x+1, y, x+y)$$

$$\text{Let } u = (x_1, y_1) \quad v = (x_2, y_2)$$

$$u+v = (x_1, y_1) + (x_2, y_2)$$

$$u+v = (x_1+x_2, y_1+y_2)$$

$$L(u+v) = L(x_1+x_2, y_1+y_2)$$

$$L(u+v) = (x_1+x_2+1, y_1+y_2, x_1+x_2+y_1+y_2)$$

$$\text{given that } u = (x_1, y_1)$$

$$L(u) = L(x_1, y_1) = (x_1+1, y_1, x_1+y_1)$$

$$L(v) = L(x_2, y_2) = (x_2+1, y_2, x_2+y_2)$$

$$L(u) + L(v) = (x_1+x_2+2, y_1+y_2, x_1+x_2+y_1+y_2)$$

Since $1 \neq 2$

Q no 4) $(-1, 3, 2)$ $n = (0, 1, -3)$

Solution:

Eqn of plane
 $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Given that

$$P = (x_0, y_0, z_0) = (-1, 3, 2)$$

$$n = (a, b, c) = (0, 1, -3)$$

So, $0(x - (-1)) + 1(y - 3) - 3(z - 2)$

$$0 \quad \underline{x+1} + 1(y-3) - 3(z-2)$$

$$0 + y - 3 - 3z + 6$$

$$\Rightarrow y - 3z - 3 + 6$$

$$\Rightarrow y - 3z + 3 \text{ Ans}$$

Q No

Sol:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

So

$$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ -2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} 0 & 1 & -1 \\ 0 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = \begin{bmatrix} 0 + 54 - 38 \\ 154 + 108 - 38 \\ 77 + 54 + 38 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} 16 \\ 8 \\ 15 \end{bmatrix}$$

$$X_2 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix} = \begin{bmatrix} 20 \\ 15 \\ 7 \end{bmatrix}$$

$$X_3 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix} = \begin{bmatrix} 18 \\ 1 \\ 16 \end{bmatrix}$$

$$X_4 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix} = \begin{bmatrix} 1 \\ 15 \\ 8 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix} = \begin{bmatrix} 1 \\ 15 \\ 18 \end{bmatrix}$$

Q. (5)
QNO

Given Value

Sol:

We know that $Ax = \lambda x$

$$\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

then
$$\begin{bmatrix} x_1 + x_2 \\ -2x_1 + 4x_2 \end{bmatrix} = \begin{bmatrix} \lambda x_1 \\ \lambda x_2 \end{bmatrix}$$

$$x_1 + x_2 = \lambda x_1 \quad \text{--- (i)}$$

$$-2x_1 + 4x_2 = \lambda x_2 \quad \text{--- (ii)}$$

So

$$\begin{aligned} x_1 - \lambda x_1 + x_2 &= 0 \\ &= (1 - \lambda) x_1 + x_2 = 0 \end{aligned}$$

$$-2x_1 + 4x_2 - \lambda x_2 = 0$$

$$= -2x_1 + (4 - \lambda) x_2 = 0$$

$$\begin{bmatrix} 1 & -\lambda & 1 \\ -2 & 4 & -\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\Rightarrow 2x_1 - x_2 = 0$$

$$\Rightarrow -2x_1 + 4x_2 = 3x_2 \quad \text{--- (ii)}$$

$$\Rightarrow -2x_1 + x_2 = 0$$

$$\Rightarrow 2x_1 - x_2 = 0$$

$$x_1 = \frac{1}{2} x_2$$

let $x_2 = x$

Where $\gamma \neq 0$

$$\text{So } x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \gamma_2 \gamma \\ \gamma \end{bmatrix}$$

eigen vector for $\lambda_2 = -2$ put
in eq 2

$$x_1 + x_2 = 2x_1 \quad \text{---(i)}$$

$$-2x_1 + 4x_2 = 2x_2 \quad \text{---(ii)}$$

$$= -x_1 + x_2 = 0$$

$$\Rightarrow \cancel{x_1} - x_2 = 0$$

$$\Rightarrow x_1 = x_2$$

$$= -2x_1 + 4x_2 = 2x_2 \quad \text{---(ii)}$$

$$= -2x_1 + 2x_2 = 0$$

$$x_1 - x_2 = 0$$

$$x_1 = x_2$$

$$x_1 = \gamma \quad \text{then} \quad x_2 = \gamma$$

$$\text{So } x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \gamma \\ \gamma \end{bmatrix}$$