Name: SAAD
Class: BS Software Engineering
ID no: 15889
University Name: Iqra ~~National~~ University
Linear Algebra

Q1: Consider the given below matrix as the augmented matrix of a linea system. Explain in your own words the next elementary row operation that should be performed in order to solve this system. Where ID_3 is the 3rd digit in your ID and ID - last is the last digit of your ID in inverse. e.g if your ID is 12345 than ID last = -5

$$\begin{bmatrix} 1 & ID_3 & 3 & 0 & 5 \\ 0 & 1 & -ID\,last & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID_3 \end{bmatrix}$$

SOLUTION

My ID is 14672

Given Matrix be A

$1x + 6y + 3z + 0t = 5$
$0x + 1y - 2z + 0t = 7$
$0x + 0y + 1z + 0t = -6$
$0x + 0y + 0z + 1t = 6$

$$\begin{bmatrix} 1 & 6 & 3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ -6 \\ 6 \end{bmatrix}$$

let the argumented Matrix be:

$$A = \left[\begin{array}{cccc|c} 1 & 6 & 3 & 0 & 5 \\ 0 & 1 & -2 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

$R_1 \sim R_1 - 6R_2$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 15 & 0 & -37 \\ 0 & 1 & -2 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

$R_2 \sim R_2 + 2R_3$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 15 & 0 & -37 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

$R_1 \sim R_1 - 15R_3$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 53 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

Hence

So we get the values of x, y, z and t

$x = 53$

$y = 5$

$z = -6$

$t = 6$

Q2a Find the elementary row operation that transform the first matrix into second and reverse row operation that transforms the 2nd Matrix into first

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

SOLUTION

let first matrix be A

let second matrix be B

Elementary Row Operation

$$A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

$$R_3 \sim R_3 - 2R_2$$

$$A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Reverse Row Operation

$$B = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$$R_3 \sim 2R_2 + R_3$$

$$B = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

Hence Proved

Q1b
Given below are some matrices. Find whether these are in the forms written in front of them or not. Explain in your own words for each of in detail

(a) $\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$ is in echelon form

SOLUTION

let $A = \begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$

Yes matrix A is in echelon from because of its definition as echelon from of a matrix states that "If a column contains a leading entry then all enteries below that leading entry are zero.
In matrix A, it satisfies the definition of echelon from of a matrix so it is in echelon form.

(b) $\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in echelon form

SOLUTION

let $B = \begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Yes matrix B is in echelon form because of its definition which states that "If a column contains a leading entry then all entries below that leading entry are zero" According to defination matrix B is columns contain leading entries as 1 and below that all enteries are zero so matrix B is in echelon form

(c) $\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form

SOLUTION

let $C = \begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

$C = \begin{bmatrix} 5/5 & 0/5 & 0/5 & 7/5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix} \quad R_1/5$

$C = \begin{bmatrix} 1 & 0 & 0 & 7/5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

Yes, matrix C is in reduced row echelon form, because reduced row echelon from state that " In reduced row echelon from the leading co-efficent must be 1 in each row is to the right of the leading co-efficent in the row above it" According to defination matrix C satisfy the defination properties so it is in reduced row echelon form

d) $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form

SOLUTION let $D = \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

No, matrix D is not in reduced row echelon form because if a matrix is in reduced row echelon form then its rows (non zero) contains its first enteries as a no "1" which is known as leading 1 i.e The first non zero entry is 1. Also if there are any rows containing only zero enteries than they are located in the bottom part of the matrix but in matrix D the zero row is located in mid of matrix, so it is not in reduced row echelon form

Q3(a):- The row echelon form is used to solve the system of linear equation What is the difference between the row echelon and reduced row echelon form? What is practical use of reduced row echelon form? Give Example

Answer

DIFFERENCE BETWEEN ROW ECHELON AND REDUCED ROW ECHELON FORM

ROW ECHELON FORM	REDUCED ROW ECHELON
Row Echelon form of a matrix is defined as the leading entry in row echelon form in each row (column) is the only non zero entry	Reduced Row Echelon form is defined as In reduced row echelon form the left-most non-zero entry of a row is

in its row (column)

(2) Echelon form of a matrix isn't unique which means there are infinite answers possible when we perform row reduction or elementary operation

(3) Each row containing a non-zero number has the number 1 appearing in the rows first non zero column. Such entry will be known as "leading entry / one".

(4) The entries only below the first leading non zero entry that must be zero not necessary for above one's

equal to 1. The leftmost non zero entry of a row is the only non zero entry in its column.

(2) Reduced Row Echelon form is unique which means when we apply elementary row operation on a matrix it will produce the same answers, no matter how we perform the same row operation

(3) In reduced row echelon form the left most non zero entry of a row is equal to 1. The leftmost nonzero entry of a row is the only non zero entry in its column

(4) The enteries above and below the first in 1 in each row must all be 0

Example

$$\begin{bmatrix} 1 & 6 & 2 & -8 \\ 0 & 1 & 14 & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & -2 & 0 & 6 \\ 0 & 1 & 7 & 0 & 1 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$$

PRACTICAL USE OF REDUCED ROW ECHELON FORM:-

1. This type of matrix is used to solve system of linear equation.
2. Reduced Row Echelon form used in balancing chemical equations
3. Such matrix is used to solve computer operations

A Example of Reduced Row Echelon Form:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$