"FINAL TERM SUMMER"

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ID : 7812

SEC : "A"

SUBJECT : ADVANCED FLUID MECHANICS

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Q No: 1 :-

Part (A) :- Drag :-

Drag is a Force acting opposite to the relative motion of an object moving with respect to a Surrounding Fluid is Called Drag.

Drag Force on Submerged body Can have Two Components

1. pressure Drag (F_p) :- It is equal to the Integration of Components in The direction of motion of all pressure force exerted on Surface of The body

$$F_p = C_p \rho \frac{v^2}{2} A$$

$\because C_p$ - depend on shape

2. Friction Drag :- It is equal to integration of Component at all Shear Stress along The Surface of The body in direction of motion

$$F_p = C_f \rho \frac{v^2}{2} BL$$

where,

$C_f \rightarrow$ depends on Viscosity

~~Friction Drag at Boundary Layer~~

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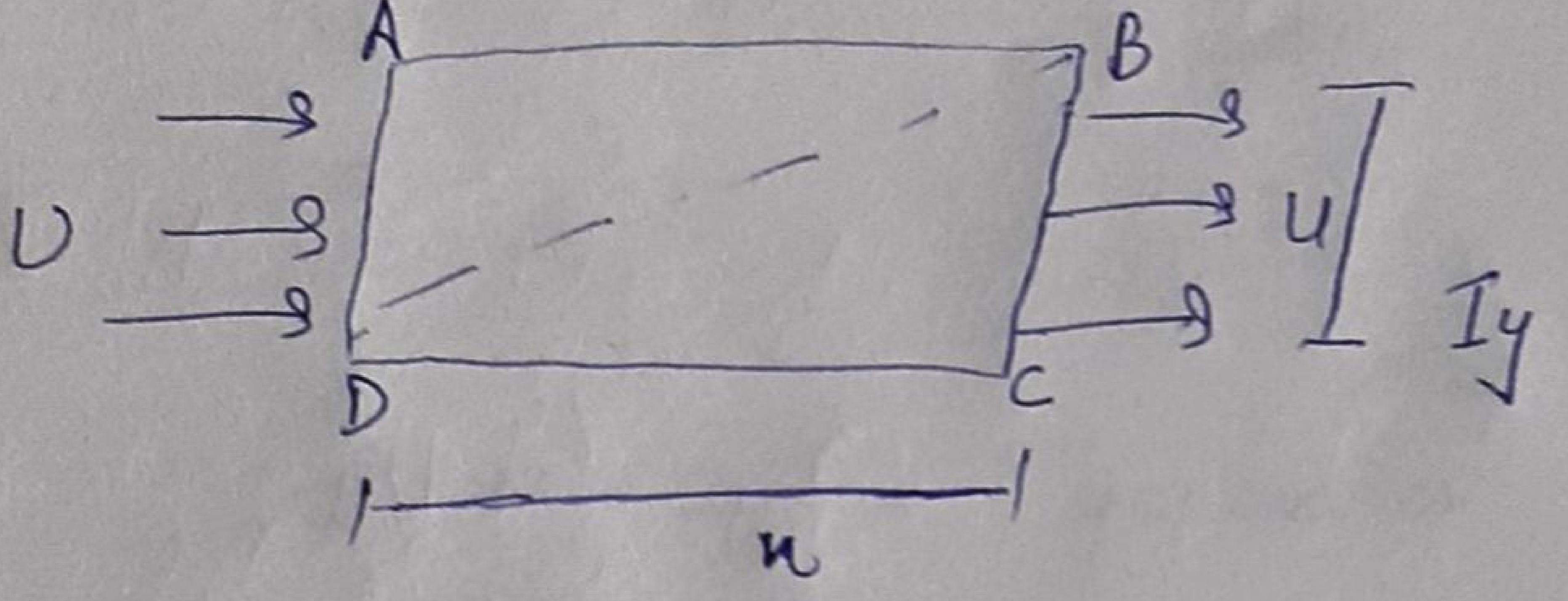
Figure Shows growth at Boundary Layer along one Side at Smooth plate in Steady Flow of incompressive Fluids Consider volume where δ is The Thickness of boundary Layer & U is undisturbed velocity.



As we have $\Sigma F_x = 0$

where

$$F_x = \frac{\Delta P}{\Delta t} = \frac{\Delta m v}{\Delta t} \quad \because m = \rho V$$

$$F_x = \frac{\Delta \rho . vol . v}{\Delta t} = \Delta \rho Q v$$

$$F_x = \Delta \rho Q v$$

$-F_x$ = rate of Chang ohm BC + AB - AD

$$AD = \rho U (U \delta B)$$

$$BC = \rho B (u^2 dy)$$

$$AB = \rho U (UB\delta) - B \int_0^\delta u dy$$

$$F_x = \rho B \int_0^\delta u (U - u) dy \quad \text{— (1)}$$

Integration b/s — (1)

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$$F_x = \int B U^2 \delta d$$

where "d" is a fention of boundary layer velocity distribution

Now to find Shear Stress

$$\tau = \frac{f_x}{A} = \frac{dF_x}{B\,dx} = \frac{d\,f_x}{B\,dx}$$

$$\tau_o = \frac{\int B U^2 d\, d\delta}{B\,dx} = \int U^2 \alpha \frac{d\delta}{dx}$$

$$\tau_o = \int U^2 \alpha \frac{d\delta}{dx} \quad - * \text{ generater}$$

Laminar Boundary Layer:

$$\frac{u}{U} = f\left(\frac{y}{\delta}\right) \quad - ①$$

$$\frac{y}{\delta} = \eta \Rightarrow y = \delta\eta$$

$$dy = \delta d\eta \quad - ②$$

$$\frac{u}{U} = f(\eta)$$

$$du = U df(\eta) \quad - ③$$

For Laminar Flow

$$\tau_o = \mu \frac{du}{dy} \quad - ④$$

$$\tau_o = \frac{\mu U df(\eta)}{\delta d(\eta)}$$

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$$\tau_0 = \frac{\mu U B}{\delta} \quad \text{--- } (5)$$

As we have $\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$

Compare ~~we~~ both

$$\rho U^2 \alpha \frac{d\delta}{dx} = \frac{\mu U B}{\delta}$$

$$\delta d\delta = \frac{\mu B}{\rho U \alpha} dx$$

Integrating on b/s

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho U \alpha} x + C \qquad \because C = 0$$

$$\delta = \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu}{\rho U} x}$$

$$\because R_n = \frac{U x 1}{\mu}$$

$B = 1.63$, $\alpha = 0.135$

$$\delta = \frac{4.91}{\sqrt{R_n}} x \quad \text{---} \quad (6)$$

where (R_n) is Local Rynold number

As we have

$$\tau_0 = \frac{\mu U B}{\delta} \quad \text{---}$$

put (6) in eq (5)

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$$\boxed{\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}}$$

Now

$$\boxed{F_f = B \int_0^{\delta} \tau_0 \, dx}$$

where $\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}$

$$\boxed{\because R_x = \frac{x U \rho}{\mu}}$$

Then putting values we have

$$\boxed{F_f = 0.664 \sqrt{\rho \mu L U^3}}$$

As we have

$$F_f = C_f \rho \frac{V^2}{2} BL$$

Thus equating b/s

$$C_f = 1.328 \sqrt{\frac{\mu}{\rho L U}} = \boxed{\frac{1.328}{\sqrt{R_x}}}$$

For Laminar $R < 500{,}000$

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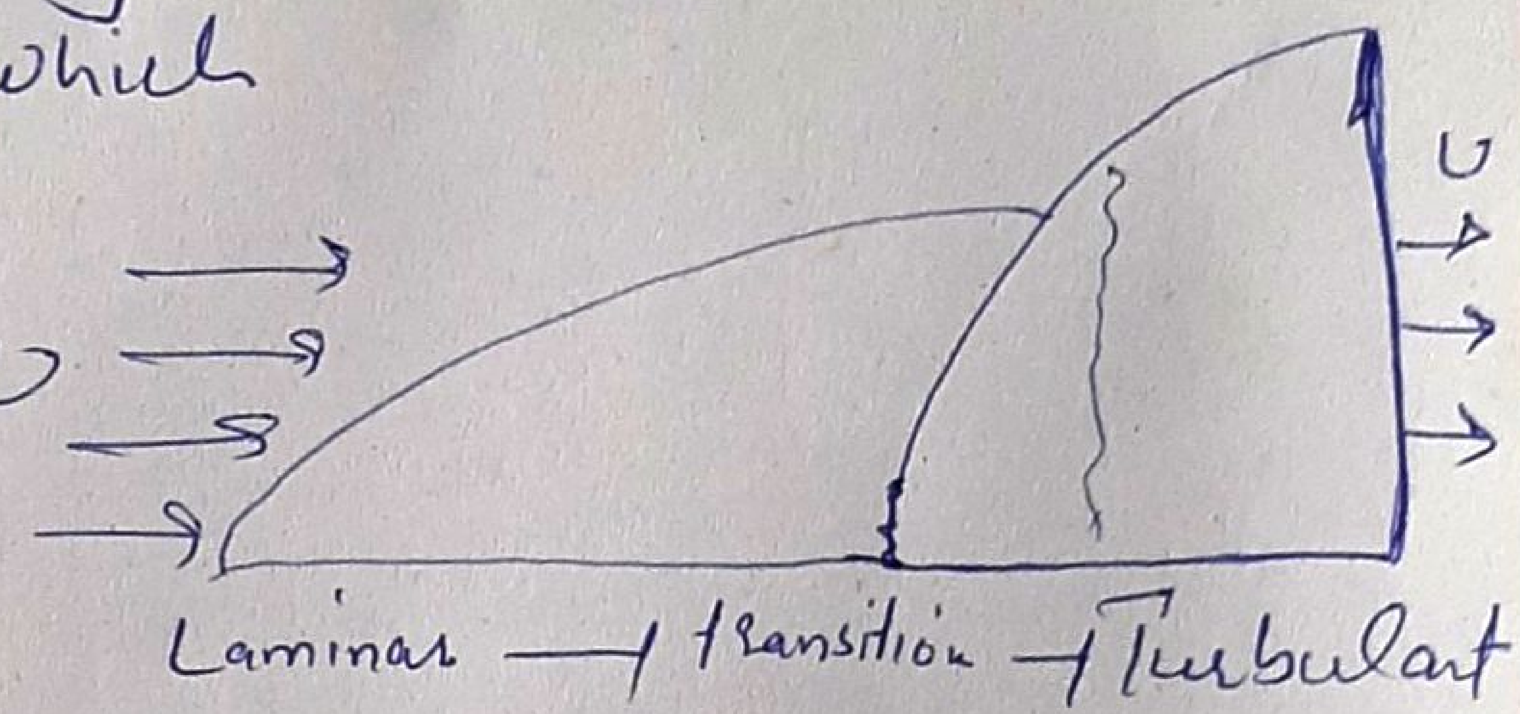
Turbulant Boundary Layer :-

This fig Show the velocity distribution of boundary Layer which is Steeper near walls & flatter Throughout Reminder at Layer



The Shear Stress is greater in turbulent Than in Laminar Thus

$$\tau_0 = \rho f \frac{v^2}{8}$$

where v is the average velocity to obtain relation b/w average and max we have

$$\frac{V}{U_{max}} = \frac{1}{1 + 1.33\sqrt{f}} \qquad = f = 0.028$$

$$\frac{V}{U_{max}} = \frac{1}{1 + 1.33\sqrt{0.028}}$$

$$U = 1.235V$$

$$\boxed{V = \frac{U}{1.235}}$$

&

$$f = \frac{0.316}{(R_n)^{3/4}} \qquad \because R_n = \left(\frac{DV}{\nu}\right)$$

$$D = 2\delta$$

$$\tau_0 = f\rho \frac{v^2}{8}$$

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$$\tau_o = \frac{0.316}{\left(\left(\frac{D}{\nu}\right)\left(\frac{U}{1.235}\right)\right)^{1/4}} \cdot \frac{1}{8}\left(\frac{U}{1.238}\right)^2$$

$$\tau_o = \frac{0.023 \rho U^2}{\left(\frac{2\delta}{\nu}\right)^{1/4}} \quad \text{—} \quad (1)$$

As we have general eq

$$\tau_o = \rho U^2 \times \frac{d\delta}{dx} \quad \text{—} (2)$$

equation (1) & (2)

$x = 0 \qquad \delta = 0$

$$\boxed{\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5}\left(\frac{\nu}{Ux}\right)^{3/5} \cdot x}$$

$d = 0.0972$

$$\boxed{\delta = \frac{0.377}{(R_x)^{2/5}} \cdot x} \quad \text{—} (3)$$

$$\boxed{\tau_o = 0.0587 \rho \frac{U^2}{2}\left(\frac{\nu}{Ux}\right)^{1/5}}$$

Now

$$F_f = B\int_0^L \tau_o \, dx$$

$$\boxed{F_f = 0.0735 \rho \frac{U^2}{2}\left(\frac{\nu}{UL}\right)^{1/5} \cdot BL}$$

$$F_f = C_f \cdot \rho \frac{U^2}{2} \cdot BL$$

Equating b/s

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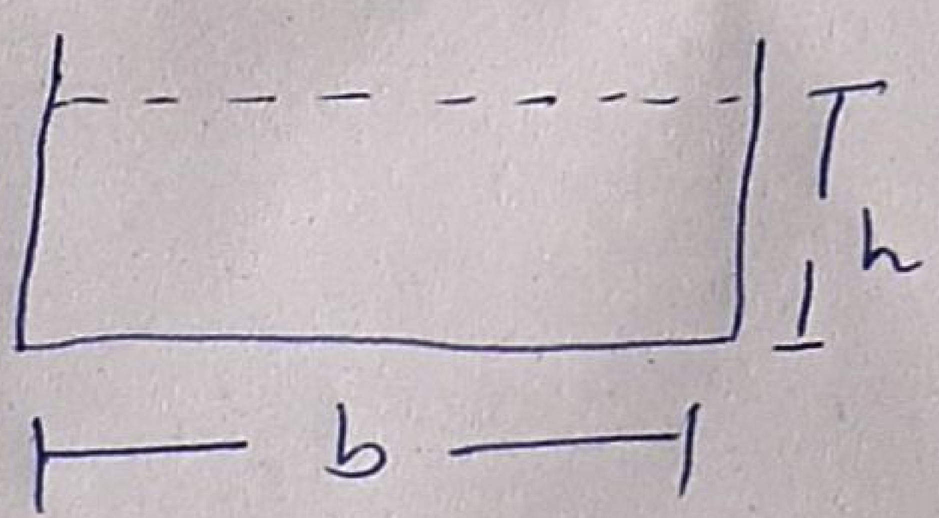
$$\boxed{C_f = \frac{0.0735}{(R)^{1/5}}} \qquad (5000,00 < R < 10^7)$$

For

& $R > 10^7$

$$\boxed{C_f = \frac{0.455}{(\log_R)^{2.52}}}$$

Part B :-

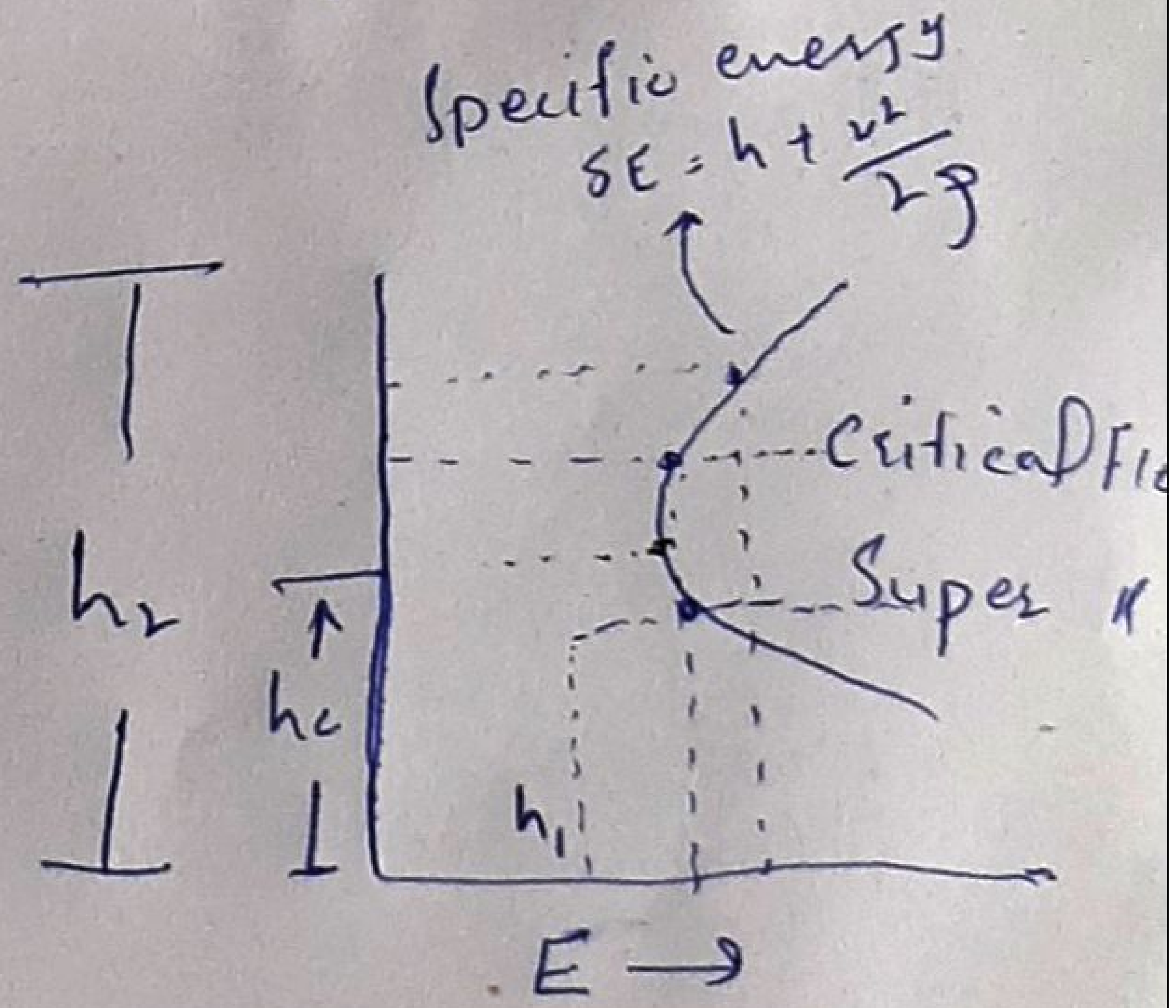


A rectangular channel having bridth (b) and height (h)

Area = $b \times h$

Specific energy equation

$$E = h + \frac{v^2}{2g} \text{ (kinetic energy)} \quad \text{—(1)}$$



As we know that

h_1 and h_2 = Alternate height

hc = Critical Depth

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$$Q = AV$$

or

$$V = \frac{Q}{A}$$

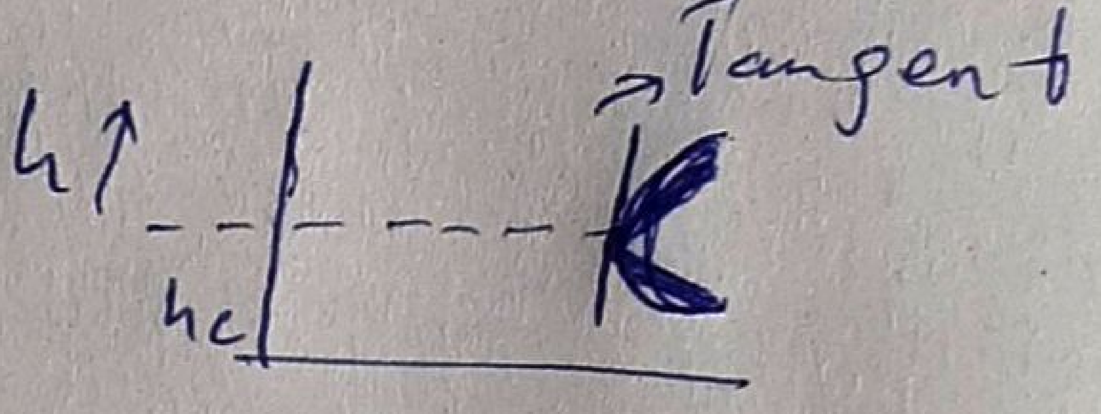
put $V = \frac{Q}{A}$ in eq (1)

we get

$$E = h + \frac{Q^2}{A^2 \cdot 2g} \qquad \text{As } A = b \times h$$

$$E = h + \frac{Q^2}{b^2 h^2 \cdot 2g} \qquad \text{As } q = \frac{Q}{b}$$

$$E = h + \left(\frac{Q}{b}\right)^2 \cdot \frac{1}{h^2 \cdot 2g}$$



So $$E = h + \frac{q^2}{2g \cdot h^2} \longrightarrow (2)$$

drive equation (2) w.r.to "h"

$$\frac{dE}{dh} = \frac{d}{dh}\left(h_c + \frac{q^2 \cdot h_c^{-2}}{2g}\right)$$

put slope = 0 (Tangent)

→ For minimum energy the height will be Critical

(For minimum Specific energy)

$$\frac{dE}{dh} = 1 + \frac{(-2)\, q^2 h_c^{-2-1}}{2g}$$

$$0 = 1 - \frac{\cancel{2} q^2 \cdot h_c^{-3}}{\cancel{2} g} \qquad \text{As } \frac{dE}{dh} = 0$$

$$0 = 1 - \frac{q^2}{h_c^3 \cdot g}$$

or $\frac{q^2}{h_c^3 \cdot g} = 1$

$h_c^3 \cdot g = q^2$

$h_c^3 = \frac{q^2}{g}$

or $\boxed{h_c = \left(\frac{q^2}{g}\right)^{1/3}}$ Critical depth

Now for Critical velocity 'V_c'

As $h_c^3 = \frac{q^2}{g} \rightarrow (3)$

by C.M

$h_c^3 g = q^2$

Now,

$\frac{q \times b}{A} = V$

$\frac{q\cancel{b}}{\cancel{b} \times h_c} = V$

So $V = \frac{q}{h_c}$

$q = V h_c$

As

$q = \frac{AV}{b}$

$qb = AV$

or $V = \frac{qb}{A}$ As $A = b \times h_c$

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From equation 3

$$q^2 = h_c^3 \cdot g$$

put $q = V_c h_c$

$$V_c^2 \cdot h^2 = h_c^3 \cdot g$$

or

$$V_c^2 = \frac{h_c^3 \cdot g}{h_c^2}$$

$$V_c^2 = g \cdot h_c$$

or

$$\boxed{V_c = \sqrt{g \cdot h_c}}$$ Critical velocity

Therefore

Critical Depth $(h_c) = \left(\frac{q^2}{g}\right)^{1/3}$

Critical velocity "V_c" $= \sqrt{g \cdot h_c}$

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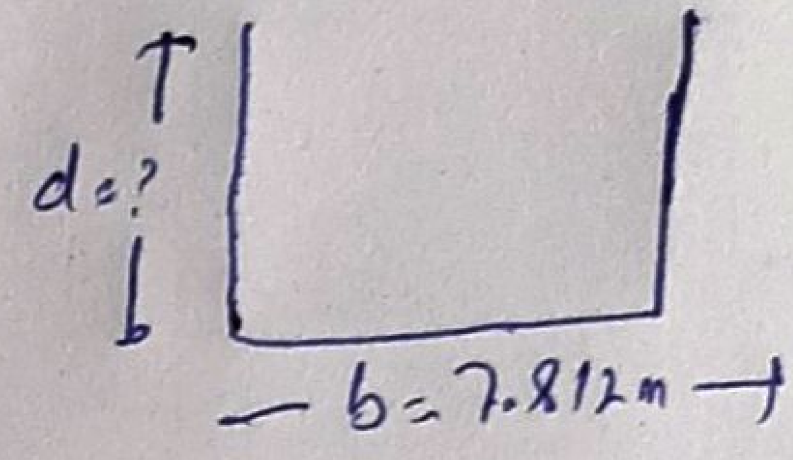
QNo : 02 :-

Given Data



Flow rate $Q = 3.5 m^3/s$

bed slop $S_0 = 0.0008$

$n = 0.0219$

width of bed = 7812mm (ID number)

$= \frac{7812}{1000} = 7.812m$

Required

depth of Rectangular channel (d) = ?

Critical Depth $h_c = ?$

Critical velocity = $V_c = ?$

Sol:- Area of Rentangular Channel

$A = b \times d$

$A = 7.812 . d$ — (1)

and perimeter

$= 2d + b$

$P = 7.812 + 2d$ — (2)

As hydraulic Radius $R_h = \frac{\text{Area}}{\text{perimeter}} = A/P$

$$= \frac{7.812d}{7.812+2d}$$

Now using Maning equation

$$Q = \frac{1}{n} A R_n^{2/3} . S_0^{1/2}$$

by putting values

$$3.8 = \frac{1}{0.0219} \times 7.812 \times d \times \left(\frac{7.812d}{7.812+2d}\right)^{2/3} \times \left(0.0008\right)^{1/2}$$

$$\boxed{d = 0.5589m}$$

put This value in equation (1)

$A = 7.812 \times 0.5589$

$A = 4.366m^2$

Now perimeter

$P = 7.812 + 2 \times 0.5589$

perimeter $P = 8.93m$

Now hydraulic Radius

$R_h = A/P$ by putting The values.

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$$R_h = \frac{4.366}{8.93}$$

$$R_h = 0.4889 m$$

Now Critical Depth

$$h_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$As = q = \frac{Q}{B} = \frac{3.5}{7.812} = 0.448 m^3/sec$$

Now,

$$h_c = \left(\frac{(0.448)^2}{9.81}\right)^{1/3} \quad As \; g = 9.81$$

$\boxed{h_c = 0.273}$ Critical Depth

As

$$0.5589 > 0.273$$

"$\boxed{\text{So the Flow is Sub-Critical}}$"

Now Critical velocity

$$V_c = \sqrt{g h_c}$$

$$V_c = \sqrt{9.81 \times 0.448}$$

$\boxed{V_c = 2.096 m/sec}$ Critical Velocity

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QNo: 03 :-

Given Data

width 'B' = 200mm

As $\boxed{1m = 1000mm}$

= 0.2m

Length "L" = 800mm

= 0.8m

Sepecific Gravity = 0.89

Undisturb Velocity u = 5m/s

Kinematic Viscosity 'v' = $0.93 \times 10^{-4} m^2/s$

Required,

Friction Drag F_f = ?

Sol:

We Know that

$$R = \frac{Lu}{v}$$

$$R = \frac{0.8 \times 5}{0.93 \times 10^{-4}}$$ By putting values

$$R = 43010.75$$

As

$$R < 500,000$$

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Now

$$C_f = \frac{1.328}{\sqrt{R}} = \frac{1.328}{\sqrt{4301075}}$$

$$C_f = 6.4 \times 10^{-3}$$

$$C_f = 0.0064$$

Now

Friction Drag

$$F_f = C_f \rho \frac{v^2}{2} B.L$$

putting the values

$$F_f = 0.0064 \times 0.925 \times 1000 \times \frac{(5)^2}{2} \times 0.2 \times 0.8$$

$$\boxed{F_f = 11.84\ N}$$

THE END