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Q1:- A man throws two fair dice. What is the conditional probability that the sum of the two dice will be 7, given that

1):- The sum is even
2):- The sum is greater than 8
3):- The two dice have the same outcome.

Solution:

Let A = {The sum is 7}
B = {The sum is even}
C = {The sum is greater than 8}.
D = {The two dice lead the same outcome}

A = {(1,6) (2,5) (3,4) (4,3) (5,2) (6,1)}
B = {(1,3) (1,5) (2,2) (2,4) (2,6) (3,1) (3,3) (3,5) (4,2)...... 6,6)}
C = {~~(...)~~ (3,6) (4,5) (4,6) (5,4) (5,5) (5,6) (6,3) (6,4) (6,5) (6,6)
D = {(1,1) (2,2) (3,3) (4,4) (5,5) (6,6)}

$(A \cap B) = \{ \quad \}$

$(A \cap C) = \{ \quad \}$

$(A \cap D) = \phi$

$P(A) = \frac{6}{36}$, $P(B) = \frac{17}{36}$, $P(C) = \frac{10}{36}$, $P(D) = \frac{6}{36}$

$P(A \cap B) = \frac{6}{36}$, $P(A \cap C) = \frac{6}{36}$ & $P(A \cap D) = 0$

Hence

$$P(A/B) = P\left(\frac{A \cap B}{P(B)}\right) = \frac{6}{36} \times \frac{36}{18} = \frac{1}{3}$$

$$P(A/C) = P\left(\frac{A \cap C}{P(C)}\right) = \frac{6}{\cancel{36}} \times \frac{\cancel{36}}{10} = \frac{3}{5}$$

$$P(A/D) = \frac{P(A \cap D)}{P(D)} = 0 \times \frac{36}{6} = 0$$

Ans

Q2:- Show that in a single throw of dice, the probability of throwing more than 7 is equal to that of throwing less than 7, & hence find the probability of throwing exactly 7. State clearly what assumptions you are making.

Ans:- Solution.

Sum of 2 has 1 way 1,1.
Sum of 3 has 2 ways 1,2 & 2,1
Sum of 4 has 3 ways 1,3; 2,2; 3,1
5 has 4 ways
6 has 5 ways
8 has 5 ways (Symmetry).
9 has 4 ways.
10 has 3 ways
11 has 2 ways
12 has 1 way
~~12 has~~

Those are 15/36 for each side with a sum of 30/36.
That leaves a 6/36 = 1/6 probability for a sum of 7.

Ans

Qs:- A & B plays a game in which A's probability of winning is 2/3. In a series of 8 games, what is the probability that A will win

1):- Exactly 4 games
2):- Atleast 4 games.
3):- From 3 to 6 games.

Ans:- **Solution:**

We are given that

$$P = \frac{2}{3}, \quad n = 8.$$

$$q = 1 - P$$

$$q = 1 - \frac{2}{3}$$

$$q = \frac{1}{3}$$

Let "x" be the number of games won by A, then

1):- Exactly 4 games $\longrightarrow P(x=4)$.

$$= \binom{8}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^4$$

$$= \frac{1120}{6561}$$

$$= \boxed{0.1707}$$

2):- Atleast 4 games $\rightarrow P(x \geq 4)$

$$1 - P(x < 4)$$

$$= 1 - \sum_{x=0}^{3} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$$

$$= 1 - \left[\left(\frac{1}{3}\right)^{8} + 8\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^{7} + 28\left(\frac{2}{3}\right)^{2}\left(\frac{1}{3}\right)^{6} + 56\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5}\right]$$

$$= 1 - \frac{1}{6561}\left[1 + 16 + 112 + 448\right]$$

$$= 1 - \frac{577}{6561}$$

$$= \frac{6561 - 577}{6561}$$

$$= \frac{5984}{6561}$$

$$= \boxed{0.9121}$$

3):- 3 to 6 games $\longrightarrow P(3 \leq x \leq 6)$

$$\sum_{x=3}^{6} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$$

$$= \binom{8}{3}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^5 + \binom{8}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^4 + \binom{8}{5}\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^3$$

$$\binom{8}{6}\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^2$$

$$= \frac{8}{(3)^8}\left[56 + 140 + 224 + 224\right]$$

$$= \frac{8 \times 644}{6561} = \frac{5152}{6561} = \boxed{0.7852}$$

Answer

Q4:- Let $C_1, C_2, \ldots CMC_1, C_2, \ldots CM$ be a partition of the sample space SS, & AA & BB be two events. Suppose we know that

★ A & B are conditionally independent given C_i for all $i \in \{1, 2, \ldots M\}$

★ B is independent of all C_is

prove that A & B are independent.

Ans:- **Proof:**

Since the C_is from a partition of the sample space we can apply the law of total probability for $A \cap B$

$$P(A \cap B) = \sum_{i=1}^{m} P(A \cap B / C_i)\, P(C_i)$$

$$P(A \cap B) = \sum_{i=1}^{m} P(A/C_i)\, P(B/C_i)\, P(C_i)$$

$\because$ (A & B are conditionally independent)

$$P(A \cap B) = \sum_{i=1}^{m} P(A/C_i)\, P(B)\, P(C_i)$$

$\because$ (B is independent of all C_i

$$P(A \cap B) = P(B) \sum_{i=1}^{m} P(A/C_i)\, P(C_i)$$

$$P(A \cap B) = P(B)\, P(A)$$

law of total probability.

Hence A & B are independent.

Ans

Q5 Derive the binomial distribution & find its mean & variance.

Ans:- **Solution**

Binomial distribution.

$$P(x=x) = \binom{n}{x} p^x (1-p)^{n-x}.$$

for $x = 0, 1, 2, \ldots\ldots, n.$

$\mu = np$
$\sigma^2 = np(1-p)$

A binomial random variable can be thought of as the sum of n independent bernoulli random variables, each with mean P & variable $P(1-P$.

Let $u_1, \ldots\ldots u$ be independent bernoulli random variables.

$$E(u_i) = P \quad \& \quad var(u_i) = P(1-P).$$

$x = u_1 + \ldots\ldots u_n$
$var(x) = var(u_1) + \ldots\ldots var(u_n)$

The binomial theorm.

$$(a+b)^m = \sum_{y=0}^{m} \binom{m}{y} a^y b^{m-y}$$

$$E(x) = \sum_x x p(x)$$

$$\sum_{x=0}^{n} x \frac{n}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-1)-(x-1)!} \cdot p^{x-1} (1-p)^{(n-1)-(}$$

$$m = (n-1) \quad , y = (x-1)$$

$$= np \sum_{y=0}^{m} \frac{m}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= np \sum_{y=0}^{m} \binom{m}{y} p^y (1-p)^{m-y}.$$

$$E(x^2) = n(n-1)p^2 + np$$

$$Var(x) = E(x^2) - [E(x)]^2$$

$$= n(n-1)p^2 + np - (np)^2$$

$$Var(x) = np[(n-1)p + 1 - np]$$

This is variable of binomial distribution.

Ans

$$Var(x) = \mathcal{E}[(x-\mu)^2] \Rightarrow \sum_x (x-\mu)^2 \, P(x).$$

$$\mathcal{E}[(x-\mu)^2] = E(x^2) - [E(x)]^2$$

$$\mathcal{E}[x(x-1)] = \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} \cdot p^x (1-p)^{n-x}$$

$$\mathcal{E}[x(x-1)] = n(n-1)p^2 \sum_{x-2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{(n-2)-(x-2)}$$

By binomial thearm.

$$\mathcal{E}(X(x-1)) = n(n-1)p^2.$$

$$\mathcal{E}(x^2 - x) = n(n-1)p^2.$$

$$\mathcal{E}(x^2) - \mathcal{E}(x) = n(n-1)p^2.$$

Since

$\mathcal{E}(x) = np$ .. Which is mean of binomial.

$$\mathcal{E}(x^2) = n(n-1)p^2.$$

Q6:- Differentiate b/w Binominal frequency distribution & binominal distribution with the help of formulas?

Ans:- **Binomial distribution:**

A binomial distribution can be though of as simply the probability of a success or failure outcome in an experiment or survey that is repeated multiple times

$$p(x-x)\ f(x) = {}^{n}C_{x}\ p^{x}\ q^{n-x}.$$

Binomial frequency distribution:

If the binomial probability distribution is multiplied by N the number of experiment or sets the delection distribution is known as binomial frequency distribution

$$N\binom{n}{x}\left(p^{x}\ q^{n-x}\right).$$

Q 7:-

Ans:-7:- Solution.

Measure	Data Set a	B	C	D
Co-efficient of variation	$CV = \frac{3}{45} \times 100$ $CV = 6.7$	$CV = \frac{11}{60} \times 100$ $CV = 18.3$	$CV = \frac{5}{50} \times 100$ $CV = 10$	$CV = \frac{15}{25} \times 100$ $CV = 60$

Ans